Measuring Mobility

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Canazei Winter School, 8 January 2019

Outline

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Introduction

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Why interest in measurement?

- improving data on intra- and inter-generational mobility
- convincing evidence needs appropriate measurement tools

Approaches to mobility

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- Variety of temporal context:
  1. inter / intra-generational
  2. long term / volatility
- Variety of analytical context:
  - in relation to a specific dynamic model
  - in relation to welfare issues
  - as an abstract distributional concept

A cause for concern?

Source: Corak (2013)

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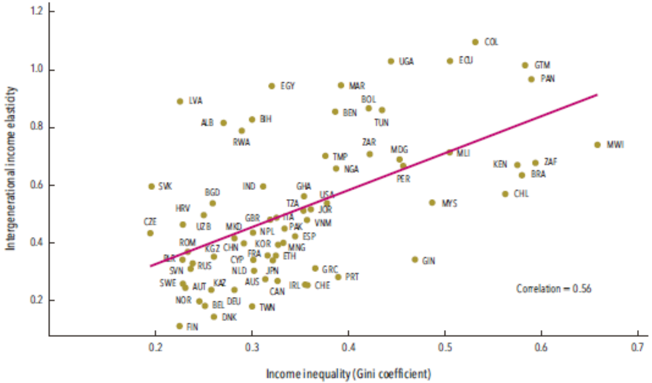
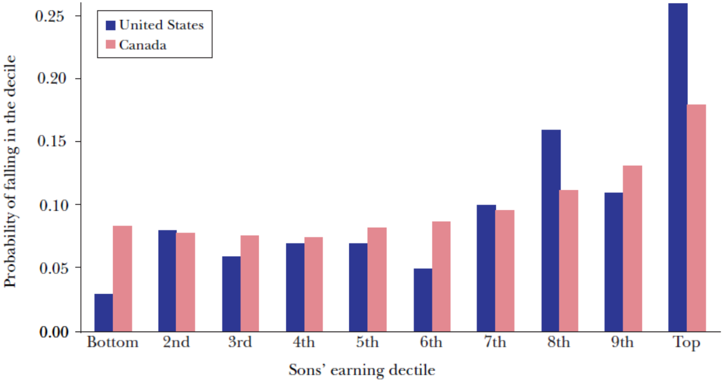


Source: Narayan et al. (2018)

Prospects for the top 10%



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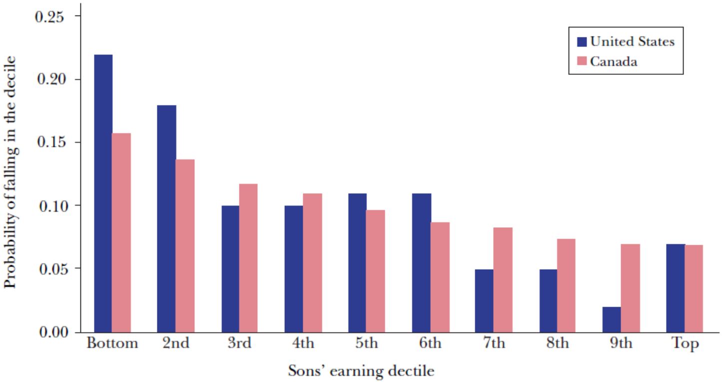

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Ingredient 1:

- Assume discrete time
- Focus on two periods: now (0) and the future (1)

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Status: classes

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First step in an approach to “status”:

- define a finite set of K classes
- $n_k \geq 0$: # in class k, $k = 1, 2, \ldots, K$
- exclusive and exhaustive
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- mobility given by $\left(x_{k^0(1)}, x_{k^0(2)}, \ldots, x_{k^0(n)}\right)$ and $\left(x_{k^1(1)}, x_{k^1(2)}, \ldots, x_{k^1(n)}\right)$

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- cardinal attribute: just aggregate the xs?
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- for example use $N^0(x_k) := \sum_{h=1}^{k} n_h^0,\ k = 1, \dots, K$
- number in or below class k using distribution at t^0

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- Revaluing the income classes: $N^1(x_k) := \sum_{h=1}^{k} n_h^1,\ k = 1, \ldots, K$

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- *static.* $z_i = \left(N^0\left(x_{k^0(i)}\right), N^0\left(x_{k^1(i)}\right)\right)$
  - cumulative numbers in class “value” the class
- *dynamic.* $z_i = \left(N^0\left(x_{k^0(i)}\right), N^1\left(x_{k^1(i)}\right)\right)$

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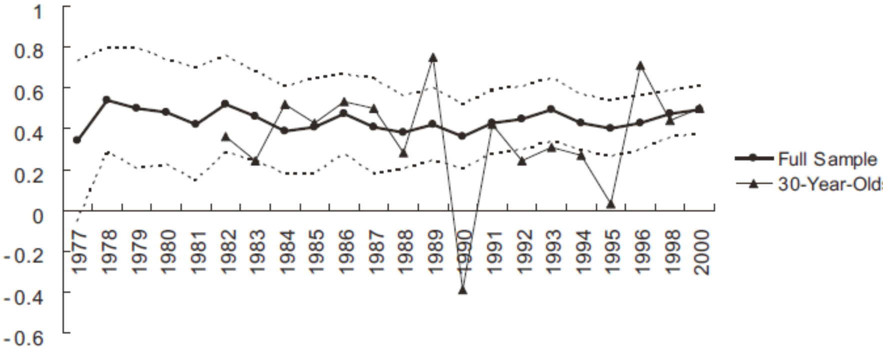
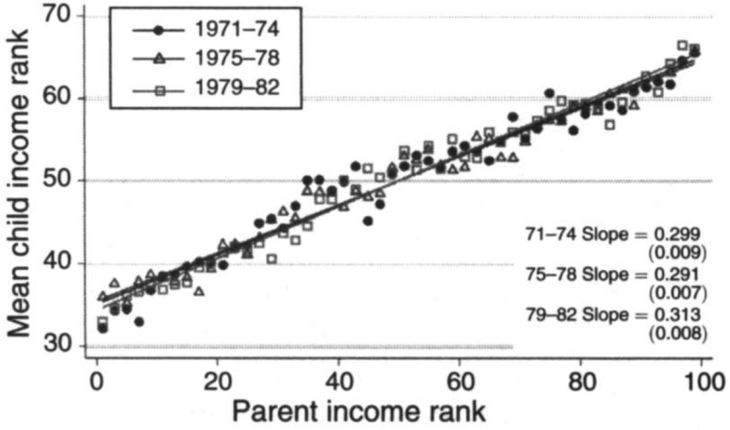
Exchange and structural mobility: (Van Kerm 2004, Tsui 2009)

Example: US Income Mobility



Intergenerational income elasticities
Source: Lee and Solon (2009).

Example: US Rank Mobility



Source: Chetty et al. (2014); see also Auten et al. (2013a, 2013b) , Chetty et al. (2014)

The approach

- Appropriate tools?
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- This presentation
  - develops ideas in Cowell and Flachaire (2017, 2018)
  - implement conventions on meaning of mobility comparisons

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- Characterise an ordering
  - formulate principles as axioms
  - develop characterisation results

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  1. mobility and unbalanced growth: (Bourguignon 2011)
  2. interpretations of "exchange mobility" (Jäntti and Jenkins 2015; Kessler and Greenberg 1981, McClendon 1977)
- Essential for mobility measurement?
  - ensures a minimum-mobility property
  - situation with some movement registers higher mobility than a situation without movement

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- Several aspects of decomposability seem to be attractive
  - decomposition by population characteristics
  - decomposition by region
- Special for mobility:
  - decompose by direction
  - mobility in terms of upward and downward movements (Bárcena and Cantó 2018)

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  - translating the distributions by the same given amount?
- Under such circumstances each pair of distributions should be ranked the same

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  - let income be y
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1. elasticity coefficient
   - linear regression of status-1 on status-0
   - $x_{1i} = \alpha + \beta x_{0i} + \varepsilon_i$
   - $1 - \hat{\beta}$ as a measure of mobility?

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2. correlation coefficient
   - use Pearson correlation coefficient $\hat{\rho}$
   - $1 - \hat{\rho}$ as a measure of mobility?

Statistical measures: elasticity coefficient

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Statistical measures: elasticity coefficient

- A high value of $1-\beta$ evidence of significant mobility?
- Low value does not necessarily imply low mobility
  - can have $1-\hat{\beta}=0$ where there is indeed mobility
  - since $\hat{\beta}=\frac{cov(\mathbf{x}_0,\mathbf{x}_1)}{var(\mathbf{x}_0)}$: $1-\hat{\beta}=0 \quad \Leftrightarrow \quad cov(\mathbf{x}_0,\mathbf{x}_1)=var(\mathbf{x}_0)$.

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- A difficulty:
  - take $\mathbf{x}_0=(x_{01},x_{01}+k,x_{01}+2k)$, $\mathbf{x}_1=(x_{11},x_{12},x_{11}+2k)$
  - we have $1-\hat{\beta}=0$, $\forall x_{01},x_{11},x_{12}$
- Example:
  - $\mathbf{x}_0=(1,2,3)$
  - $\mathbf{x}_1\in\{(2,0,4),(2,1,4),(2,1760,4),(2100,1,2102),\dots\}$
  - zero mobility *in all cases*?

Statistical measures: correlation coefficient

- Both scale and translation independent:
  - if $x_1 = ax_0 + b$, then $\hat{\rho} = 1 \quad \Leftrightarrow \quad 1 - \hat{\rho} = 0$
  - so $\mathbf{x}_0 = (1, 2, 3)$ and $\mathbf{x}_1 = (0, 2, 4)$ imply $x_1 = 2x_0 - 2$; $1 - \hat{\rho} = 0$
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- Measure can behave strangely:
  - take equidistant status
  - $\mathbf{x}_0 = (x_{01}, x_{01} + k, x_{01} + 2k)$, $\mathbf{x}_1 = (x_{11}, x_{12}, x_{11})$
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  - Get $1 - \hat{\rho} = 1$ and $1 - \hat{\beta} = 1$, $\forall x_{01}, x_{11}, x_{12}$
- Example
  - $\mathbf{x}_0 = (1,2,3)$
  - $\mathbf{x}_1 \in$ $\{(3,2,3),(3,0,3),(3,100,3),(1,2,1),(10,1,10),(2,1,2),\ldots\}$
  - in all cases $1 - \hat{\rho} = 1$ and $1 - \hat{\beta} = 1$

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Other measures:

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  - $FO_2 = \frac{1}{n} \sum_{i=1} \mid \log y_{1i} - \log y_{0i} \mid$

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- Fields and Ok (1999b) measure based on log-income differences:
  - $FO_2 = \frac{1}{n}\sum_{i=1} \mid \log y_{1i} - \log y_{0i} \mid$
- Shorrocks (1978) measures related to inequality:
  - $S_I = 1 - \frac{I(y_0+y_1)}{\frac{\mu_{y_0}}{\mu_{y_0+y_1}} I(y_0) + \frac{\mu_{y_1}}{\mu_{y_0+y_1}} I(y_1)}$
  - where $I(.)$ is a predefined inequality measure

Comparative performance: China

- Intragenerational income mobility in China
- Did it rise or fall around the millennium?
- Example based on Chen and Cowell (2017)

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	1989-2000	*2000-2011*
$1-\beta$	0.7564	0.6928
$1-\rho$	0.7947	0.7257
FO_1	6506.5	16979.62
FO_2	0.9619	1.1726

Outline

Axiomatic approach

Axiomatic approach

- Basic concepts
  - status
    - individual observation
    - derived from distribution
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- Key axioms:
  - correspond to main principles
  - movement, decomposition consistency
  - do this in two stages

Monotonicity Axiom 1

[Monotonicity] If $\mathbf{z}, \mathbf{z}' \in Z^n$ differ only in their ith component and $u_i' = u_i$ then, if $v_i > v_i' \geq u_i$, or if $v_i < v_i' \leq u_i$, $\mathbf{z} \succ \mathbf{z}'$

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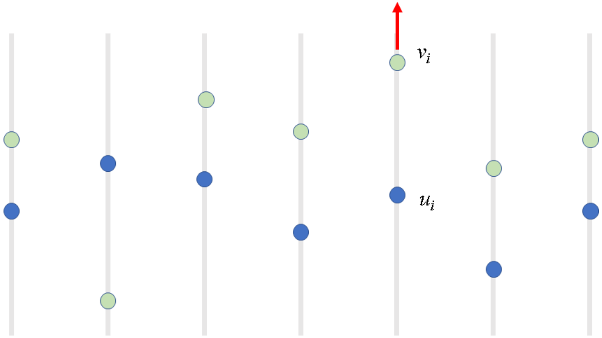

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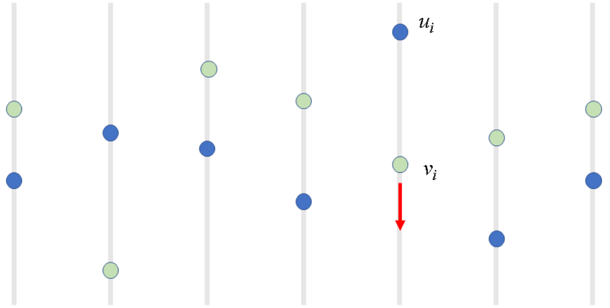

Monotonicity Axiom 2

[Monotonicity-2] If $\mathbf{z}, \mathbf{z}' \in Z^n$ differ only in their ith and jth components and $u_i' = u_i$, $u_j' = u_j$, $v_i' - v_i = v_j - v_j'$ then, if $v_i > v_i' \geq u_i$ and if $v_j < v_j' \leq u_j$, $\mathbf{z} \succ \mathbf{z}'$

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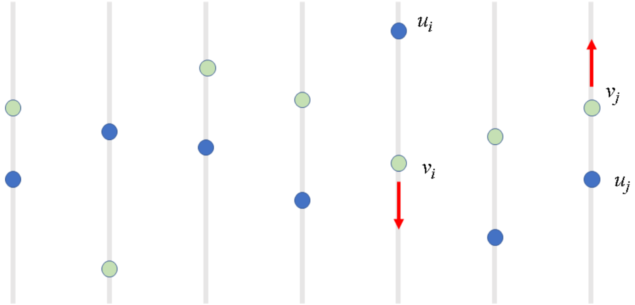

Independence Axiom

[Independence] Let $\mathbf{z}(\zeta, i)$ be profile formed by replacing the ith component of $\mathbf{z}$ by the history $\zeta \in Z$ and let $\hat{Z}_i := \left[u_{(i-1)}, u_{(i+1)}\right] \times \left[v_{(i-1)}, v_{(i+1)}\right]$ For $\mathbf{z}, \mathbf{z}' \in Z^n$ suppose that $\mathbf{z} \sim \mathbf{z}'$ and $z_i = z_i'$ for some $i \in 2, \ldots, n-1$: then $\mathbf{z}(\zeta, i) \sim \mathbf{z}'(\zeta, i)$ for all $\zeta \in \hat{Z}_i$

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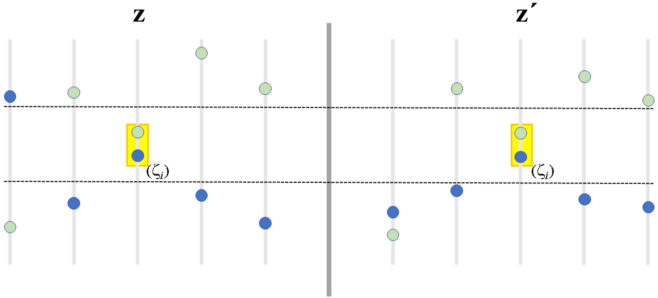

Scale-irrelevance Axiom

[Status scale irrelevance] For any $\mathbf{z}, \mathbf{z}' \in Z^n$ such that $\mathbf{z} \sim \mathbf{z}'$, $\mathbf{z} \times (\lambda_0, \lambda_1) \sim \mathbf{z}' \times (\lambda_0, \lambda_1)$, for all $\lambda_0, \lambda_1 > 0$

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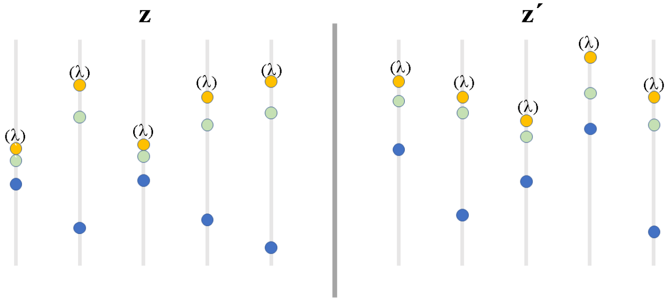

Outline

Results: stage 1

- ***[Continuity]*** $\succeq$ is continuous on Z^n
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- ***[Local immobility]*** Let $\mathbf{z}, \mathbf{z}' \in Z^n$ where for some i, $u_i = v_i$, $v_i' = u_i'$ and, for all $j \neq i$, $u_j' = u_j$, $v_j' = v_j$. Then $\mathbf{z} \sim \mathbf{z}'$

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<u>**Theorem 1:**</u> given these axioms then $\forall \mathbf{z} \in Z^n$ the mobility ordering $\succeq$ is an increasing monotonic transform of $\sum_{i=1}^{n} \phi_i(z_i)$

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Theorem 2: Given the above Axioms $\succeq$ is representable by the form in Theorem 1 where ϕ_i is given by

- either (1) $\phi_i(u,v) = c_i \left[u^\alpha v^{1-\alpha} - \alpha u - [1-\alpha] v\right]$
- or (2) $\phi_i(u,v) = a_i [b_i v - u]$,where $\alpha, a_i, b_i, c_i \in \mathbb{R}$

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  - $\psi\left(\frac{1}{n}\sum_{i=1}^{n}u_i^{\alpha}v_i^{1-\alpha}-\theta(\mu_u,\mu_v),\mu_u,\mu_v\right)$

Mobility indices: Class 1

Corollary

$$M_\alpha := \frac{1}{\alpha[\alpha-1]n} \sum_{i=1}^{n} \left[\left[\frac{u_i}{\mu_u} \right]^\alpha \left[\frac{v_i}{\mu_v} \right]^{1-\alpha} - 1 \right]$$

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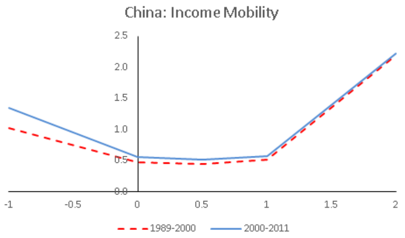

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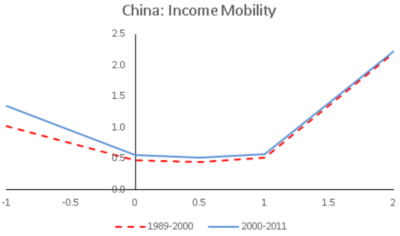

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  - overall mobility index $\sum_{i=1}^{n} a_i d_{(i)}$
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- **Special Case (b): linear ϕ**
  - weights are: $a_i = \frac{i}{n} - p - \frac{1}{2n}$
  - so $\Gamma_1 := \frac{1}{n}\sum_{i=1}^{n} \frac{i}{n} d_{(i)} - \left[p + \frac{1}{2n}\right] \mu_d$,
  - $\Gamma_1 = {}^1\!/_2 G + \mu_d \left[\frac{1}{2} - p\right]$

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  - aggregation over mean changes of groups
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- Partition population upward U and downward D groups:
  - $M_\alpha = w^{\mathsf{U}} M_\alpha^{\mathsf{U}} + w^{\mathsf{D}} M_\alpha^{\mathsf{D}} + M_\alpha^{\mathsf{btw}}$

Decomposition: Class 1 (Example)

- Key feature in China data: Rural/Urban breakdown

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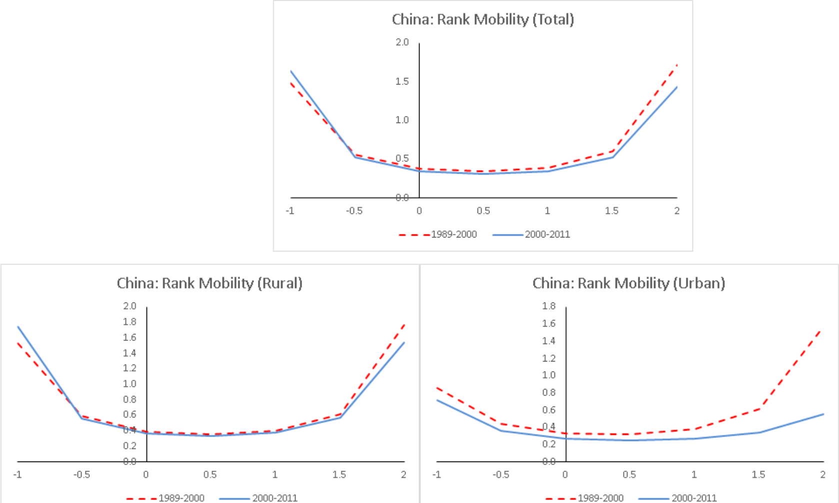

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    - $\Gamma_0 = -pd^{\mathsf{D}} + [1-p]\, d^{\mathsf{U}}$

Decomposition: Class 2

- Exact decomposition by subgroups not possible for arbitrary partition
- But U/D decompositions work
- FO indices:
  - $\Gamma_0 = -pd^{\mathsf{D}} + [1-p]\, d^{\mathsf{U}}$
- For general case $a_i = \phi\left(\frac{i}{n} - p - \frac{1}{2n}\right)$
  - $\Gamma_\gamma = p^{\gamma+1}\Gamma_\gamma^{\mathsf{D}} + [1-p]^{\gamma+1}\Gamma_\gamma^{\mathsf{U}}$
  - $\phi(x) = x^\gamma$

Class 2: individual weights

- Mobility index: $\sum_{i=1}^{n} a_i d_{(i)}$. Plot a_i against i

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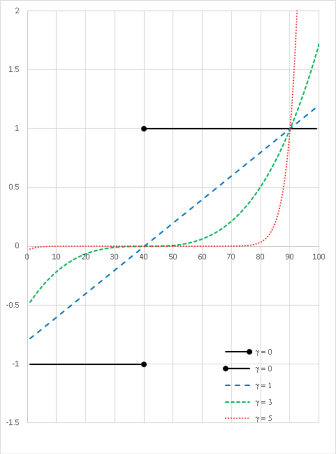
- Mobility index: $\sum_{i=1}^{n} a_i d_{(i)}$. Plot a_i against i



Outline

Summary

- The approach:
  - separate “status” from “aggregation” issues
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Summary

- The approach:
  - separate "status" from "aggregation" issues
  - focus on meaning of mobility comparisons.
  - characterise "suitable" measures
- The results
  - two broad classes of mobility indices
  - each class satisfies the minimal set of requirements for mobility comparisons
  - each of these classes has a natural interpretation in terms of distributional analysis

Outline

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