Inequality of opportunity:
Concepts and Measurement

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The first part of this lecture is based on my chapter with Vito Peragine, "Individual Responsibility and Equality of Opportunity" (Ch. 25) in Adler and Fleurbaey (eds.), 2016, *Oxford Handbook of Well-Being and Public Policy.* It also draws on insights and inputs generously provided by Paolo Brunori. But neither of them is to blame for my errors!

Outline

1. **Equality of opportunity: Motivation and background**
2. Economic models of equality of opportunity
3. Measuring inequality of opportunity
4. Empirical applications
    i. ‘First generation’ between-types approach
    ii. ‘Second-generation’ between-types approach
5. Concluding remarks

1a. Politics and policy

"We know that equality of individual ability has never existed and never will, but we do insist that equality of opportunity still must be sought"

(Franklin D. Roosevelt, second inaugural address, 20 January 1937)

"The rise in inequality in the United States over the last three decades has reached the point that inequality in incomes is causing an unhealthy division in opportunities, and is a threat to our economic growth"

(Alan Krueger, Center for American Progress, 12 January 2012)

If these concepts matter for policy, can they be rigorously defined and measured?

1b. Normative arguments

Political philosophers and economists have argued that outcomes alone are not a sufficient informational basis for the assessment of social justice

- John Rawls (1971): *A Theory of Justice* (Harvard University Press)
- Amartya Sen (1980): "Equality of what?" in McMurrin (ed.), *The Tanner Lectures on Human Values*
- Ronald Dworkin (1981): "What is Equality? Part 1: Equality of Welfare; Part 2: Equality of Resources", *Philos. Public Affairs*, **10**, pp.185-246; 283-345.
- Richard Arneson (1989): "Equality of Opportunity for Welfare", *Philosophical Studies*, **56**, pp.77-93.
- Gerald Cohen (1989): "On the Currency of Egalitarian Justice", *Ethics*, **99**, pp.906-944.

This approach "… performs for egalitarianism the considerable service of incorporating within it the most powerful idea in the arsenal of the anti-egalitarian right: the idea of choice and responsibility" (Cohen, 1989, p.993)

1c. Empirical evidence on preferences

1. It is now well-established that individuals value ‘fairness’, in the specific sense that they are prepared to give up private monetary gains to achieve what they perceive as a just allocation.
   – Fehr and Gachter (2000); Fehr and Fischbacher (2003); Henrich et al. (2004)

2. There is also evidence that most people are neither strict egalitarians or libertarians: in forming their views of just dessert, they tend to hold people responsible for effort, but not for purely exogenous shocks.
   – E.g. Cappelen, Sorensen and Tungodden (2010) on Norwegian business students and alumni

Preference groups	Responsibility sets	Frequency in sample
Strict egalitarians	$\mathcal{R}^{SE} = \emptyset$	0.18
Choice egalitarians	$\mathcal{R}^{CE} = \{q\}$	0.05
Meritocrats	$\mathcal{R}^{M} = \{q, a\}$	0.47
Libertarians	$\mathcal{R}^{L} = \{q, a, p\}$	0.30

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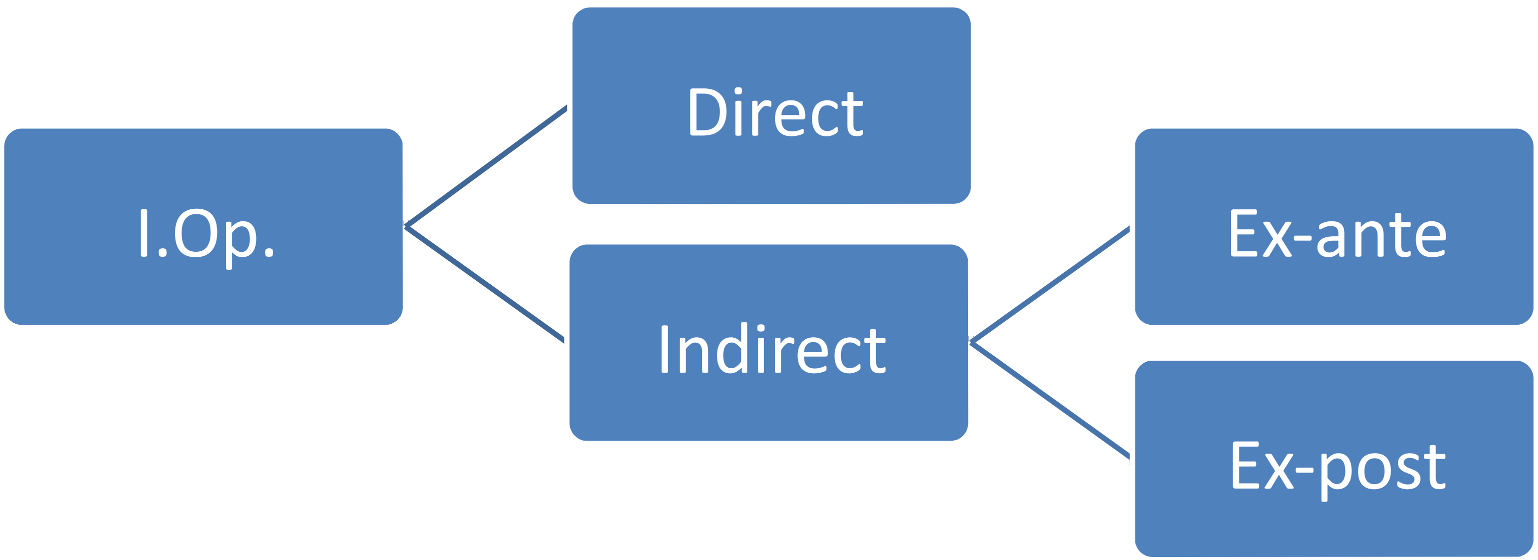
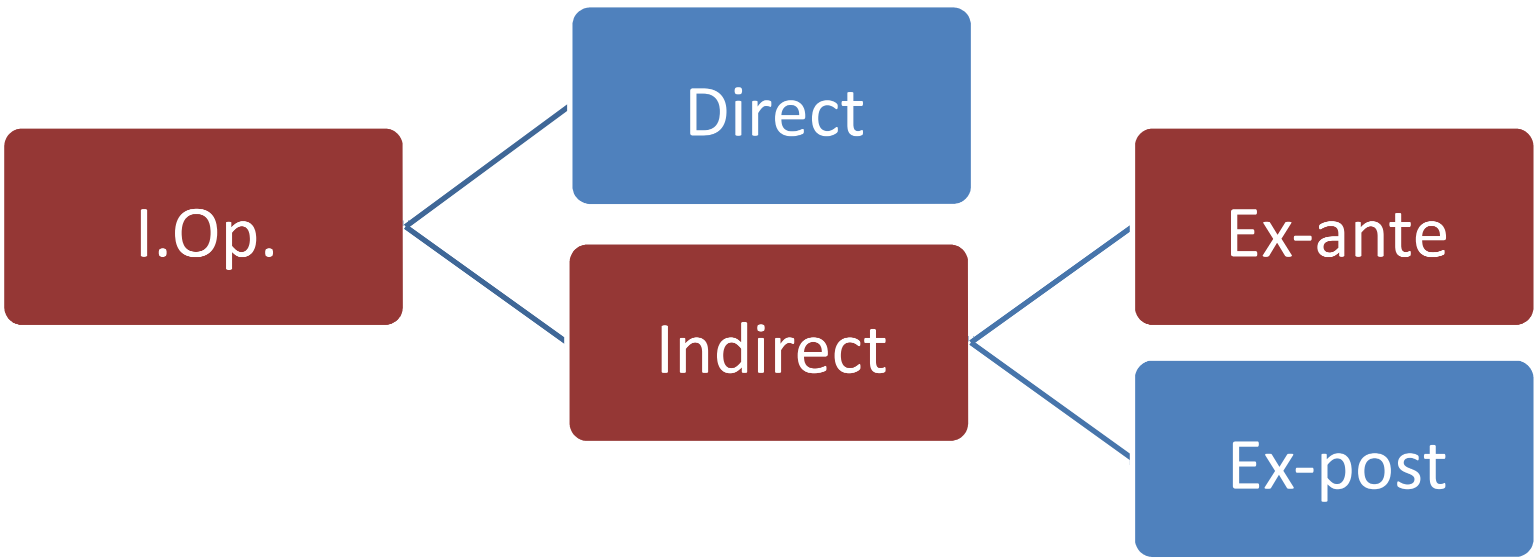
2. Economic models



2. Economic models



2. Economic models

- **Direct approaches**
  - Sought to model opportunity sets explicitly
  - Ranking / ordering opportunity sets
    - Pattanaik and Xu (1990): the cardinality ordering
    - Weymark (2003): the set inclusion ordering
    - Barberà et al. (2004): a survey
  - Ranking / ordering profiles of opportunity sets
    - Kranich (1996) – cardinality difference relation
    - Weymark (2003) – generalized Gini orderings
    - Savaglio and Vannucci (2007)

2. Economic models

- **Indirect approaches**
  - Build primarily on the Arneson / Cohen "control view" of equality of opportunity.
  - Pioneering economists who adopted and built on these ideas include:
    - John Roemer (1993, 1998)
    - Dirk van de Gaer (1993)
    - Marc Fleurbaey (1994, 2008)
  - In essence, equality of opportunity is defined as a situation in which the outcome of interest is distributed independently of (predetermined) circumstances for which the individual ought not to be held responsible:

$$F(x|C) = F(x)$$

  - This is often expressed in terms of two central principles:
    - Principle of compensation: outcome differences due to factors beyond an individual's responsibility ("circumstances") are unfair, and should be compensated.
    - Principle of reward: outcome differences reflecting differential reward to individual responsibility and effort are ethically legitimate, and should be preserved.

2. Economic models

- A simple "canonical" model
- Let each and every individual be fully characterized by the triple *(x, C, e)*, and

$$C \in \Omega$$

$$e \in \Theta$$

$$x = g(C, e)$$

$$\boldsymbol{g}: \boldsymbol{\Omega} \times \boldsymbol{\Theta} \Rightarrow \mathbb{R}$$

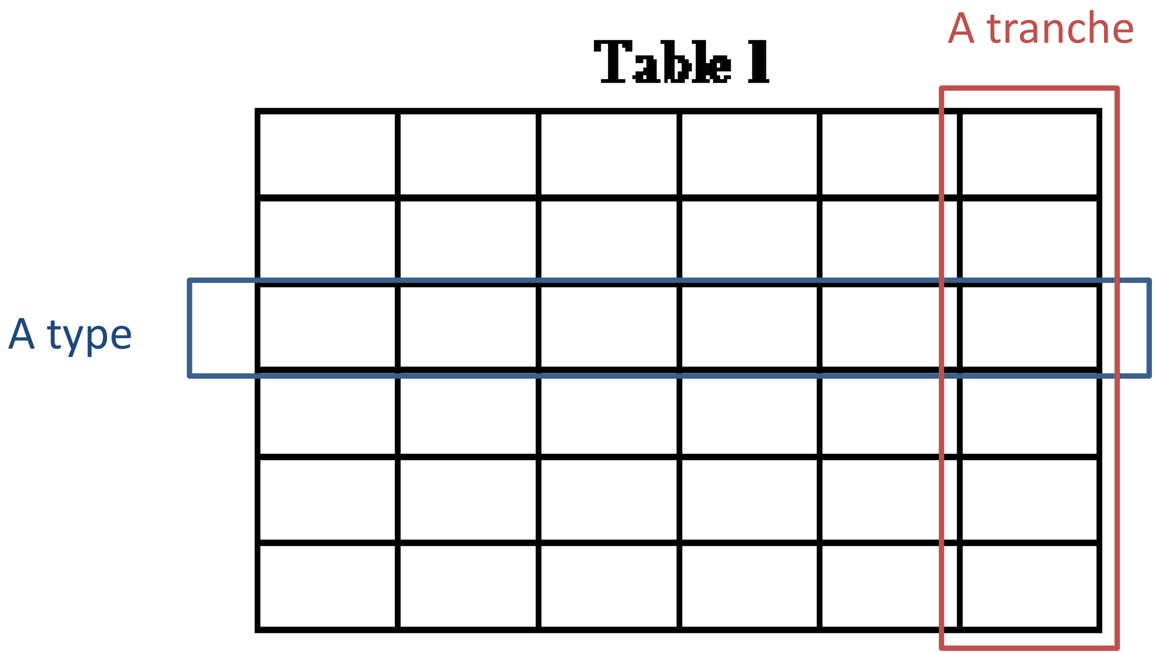
2. Economic models

- Let all elements of the vector **C, as well as *e*, be discrete.**
- Let $x_{ij} = g(C_i, e_j)$
- $x_{ij}(C_i, e_j) \le x_{ik}(C_i, e_k), e_j < e_k, \forall i, j$
- Let a *type* consist of all individuals with identical circumstances
- Let a *tranche* consist of all individuals with identical effort levels
- Let there be *n* types and *m* tranches
- Then the population can be represented by the *n x m* matrix $[X_{ij}]$ below.
- To $[X_{ij}]$, let there be associated another ***n x m* matrix $[P_{ij}]$, whose elements** p_{ij} denote the proportion of the total population with circumstances C_i and effort level e_j.

2. Economic models

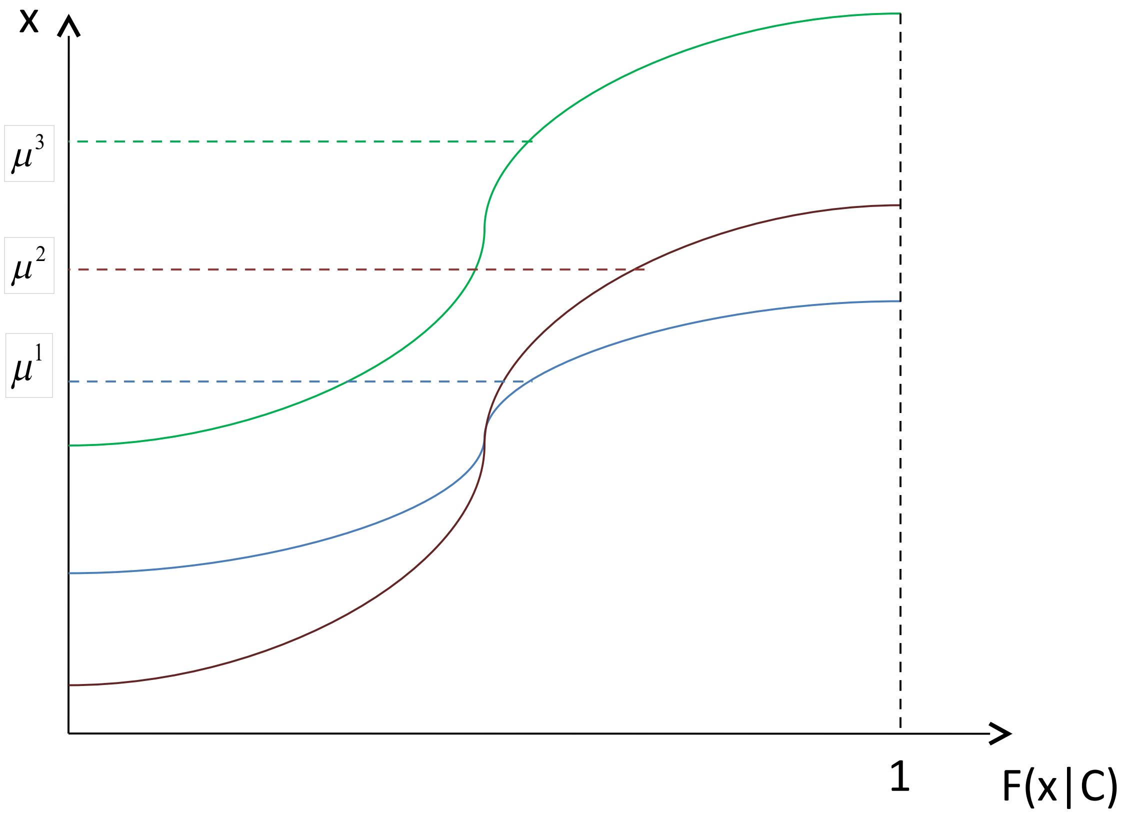
Table 1

2. Economic models



2. Economic models

When effort is continuous, n=3



2. Economic models

- Two central principles:
  - Principle of compensation: outcome differences due to factors beyond an individual's responsibility (circumstances) are unfair, and should be compensated
    - Ex-ante (van de Gaer, 1993): Eliminate inequality across types before effort is realized, by equating values of opportunity sets (defined in terms of the distribution of x conditional on C).
    - Ex-post (Roemer, 1993): Eliminate inequality across types after effort is realized, by eliminating inequality among people exerting the same degree of effort. (i.e. eliminate inequality within tranches).
  - Principle of reward: outcome differences reflecting differential reward to individual responsibility and effort are ethically legitimate, and should be preserved.
    - Liberal reward
    - Utilitarian reward
    - Etc.

2. Economic models

- Key results (Fleurbaey and Peragine, 2013):

  1. In general, the ex-ante and ex-post compensation principles are inconsistent
  2. In general, the ex-post compensation principle is inconsistent with reward principles
  3. The ex-ante compensation principle and the reward principles are consistent.

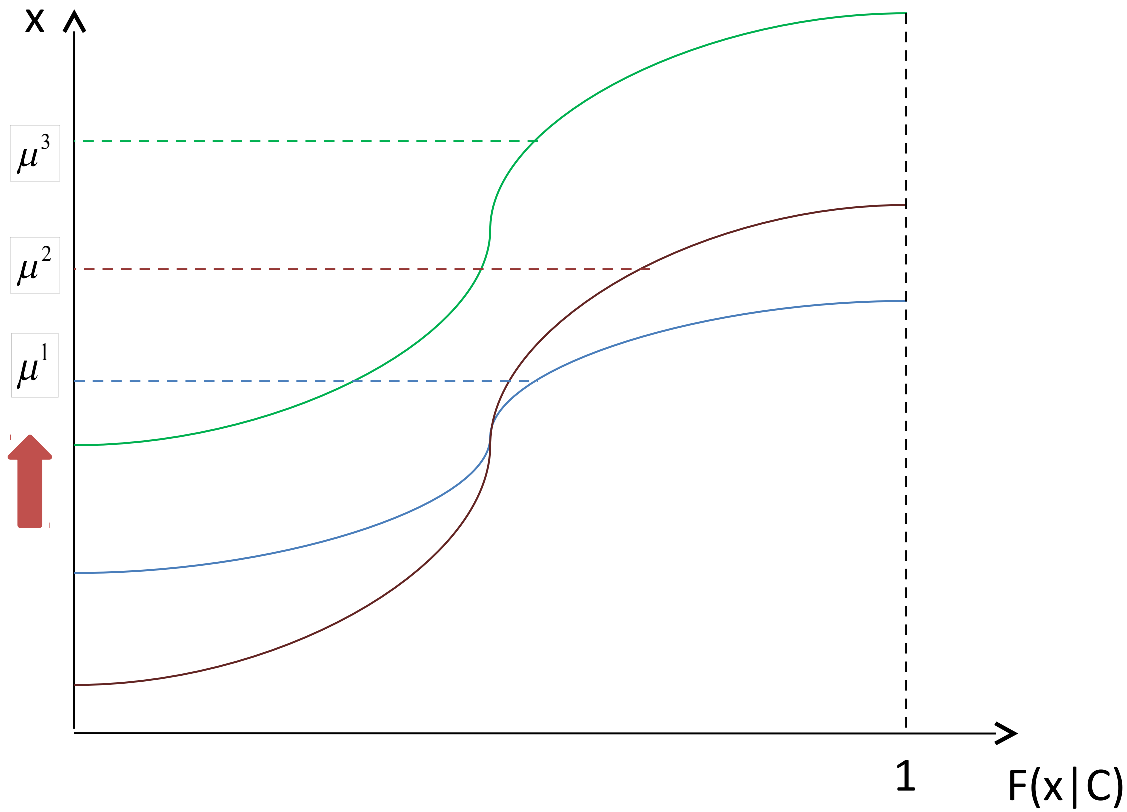
- Variations of this framework have been used to propose:

  i. Social orderings and allocation rules
     - When feasible resource transfers are introduced in the model

  ii. Measures of inequality of opportunity

2. Economic models

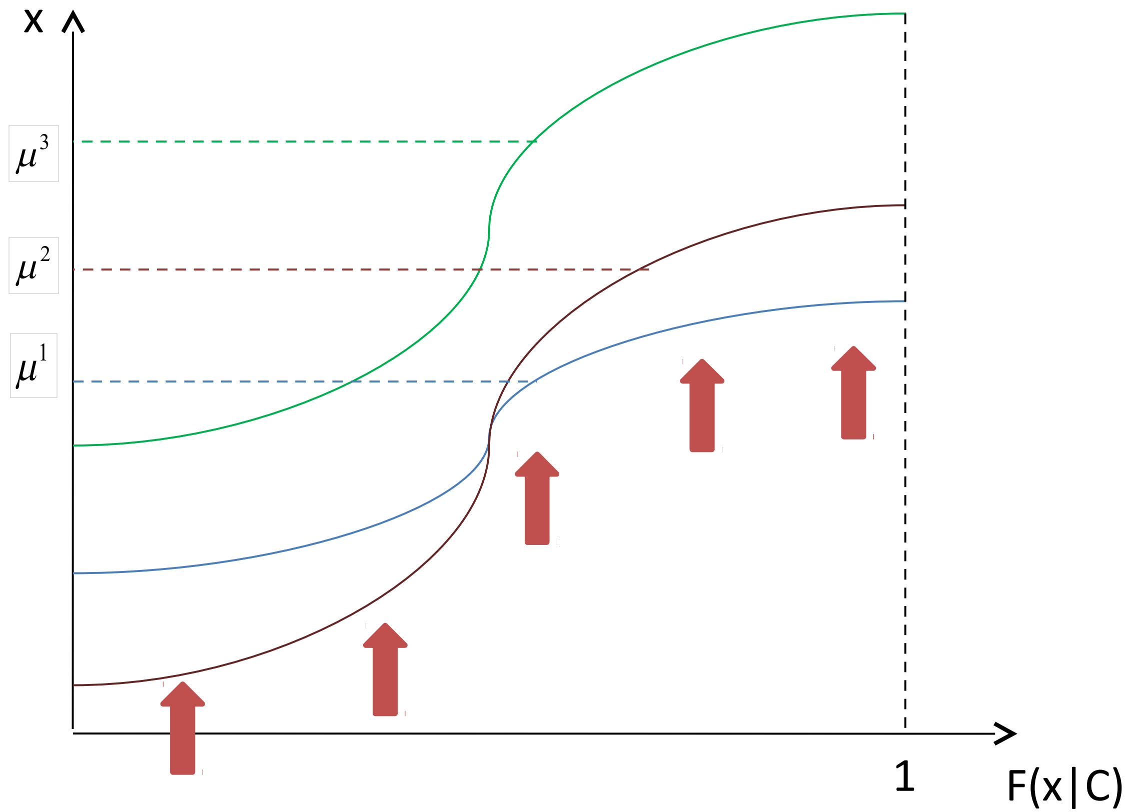
Allocation rules: (i) van de Gaer’s “min of means” (satisfies ex-ante compensation and reward)



$$\min_i (\mu_1, \ldots, \mu_n)$$

2. Economic models

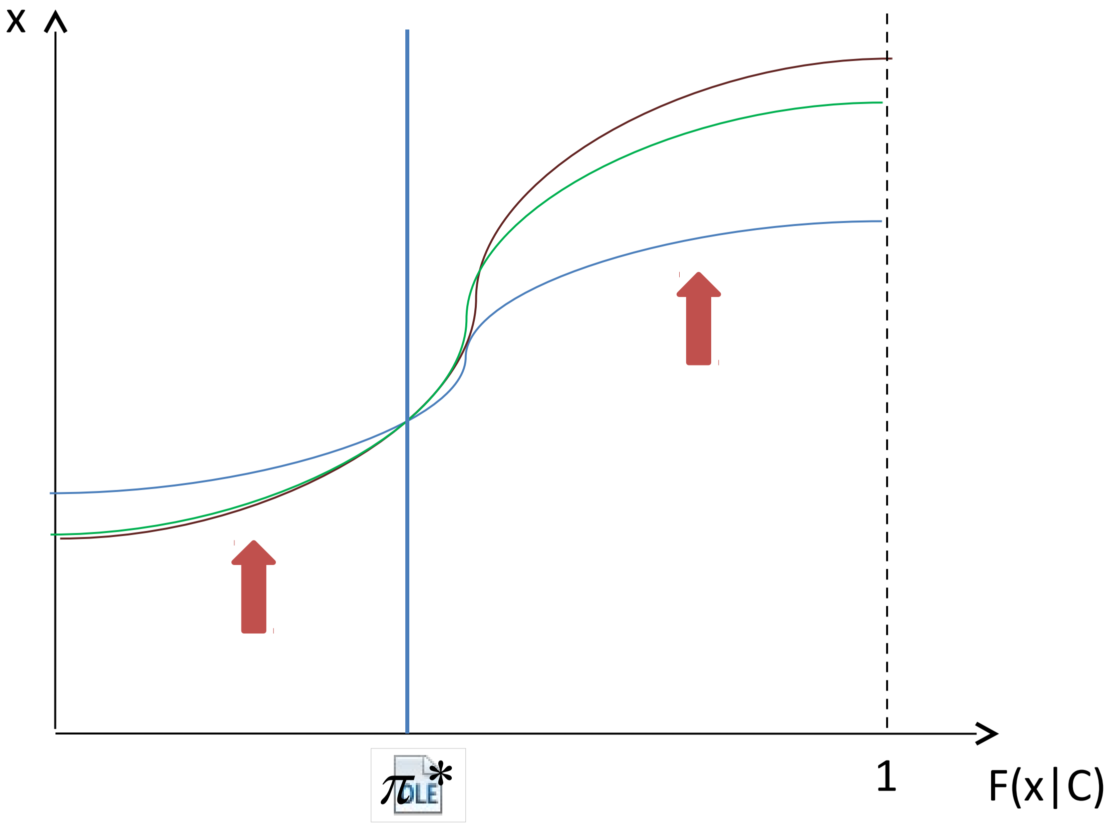
Allocation rules: (ii) Roemer's "mean of mins" (satisfies ex-post compensation)



$$\left(\frac{1}{m}\sum_{j=1}^{m} \min_i (x_{1j}, \ldots, x_{nj})\right)$$

2. Economic models

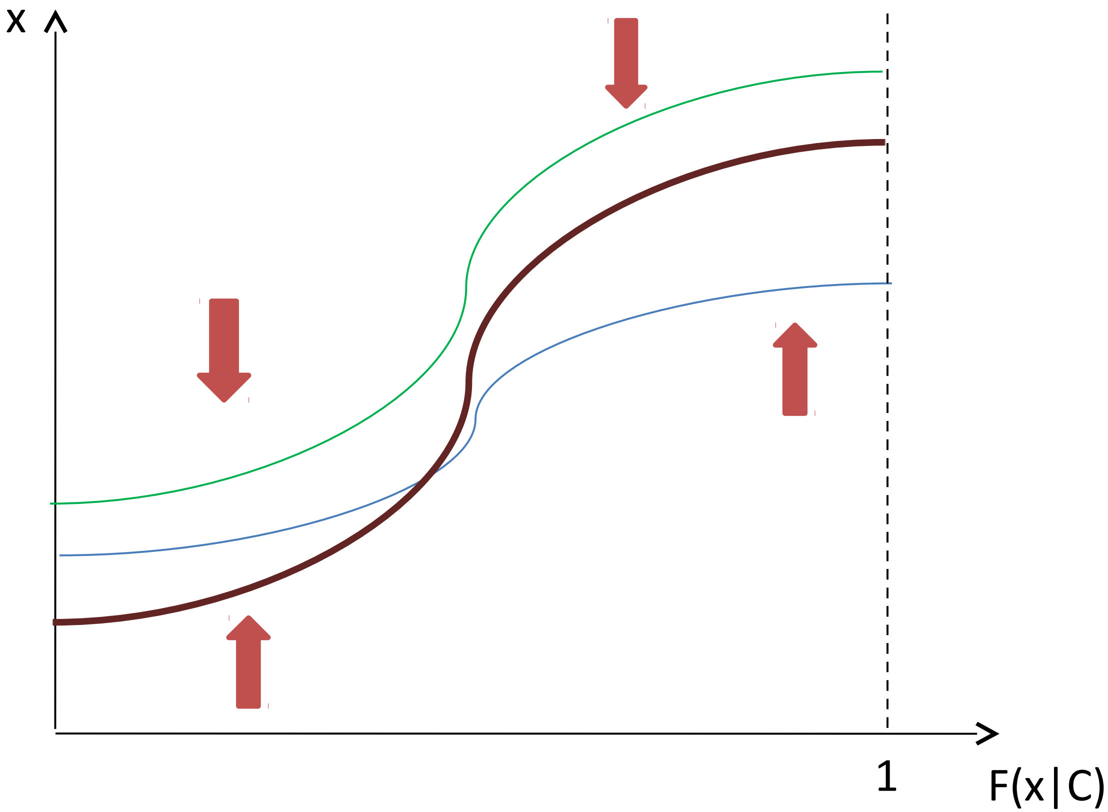
Allocation rules: (iii) Conditional equality (seeks a compromise between ex-post compensation – satisfied only for a reference effort level - and reward.



See Fleurbaey (2008).

2. Economic models

Allocation rules: (iv) Egalitarian equivalence (seeks a compromise between ex-post compensation and reward – satisfied only for a reference type).



See Pazner and Schmeidler (1978), and Fleurbaey (2008).

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3. Measuring inequality of opportunity

In essence, the measurement of inequality of opportunity can be thought of as a two-step procedure:

1. First, the actual distribution $[X_{ij}]$ is transformed into a counterfactual distribution $[\tilde{X}_{ij}]$ that reflects *only and fully* the unfair inequality in $[X_{ij}]$, while all the fair inequality is removed.

2. In the second step, a suitable measure of inequality is applied to $[\tilde{X}_{ij}]$.

3. Measuring inequality of opportunity

Between types *($\tilde{X}_{BT}$): For all $j \in \{1,\dots,m\}$ and for all $i \in \{1,\dots,n\}$, $\tilde{x}_{ij} = \mu_i$.*

Table 2: Between-types inequality *(n=m=3)*

	μ_1	μ_1	μ_1
	μ_2	μ_2	μ_2
	μ_3	μ_3	μ_3

Draws on the min of means approach. Satisfies ex-ante compensation and reward.

3. Measuring inequality of opportunity

Within tranches ($\tilde{X}_{WTR}$)*: For all* $j \in \{1,\ldots,m\}$ *and for all* $i \in \{1,\ldots,n\}$, $\tilde{x}_{i,j} = g(c_i, e_j)/V_j$.

Table 4: Within tranches inequality *(n=m=3)*

	V_1	V_2	V_3
	V_1	V_2	V_3
	V_1	V_2	V_3

Draws on the mean of mins approach. Satisfies ex-post compensation everywhere, but not the reward principle.

3. Measuring inequality of opportunity

Direct unfairness ($\tilde{X}_{DU}$): *take* $\tilde{e}$ *as the reference effort. Then* $\tilde{x}_{ij} = g(c_i, \tilde{e})$, $\forall i \in \{1, ..., n\}$ *and* $\forall j \in \{1, ..., m\}$.

Table 3: Direct unfairness (with $\tilde{e}$=1 and n=m=3)

Draws on the conditional equality compromise. Satisfies ex-ante compensation and reward; and ex-post compensation only for Tranch 1.

3. Measuring inequality of opportunity

Fairness gap *($\tilde{X}_{FG}$): take $\tilde{c}$ as the reference circumstance. Then let* $\tilde{x}_{i,j} = g(c_i, e_j) / g(\tilde{c}, e_j)$,

$\forall i \in \{1,\ldots,n\}$ *and* $\forall j \in \{1,\ldots,m\}$.

Table 5: Fairness gap (with $\tilde{c}$=1 and n=m=3)

Draws on the egalitarian equivalence compromise. Satisfies ex-post compensation everywhere, but liberal reward only for Type 1.

3. Measuring inequality of opportunity

A summary of the four indirect approaches to measuring I. Op.

Table 6: Welfare criteria, allocation rules and inequality measures

Approaches		
Ex ante		

3. Measuring inequality of opportunity

- **Partial orderings** can be sought instead of complete orderings.

i. <u>To define and test for E.Op.</u> (Lefranc, Pistolesi, Trannoy, RIW, 2008)

- Partition society into types s ($s \in S$). Define E.Op. as a situation where there is no second-order stochastic (SSD) dominance between $F(x|s)$ *and* $F(x|s'), \forall s, s' \in S$.
- Test for this using Davidson and Duclos (2000) tests for statistically significant SSD, in nine rich countries.

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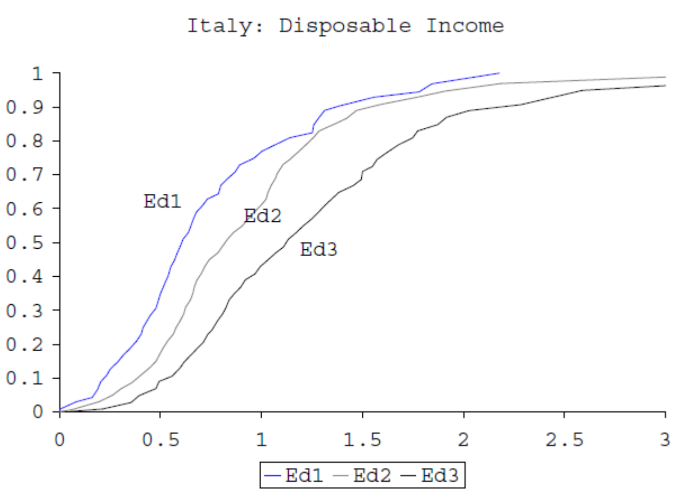

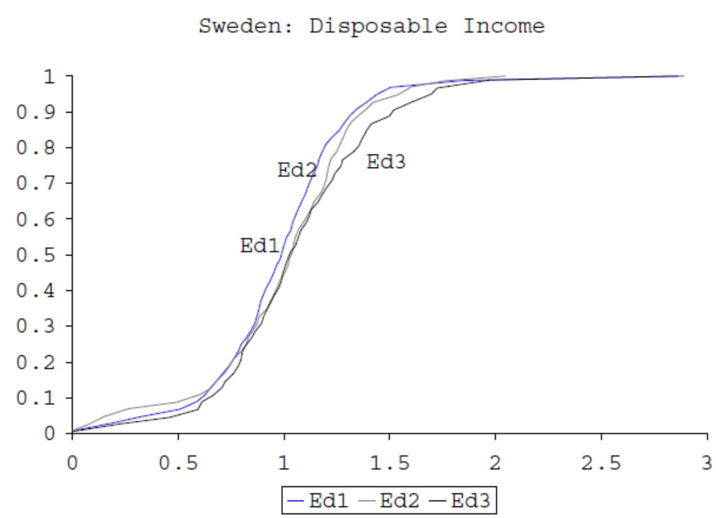

3. Measuring inequality of opportunity

- **Partial orderings** can be sought instead of complete orderings.

i. <u>To rank 'social states' by I.Op.</u> (Peragine, *JoEI*, 2004)

Proposes two ways in which income distributions can be (welfare) ranked according to inequality of opportunity:

1. "Types approach" : Define a types-mean distribution as

$$X_{\mu} = \{p_1^X \mu_1^X, \dots, p_i^X \mu_i^X, \dots, p_n^X \mu_n^X\}$$

Then for two distributions X, Y, and for all W in a class of welfare measures satisfying MON, SepBT, SymWT, INWT and IABT, $W(X) \geq W(Y)$ if and only if $X_{\mu} \succcurlyeq_{GL} Y_{\mu}$.

2. "Tranches approach": Let each tranche of *[X_ij]* be denoted , j = 1,…,m. Then for all W satisfying MON, AddBTr, SymWTr, IAWTr, if and only if .

2. "Tranches approach": Let each tranche of $[X_{ij}]$ be denoted χ_j^X, j = 1,…,m. Then for all W satisfying MON, AddBTr, SymWTr, IAWTr, $W(X) \geq W(Y)$ if and only if $\chi_j^X \succcurlyeq_{GL} \chi_j^Y, \forall j \in \{1, \dots, m\}$.

3. Measuring inequality of opportunity

- Partial orderings can be sought instead of complete orderings.

i. To rank 'social states' by I.Op. (Andreoli et al., *REStat*, 2018)

- Look for dominance not of $F(x|s)$, but of its difference, the gap curve: $\Gamma(F, F', p) = F^{-1}(p) - F'^{-1}(p)$
- IOp higher in state 0 than in 1 - for all preferences in the Yaari (1987) rank-dependent family of preferences - when $|\Gamma(F_1, F'_1, p)| \leq |\Gamma(F_0, F'_0, p)|, \forall p \in (0,1)$
- When there is no FSD between types, look for progressively higher-order dominance relations, to obtain rankings for progressively narrower subclasses of the Yaari family of preferences.
- When there are more than two types, require this for all possible pairwise combinations of types (!) – anonymously or non-anonymously

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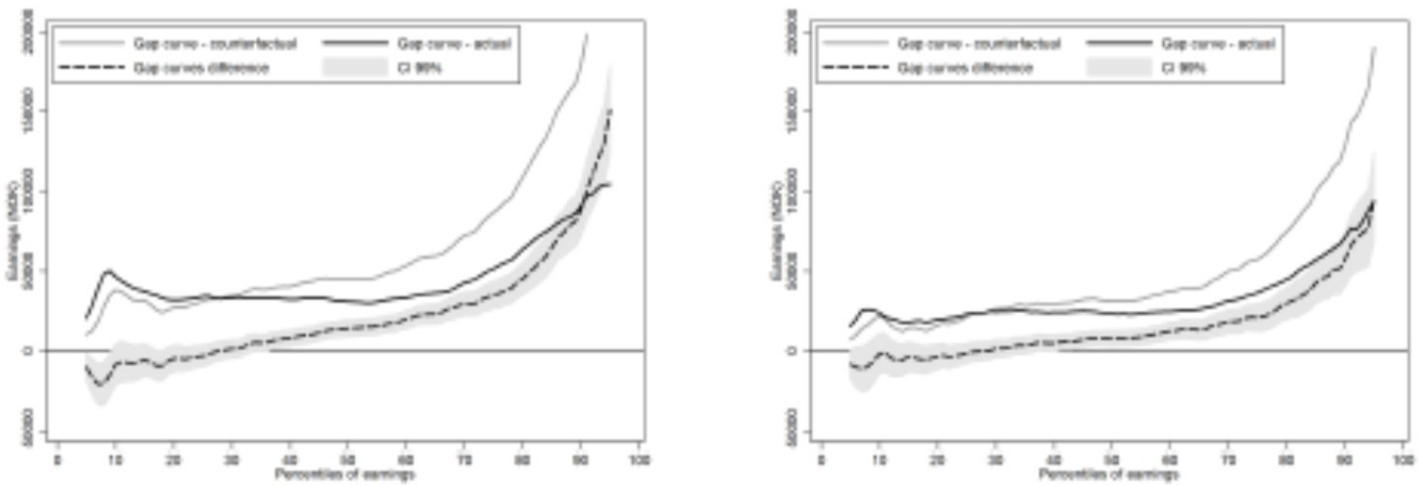
3. Measuring inequality of opportunity

- Partial orderings can be sought instead of complete orderings.
  i. <u>To rank 'social states' by I.Op.</u> (Andreoli et al., *REStat*, 2018)
     - Nice application to evaluation of impact of a child care reform in Norway, using QTEs.

B - Gap curves



(e) Lower vs upper class

(f) Middle vs upper class

- Results become inconclusive with many types. Revert to scalar indices.

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4. Empirical applications

- I am not aware of any empirical applications of the direct approach.
- Empirical applications exist of all four indirect approaches reviewed above (e.g. Almas et al., 2011; Checchi and Peragine, 2010; Devooght, 2008)
- To my knowledge, only the between-types approach - $I(\tilde{x}_{BT})$ - has been applied sufficiently widely so as to permit international comparisons.
- There are two versions of this index, absolute and relative:
- IOL: $\theta_a = I(\tilde{x}_{BT})$
- IOR: $\theta_r = \frac{I(\tilde{x}_{BT})}{I(x)}$
- Non-parametric estimation of these indices, using the (path-independent) decomposable MLD index, was pioneered by Checchi and Peragine (2010).

4. Empirical applications

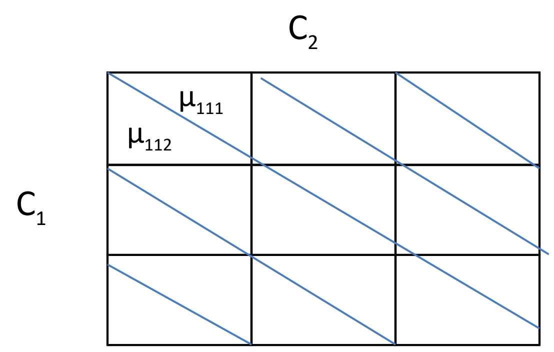
- **Statistical challenges: A tale of two biases**

1. A downward bias arises from the partial observability of circumstances

$$\Omega_{observed} \subset \Omega$$

- Omitted circumstances can only lead to a finer partitioning of the rows in *[X_ij]*, which can not reduce, but may increase measure.
- Implication (i): $I(\tilde{\mu}_{BP})$ is biased downwards
- Implication (ii): causal attribution to specific variables is unwarranted.

	C2		
C1	μ11	μ12	μ13
	μ21	μ22	μ23
	μ31	μ32	μ33



- See discussion in Ferreira and Gignoux (2011).

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4. Empirical applications

- Statistical challenges: A tale of two biases

2. An upward bias arises from the sampling variance within types
   - Sampling variation in the estimation of type means inflates measures of inequality across them.
   - Analogous to the Chakravarty and Eichhorn (1994) result for inequality measurement when income is measured with error.
   - The issue was a key reason why Bourguignon et al. (2007) and Ferreira and Gignoux (2011) first proposed a parametric approach:

> "*As the number of types increases, the frequency of sample observations per type tends to diminish quite rapidly [...] causing the precision of the estimates of the mean advantage per type to become unacceptably low. As is often the case when sample sizes are insufficient for fully flexible, non-parametric estimation, a parametric alternative is available that permits efficient estimation, at the cost of some functional form assumptions*" (FG 2011, p. 633)

   - But the upward bias implications was first recognized by Brunori, Peragine and Serlenga (2018).

4. Empirical applications

- When the information on circumstances is rich enough for a given sample size, the number of types may become too great to estimate either IOL or IOR non-parametrically.
- Bourguignon et al. (2007) and Ferreira and Gignoux (2011) propose a simple model:

$$x = g(C, e, u)$$
$$e = f(C, v)$$

- For the purpose of simply measuring inequality of opportunity, it suffices to estimate the reduced form:

$$x = \phi(C, \varepsilon)$$

- Say, by OLS: $x = C\psi + \varepsilon$
- Can then compute "parametrically smoothed distribution": $\tilde{x}_i = C_i\hat{\psi}$
- Leading to the parametric estimate: $IOL = I(\tilde{x}_i)$

4. Empirical applications

TABLE 1
HOUSEHOLD SURVEY NAMES, DATES, AND SAMPLE SIZES

	Brazil	Colombia	Ecuador	Guatemala	Panama	Peru
Survey	PNAD 1996	ECV 2003	ECV 2006	ENCOVI 2000	ENV 2003	ENAHO 2001
Sample of 30 to 49 year-olds	85,692	22,517	12,650	6,956	6,339	17,030
Sample of heads and spouses, aged 30 to 49 years	73,847	18,069	10,719	6,067	5,105	13,947
Of those, observations with income/consumption and circumstances	70,521	17,979	10,719	5,988	4,556	13,621
(share of original sample)	0.823	0.798	0.847	0.861	0.719	0.800

Source: Ferreira and Gignoux, 2011

4. Empirical applications

TABLE 3

DEFINITION OF CIRCUMSTANCE VARIABLES, BY COUNTRY

	Brazil	Colombia	Ecuador	Guatemala	Panama	Peru
Ethnicity						
Category 1	Self reported white ethnicity	Other	Self-reported ethnicity: white, mixed blood ("mestizo") or other	European maternal language	Other	European maternal language
Category 2	Self reported black ("negro") and mixed blood ("pardo") ethnicity	Self-reported minority ethnicity: "indígena, gitano, archipiélago o palenquero"	Self-reported ethnicity: indigenous, black ("negro" or "mulato")	Indigenous maternal language	Speaks indigenous language	Indigenous maternal language
Father's occupation						
Category 1	Agricultural worker	Missing	Agricultural worker or domestic worker	Agricultural worker	Agricultural worker	Missing
Category 2	Other		Other	Other	Other	
Mother's and father's education						
Category 1	None or unknown	None or unknown	None or unknown	None or unknown	None or unknown	None or unknown
Category 2	Completed grade 1 to 4	Primary incomplete	Primary	Primary incomplete	Primary	Primary incomplete
Category 3	Completed grade 5 or more	Primary complete or more	Secondary or more	Primary complete or more	Secondary or more	Primary complete or more
Birth region						
Category 1	Sao Paulo and Federal district	Departments at the periphery	Sierra and Amazonia provinces	Guatemala City, Northeast departments and El Petén	Cities and intermediate urban centers	Inland non-southern departments
Category 2	South East, Center-West, and South	Central departments(a)	Costa and Insular provinces	North and Northwest departments	Other urban centers	Southern and other costal departments
Category 3	North-East, North or missing	Bogota, San Andres, and Providencia islands and foreign country	Pichincha province (with Quito) and Azuay province	Southeast, Southwest, and Center departments	Rural areas	Arequipa, Callao, and Lima

Note: Central departments are Boyaca, Caldas, Caqueta, Cundinamarca, Huila, Meta, Norte de Santander, Quindio, Risaralda, Santander, Tolima, and Valle del Cauca.

Source: Ferreira and Gignoux, 2011

4. Empirical applications

TABLE 6

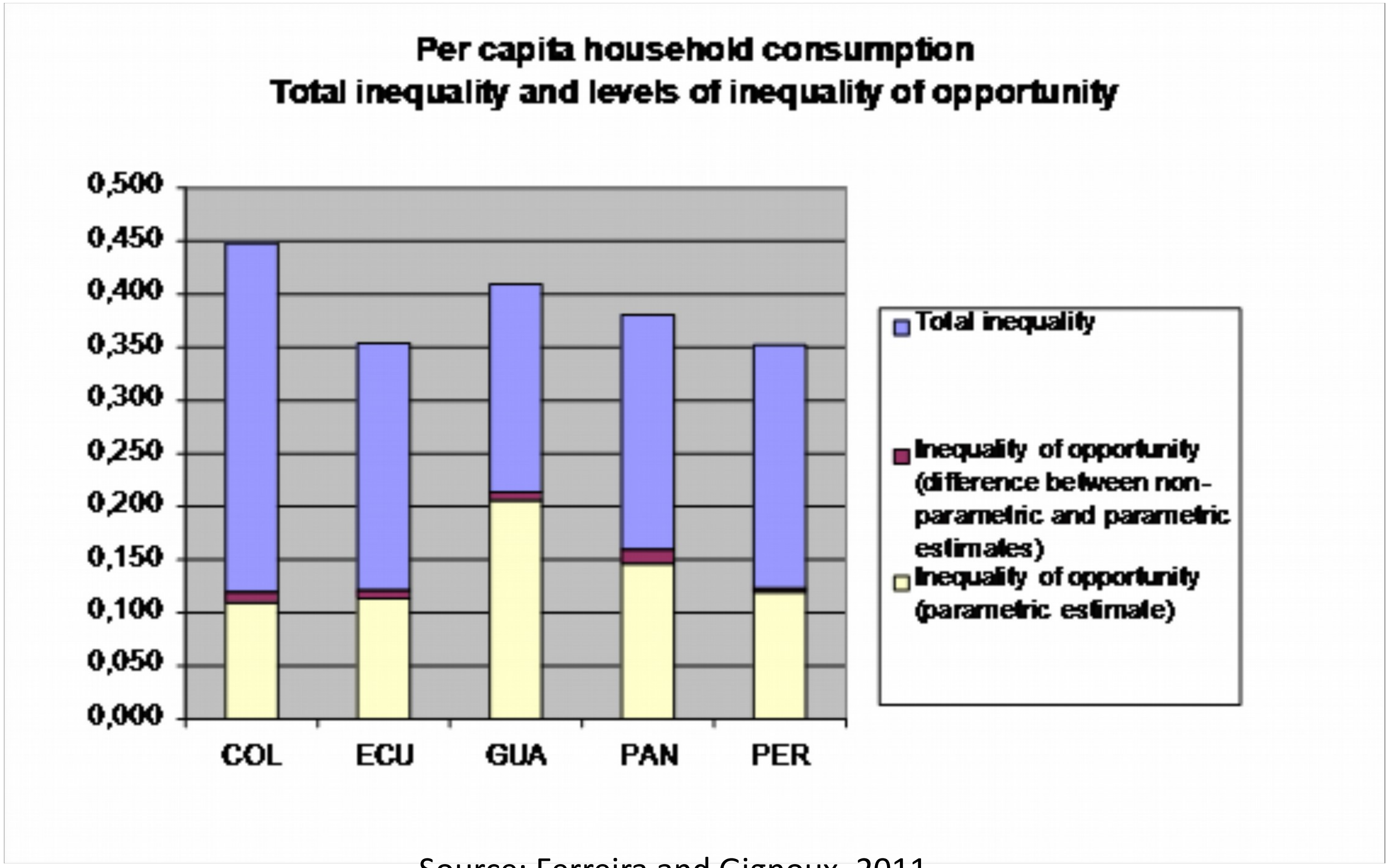
SCALAR INDICES OF INEQUALITY OF OPPORTUNITY

	Brazil	Colombia	Ecuador	Guatemala	Panama	Peru
Panel A: Household income (per capita)						
Total inequality (E_0)	0.692	0.572	0.580	0.593	0.630	0.557
	(0.013)	(0.033)	(0.028)	(0.036)	(0.029)	(0.022)
Non-parametric estimates						
IOL	0.227	0.144	0.164	0.213	0.213	0.163
	(0.008)	(0.023)	(0.022)	(0.031)	(0.024)	(0.015)
IOR	0.329	0.252	0.283	0.359	0.338	0.293
	(0.008)	(0.026)	(0.023)	(0.030)	(0.026)	(0.018)
Parametric estimates						
IOL	0.223	0.133	0.150	0.199	0.190	0.156
	(0.008)	(0.019)	(0.020)	(0.028)	(0.023)	(0.014)
IOR	0.322	0.232	0.259	0.335	0.301	0.279
	(0.008)	(0.023)	(0.023)	(0.030)	(0.028)	(0.018)
Panel B: Household consumption expenditures (per capita)						
Total inequality (E_0)		0.462	0.359	0.415	0.381	0.351
		(0.018)	(0.015)	(0.025)	(0.018)	(0.013)
Non-parametric estimates						
IOL		0.123	0.124	0.221	0.156	0.123
		(0.015)	(0.013)	(0.024)	(0.016)	(0.010)
IOR		0.265	0.346	0.532	0.409	0.351
		(0.021)	(0.021)	(0.023)	(0.025)	(0.018)
Parametric estimates						
IOL		0.114	0.117	0.213	0.144	0.119
		(0.014)	(0.012)	(0.022)	(0.015)	(0.009)
IOR		0.247	0.326	0.514	0.377	0.339
		(0.021)	(0.022)	(0.022)	(0.026)	(0.017)

Notes: Sample: household heads and spouses, aged 30–49, with positive income and information on a set of circumstances; bootstrap standard errors (taking into account stratification and clustering) in parentheses; father's occupation missing for Colombia and Peru.

4. Empirical applications

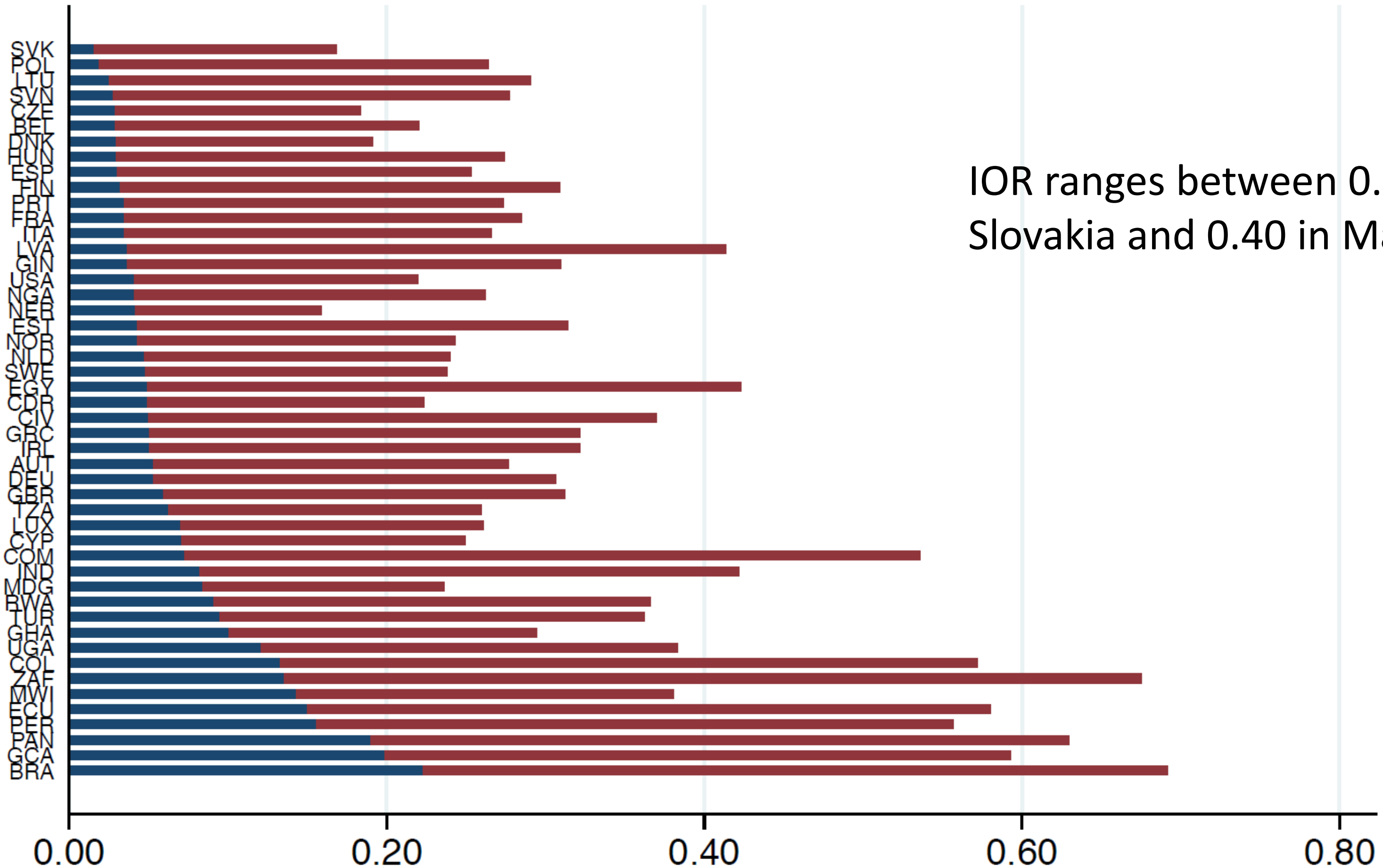
1. In Latin America, inequality of economic opportunity:
   - ranges from 23% to 35% for income per capita.
   - ranges from 24% to 50% for consumption per capita.



Source: Ferreira and Gignoux, 2011

4. Empirical applications

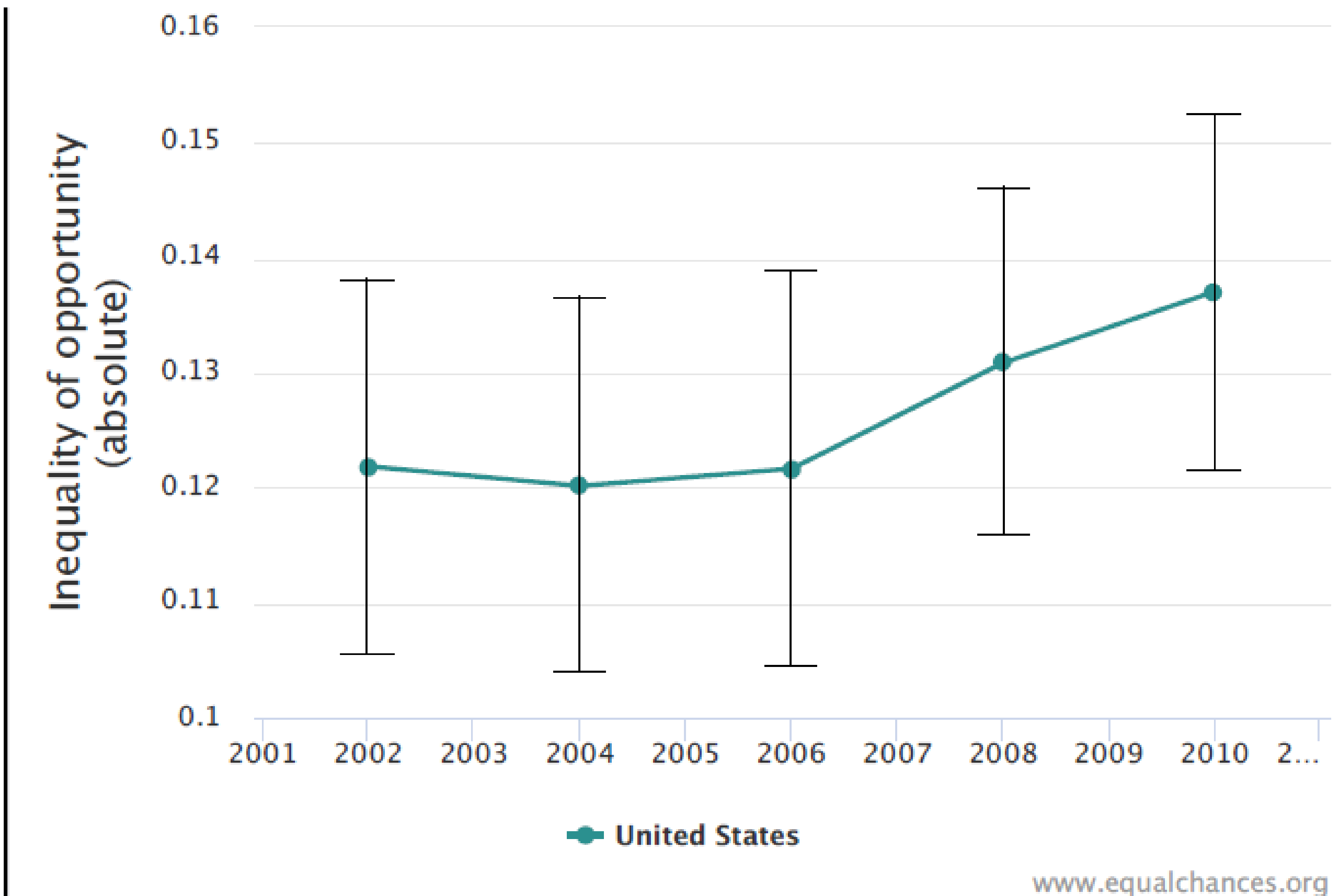
Total inequality GE(0) and IOp (absolute)
ordered according to IOL



IOR ranges between 0.05 in Slovakia and 0.40 in Malawi.

Note: Estimates come from different studies and are not strictly comparable.
Source: Brunori et al. (2015)

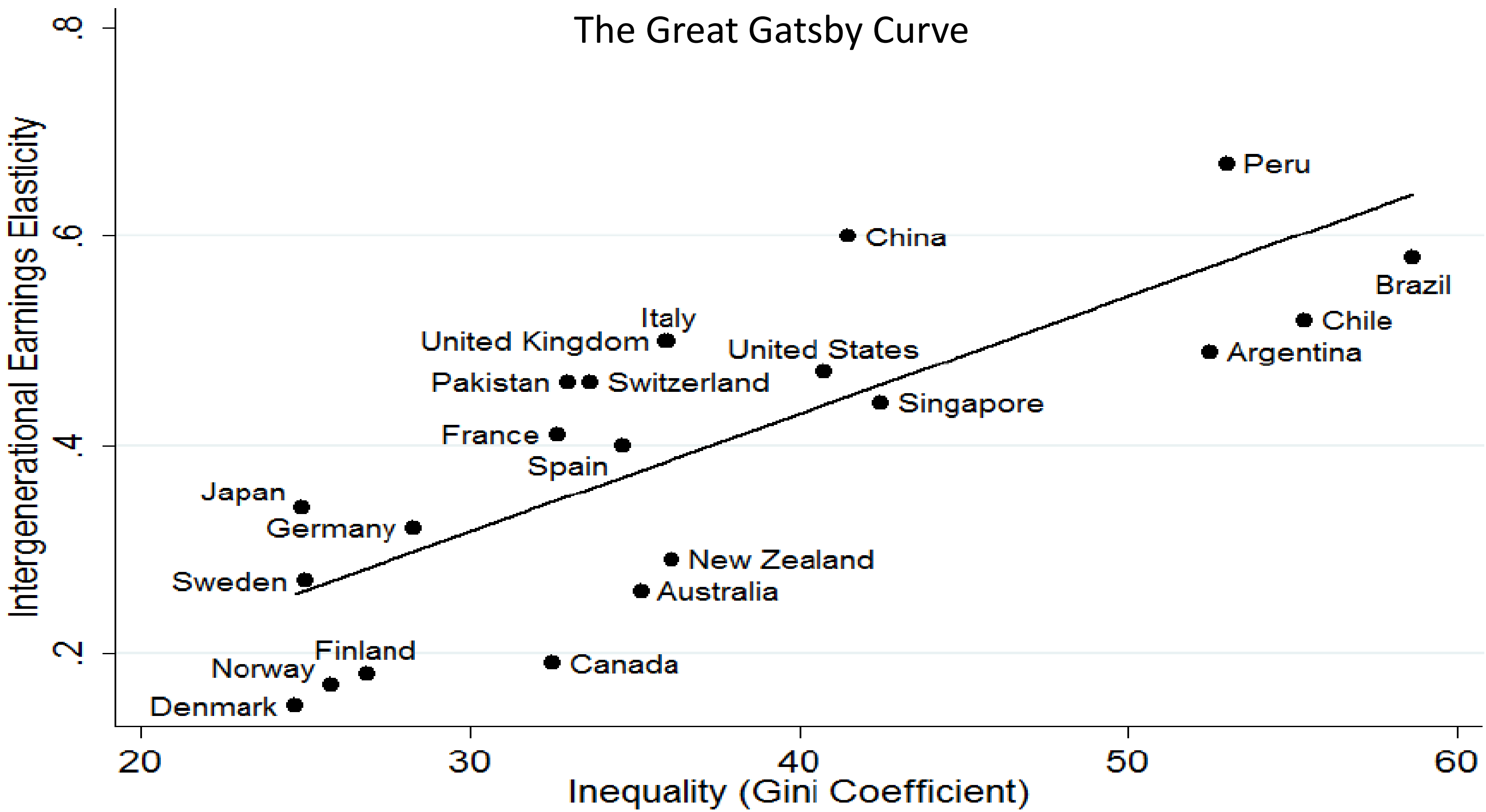
4. Empirical applications



4. Empirical applications

Inequalities of outcome and opportunity: strong correlation

4. Empirical applications



Source: Corak (2012)

Outline

1. Equality of opportunity: Motivation and background
2. Economic models of equality of opportunity
3. Measuring inequality of opportunity
4. Empirical applications
   i. 'First generation' between-types approach
   ii. 'Second-generation' between-types approach
5. Concluding remarks

4. Empirical applications

1. Those first-generation studies typically used parsimonious parametric models (with purely linear specifications, omitting higher-order polynomial terms or interactions) or non-parametric estimation (with relatively coarse partitions).

2. The resulting estimates were – perhaps strictly incorrectly – interpreted as lower-bound estimates.
    1. Though it is likely that in most of those studies the downward bias outweighed the upward bias.

3. Some of the IOR estimates, particularly for richer countries, were judged to be uninformatively low, and the usefulness of the lower-bound results was criticized (e.g. Kanbur and Wagstaff, 2016)

4. So people started looking for finer partitions, or enriching their parametric specifications.

4. Empirical applications

1. 'Second-generation' between-types approach: looking for upper-bound estimates (Niehues and Peichl, SCW 2014)

- Two-stage estimator using panel data:

  i. Estimate $\ln w_{it} = \beta E_{it} + c_i + u_t + \varepsilon_{it}$

  ii. Back in cross-section, estimate $\ln w_{is} = \varphi \hat{c}_i + \upsilon_i$

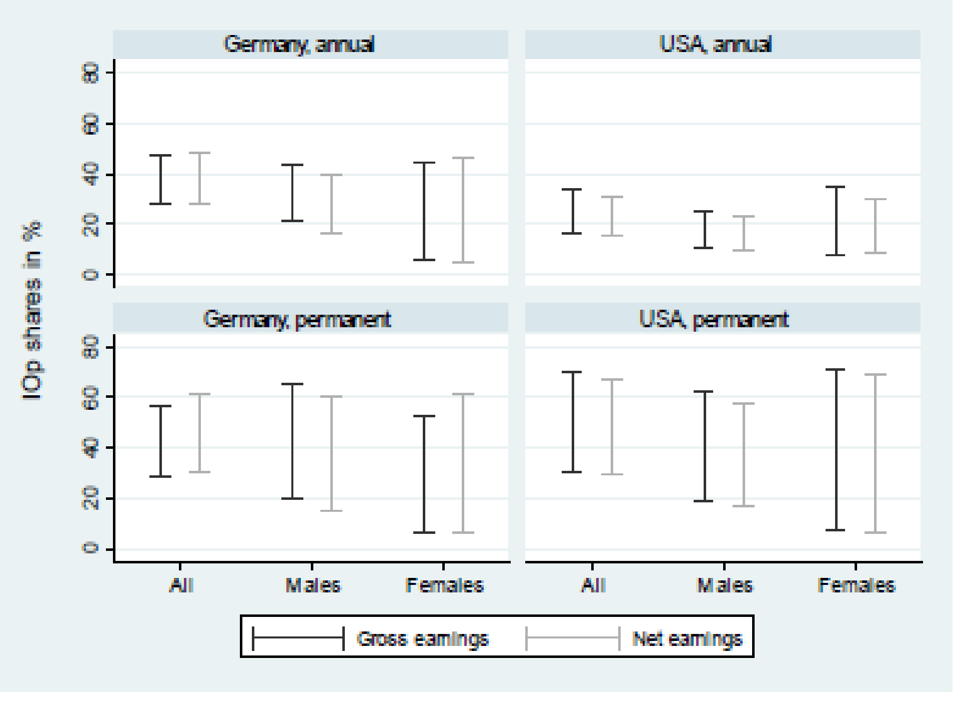
  Construct $\tilde{\mu}^{UB} = \exp(\hat{\varphi}\widehat{c_i} + \sigma^2/2)$

  – Application to Germany (SOEP) and the US (PSID), for both current and permanent incomes

4. Empirical applications

1. 'Second-generation' between-types approach: looking for upper-bound estimates (Niehues and Peichl, SCW 2014)

Figure 2: IOp shares in outcome inequality



Source: Own calculations based on SOEP and PSID. The two graphs on the top ustrate IOp shares in annual incomes (2009 for Germany, 2007 for the US); the grapl at the bottom IOp shares in permanent incomes.

4. Empirical applications

1. ‘Second-generation’ between-types approach: enlarging the circumstance set through admitting an “age of consent” (Hufe, Peichl, Roemer and Ungerer; 2017)
   – Use National Longitudinal Survey of Youth (NLSY -79) for the US and British Cohort Study (BCS – 70) for the UK

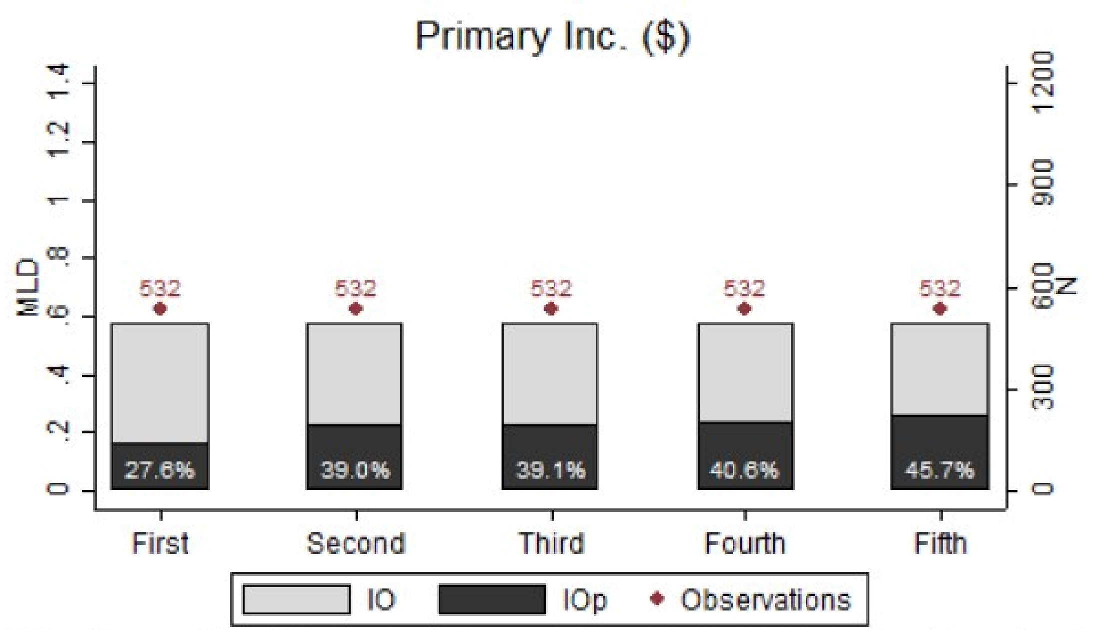
Scenario	Circumstance Set	Circumstance Var.
First	Base	Sex, Country of Birth, Ethnic Affiliation, Cohort, Age, Academic Achievement Mother, Occupation Code Mother, Rural/Urban, Height (16), Family Income
Second	Ability	PIAT Math, PIAT Reading
Third	Behavioral Problems	Behavioral Problems Index (BPI)
Fourth	Child-Parent Relationship	Play/Schoolwork w/ Parents, Perceived Quantity of Time w/ Parents, Parents Split, Parental Income
Fifth	Health-Related Behavior	Smoking Habits Mother, Drinking Habits Mother, Health Restrictions Child
Sixth	Survey Specifics	Specific to NLSY79 and BCS70. See text for more information.

Table 1: Overview of Circumstance Scenarios

4. Empirical applications

1. Hufe, Peichl, Roemer and Ungerer (2017) find that the lower-bound IOR can be as high as 45% in the US and 31% in the UK when using this extended circumstance set.

Figure 2: IOp with varying circumstance sets (NLSY79), comparable sample, average income



Note: The overall bar yields the extent of outcome inequality IO. The black colored share of each bar yields inequality attributed to circumstances, i.e. the lower bound absolute measure of inequality of opportunity IOp. The residual gray colored share of each bar can be interpreted as an upper bound measure of inequality attributed to differential efforts. The white labels at the bottom of each bar indicate the share of IOp in IO, i.e. the relative measure of inequality of opportunity r.

4. Empirical applications

1. But, in general, refining type partitions - e.g. by adding interaction terms to parametric models, or refining categories for each circumstance variable - alleviates the downward bias (from partial observability) at the expense of increasing the upward bias (from within-type sampling variance).

2. Given a certain set of observed circumstance variables, and a sample of observations, choices of model specification between the simplest linear specification (where the impact of circumstances is restricted to be linear and additive), and a fully interacted model (which is equivalent to the non-parametric estimate) have so far been made arbitrarily.

3. Is there a meaningful criterion that can help practitioners choose an "optimal" specification, given the trade-off between the two biases?

4. Empirical applications

1. Brunori, Peragine and Serlenga (2018) propose choosing the specification $\hat{g}(C)$ that minimizes the mean squared error of out-of-sample predictions:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{g}(C_i)\right)^2$$

2. Which is decomposable as follows:

2. Which is decomposable as follows:

$$E\left(y_0 - \hat{g}(C_0)\right)^2 = Var(\hat{g}(C_0)) + \left[Bias\left(\hat{g}(C_0)\right)\right]^2 + Var(u)$$

Captures the upward bias from sampling variation

Captures the downward bias from mis-specification

4. Empirical applications

3. The procedure uses k-fold cross-validation. The average MSE for the k test samples is computed for each model specification, and the specification with the lowest MSE is chosen.

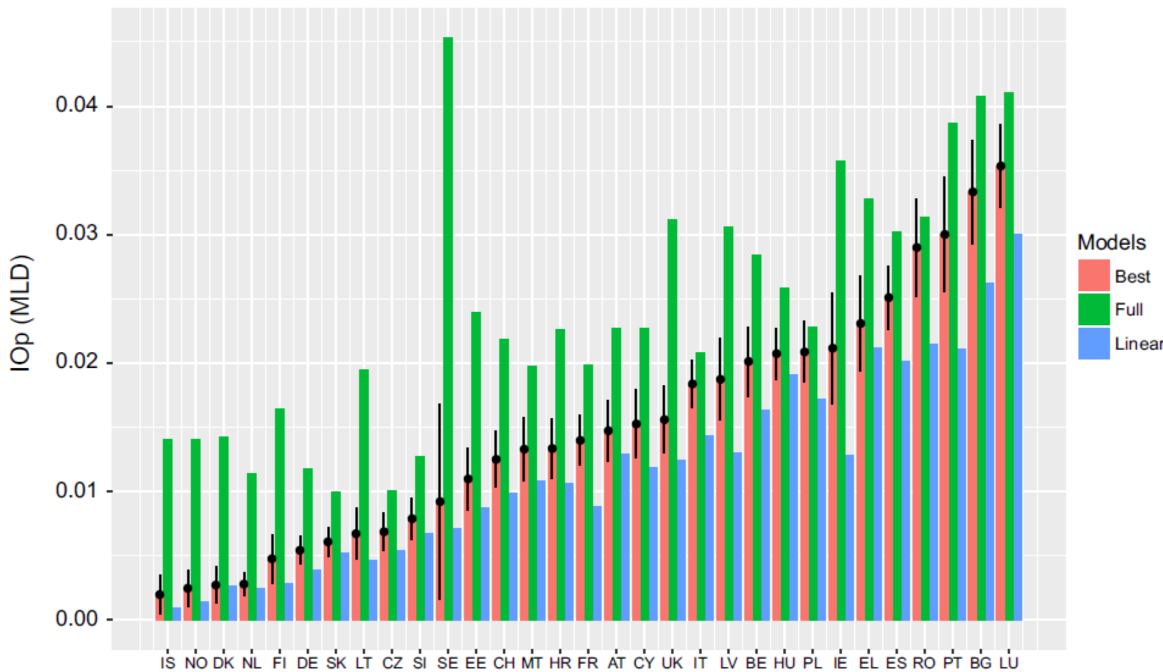


Fig. 1 IOp in 31 European countries under different model specifications. The Figure shows each country's IOp measure obtained with the three alternative methods: (i) the linear, most parsimonious case (*linear*), (ii) the fully interacted model (*full*); (iii) the best model selected (*best*). Countries are ordered according to the IOp level based on the *best* model specification with 95% confidence intervals. Table 2 in the Appendix contains IOp estimates and relative bootstrapped standard errors based on 500 replications for the three alternative model specifications . *Source: EU-SILC, 2011*

4. Empirical applications

3. An alternative approach to "let the data choose the model specification" is proposed by Brunori, Hufe and Mahler (2018), using conditional inference trees and forests.

   – A conditional inference tree consists of a set of terminal nodes (leaves) obtained by recursive binary splitting, as follows.

   – Given a set of circumstance variables and categories, the algorithm splits the sample in all possible partitions [C], and computes the p-value for the null hypothesis that the statistic of interest (e.g. the mean) in the two sub-samples is identical.

   – [C]* is chosen as $[C]^* = \{[C]: argmin\ p_{adj}^{[C]}\}$, where the adjustment is a Bonferroni correction (for multiple hypothesis testing).

   – A critical significance level α can be chosen so that if $p_{adj}^{[C]^*} > \alpha$, one exit the algorithms, and otherwise [C]* is chosen as splitting variable.

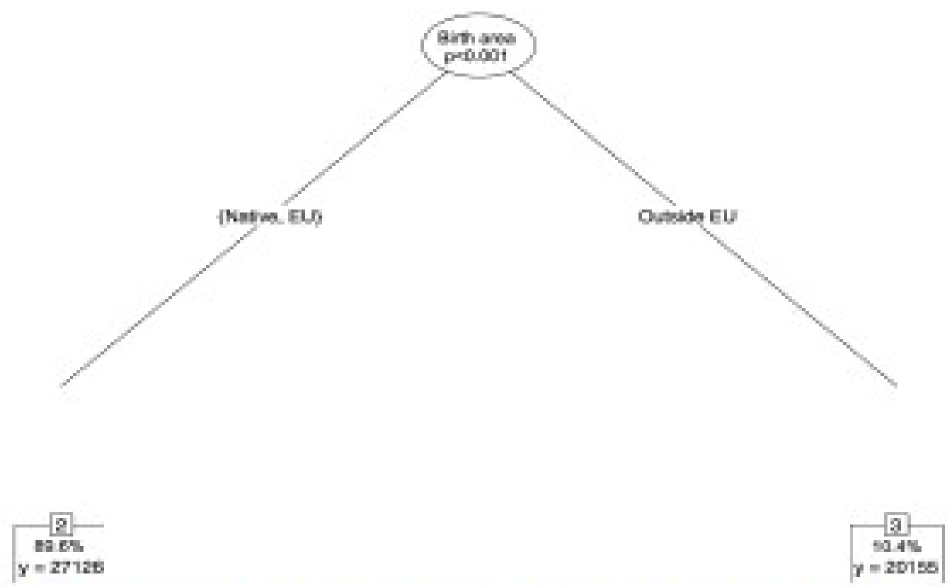
   – Repeat the algorithm for each node (sub-sample), until one has exited everywhere.

   – A conditional inference forest is basically a set of trees estimated on random subsamples of the original data, in each case using a different subset of circumstance variables. The size of the subsets of circumstances is chosen by minimizing the "out-of-the-bag" MSE.

4. Empirical applications

3. Although forests outperform trees in terms of out-of-sample prediction, trees can be visually informative of the 'structure' of inequality of opportunity in different countries.

Figure 3: Opportunity Tree: Sweden



Note: Opportunity tree for Sweden. White rectangular boxes indicate terminal nodes. The first number inside the rectangular boxes indicates the share of the population belonging to this group, while the second number indicates the predicted income.

4. Empirical applications

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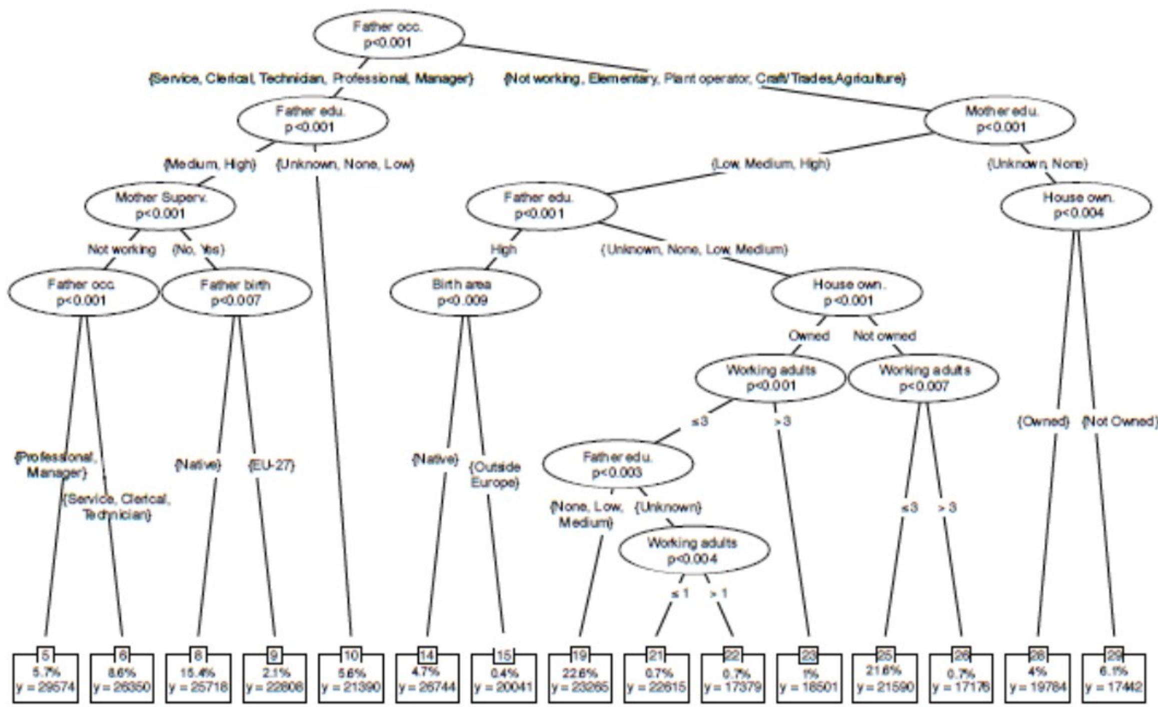
Figure 4: Opportunity Tree: Germany



Note: Opportunity tree for Germany. White rectangular boxes indicate terminal nodes. The first number inside the rectangular boxes indicates the share of the population belonging to this group, while the second number indicates the predicted income. Occupation refers to ISCO-08 one digit codes. All variables describing household characteristics refer to the period in which the respondent was about 14 years old. See Table 1 for details.

4. Empirical applications

1. A visually appealing, didactic set of illustrations of some of these approaches, for the case when the only circumstances are parental education and occupation. Courtesy of Paolo Brunori.

$$y_i = f(ED_i, OC_i, e_i)$$

Figure 1: the "space" of circumstances in a simplified model *à la* Roemer

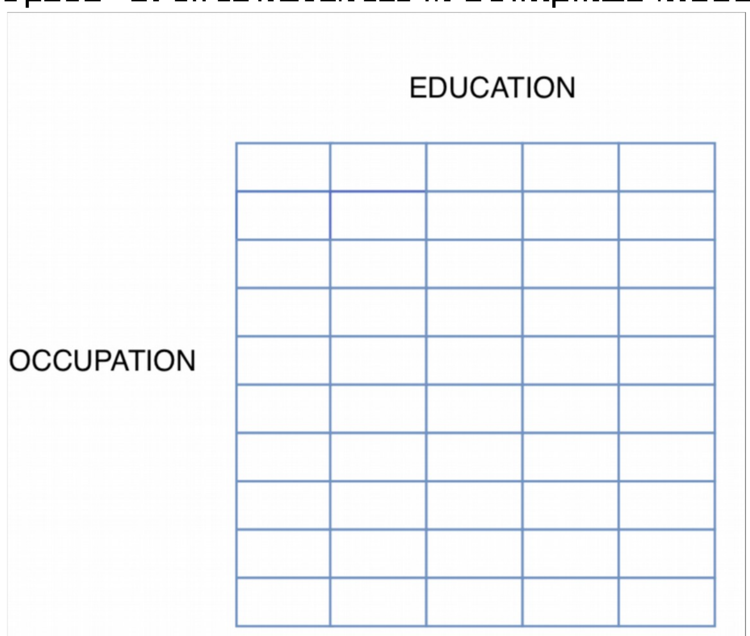


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Figure 2: non-parametric estimation:

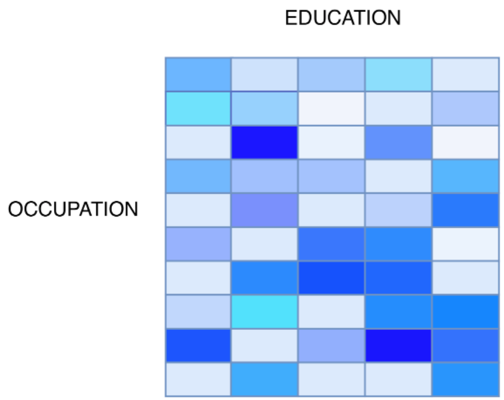

Figure 3: parametric estimation:

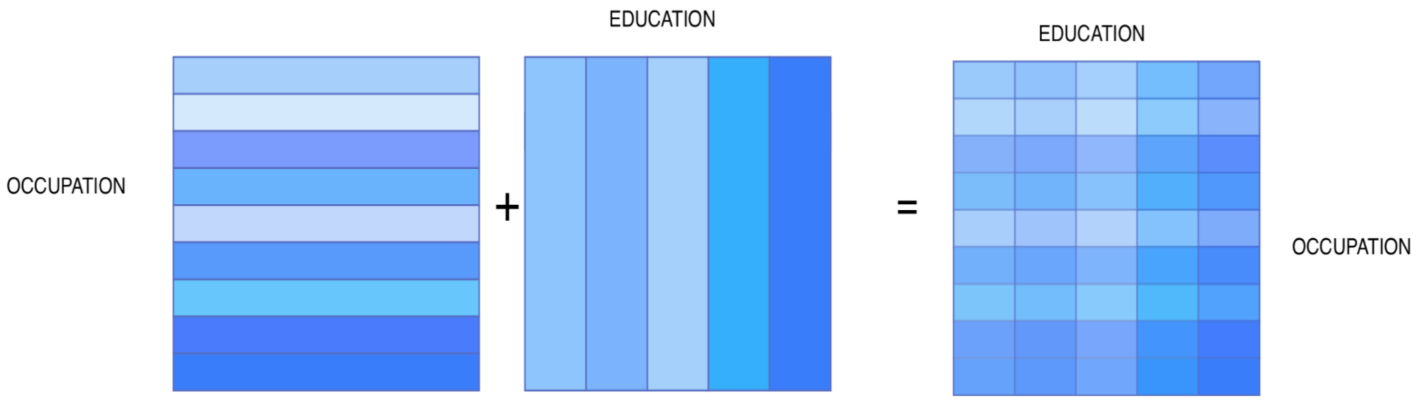

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Figure 4: data-driven non-parametric e.g. conditional inference tree:

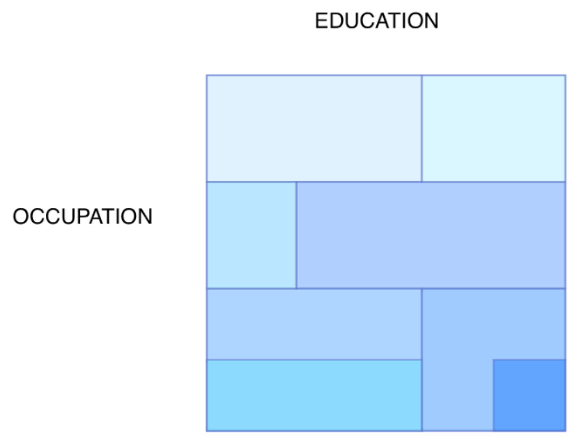
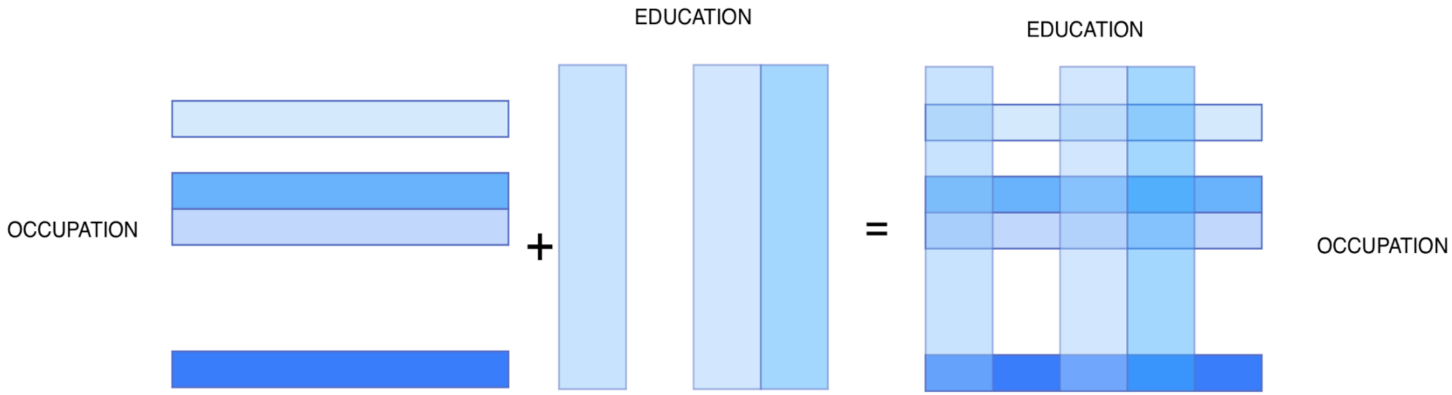


Figure 5: data-driven parametric, e.g. minimizing MSE:



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5. Concluding remarks

- Inequality of opportunity remains an active area of research in economics – likely because it matters…
  - Intrinsically (both normatively and psychologically)
  - Instrumentally
- But the field still struggles with challenges…
  - Conceptually, because there are multiple ways of operationalizing the principles of compensation and reward, and these sometimes clash
    - And because of the materialist 'causal thesis' and 'incompatibilist' views.
  - Empirically, because of data limitations
    - Partial observability of circumstances (downward bias)
    - Sample size limits and sampling variation (upward bias)
- Nonetheless recent efforts to use richer data and new econometric methods, including from machine learning, hold promise.
- Need a discussion of what society chooses to classify as circumstances, particularly as data on (epi)genetics become more widely available.
  - Recall that value judgments are inherent to inequality analysis, even when one is just looking at incomes (Atkinson, 1970).