Workshop LAMETA
Inequality and social welfare:
preferences, behavior and measurement

Ranking Distributions of an Ordinal Attribute

Nicolas Gravel (Aix-Marseille Université, AMSE, France)
Brice Magdalou (Univ. de Montpellier, LAMETA, France)
Patrick Moyes (Univ. de Bordeaux, GREThA, France)

Montpellier, November 17-18, 2016

Introduction

Objective of the paper

A real demand for new social welfare indicators

- « for a better life index » initiative (OECD, 2011) ...

Most of the social welfare dimensions are ordinal attributes

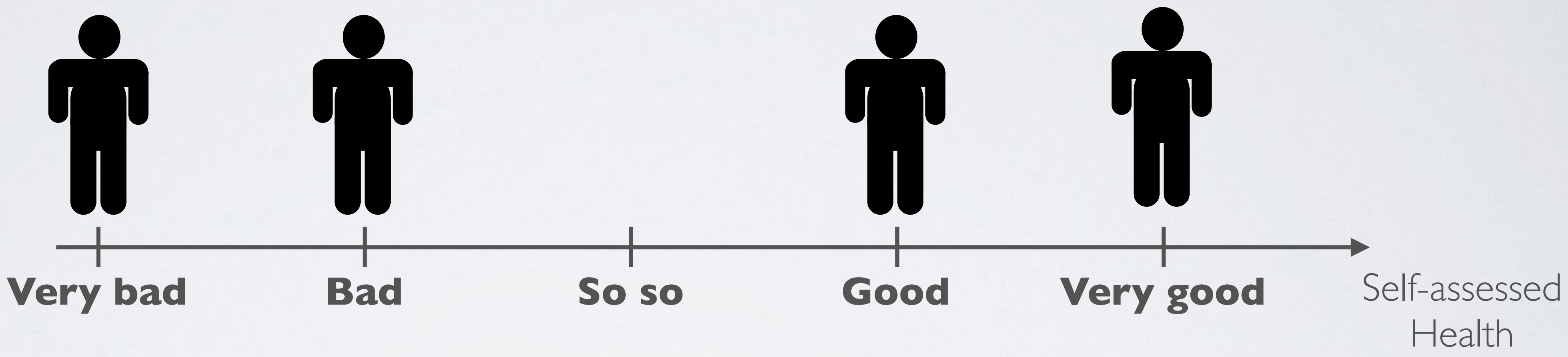
- examples: development (access to housing), health (health status, body mass index, QALY), subjective well-being (life satisfaction, happiness), education (Pisa scores, years of schooling, IQ), ...

Our objective: To propose criteria to rank such distributions

- with transparent ethical foundations : efficiency / equity
- empirically implementable (ex: Lorenz dominance)
- Hardy-Littlewood-Polya theorem (Kolm 1966, Atkinson 1970, ...)

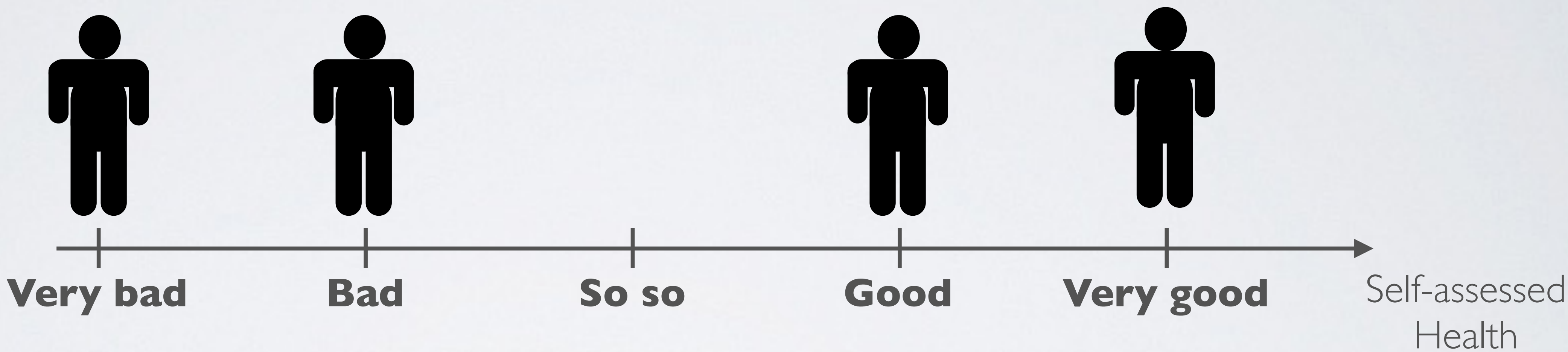
Specificities of ordinal measurement

A scale is a list of ordered categories



Specificities of ordinal measurement

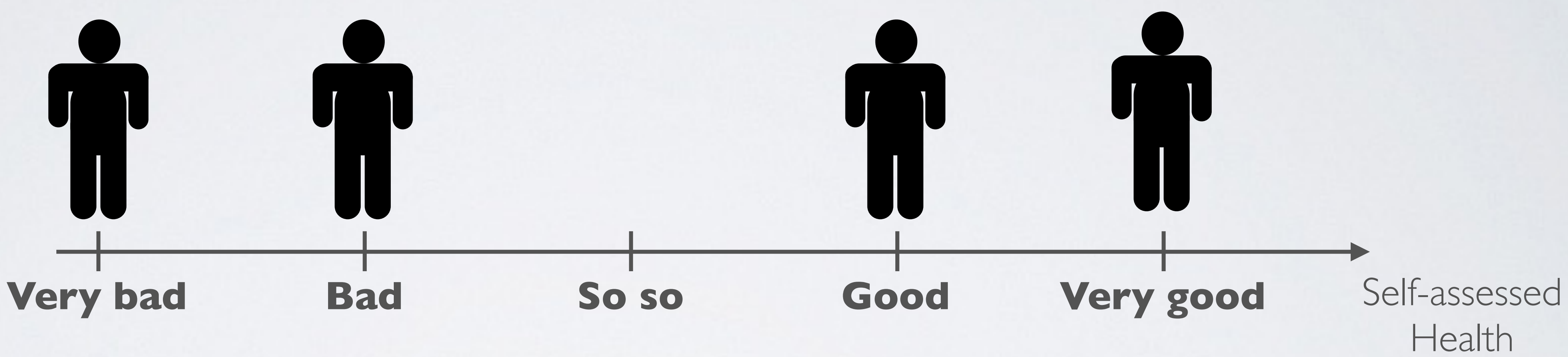
A scale is a list of ordered categories



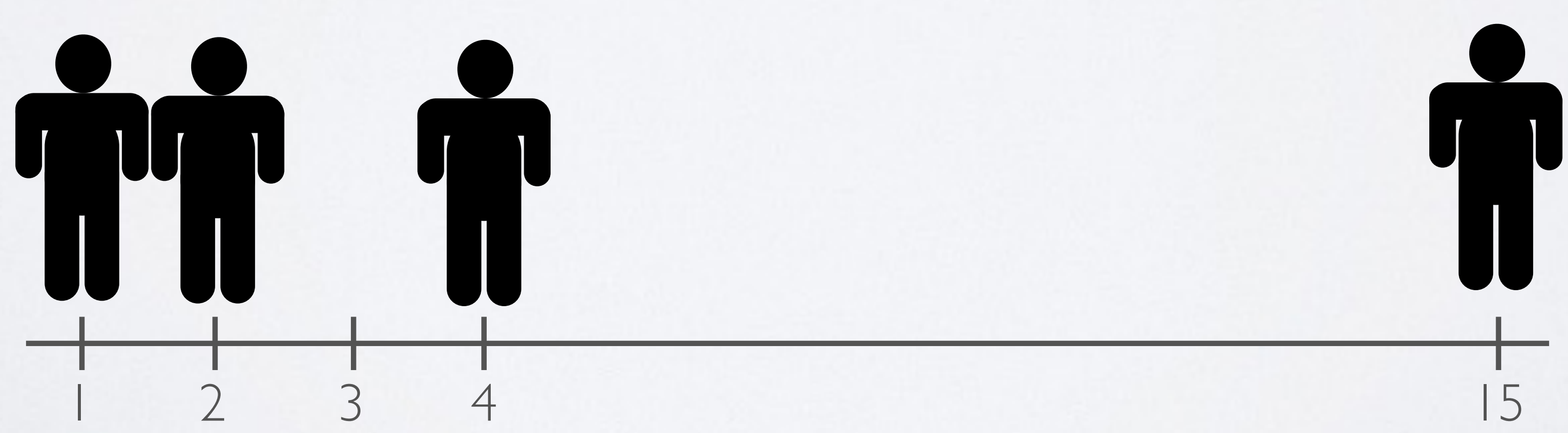
Numerical representations of the scale are allowed, but defined up to a strictly increasing transformation

Specificities of ordinal measurement

A scale is a list of ordered categories

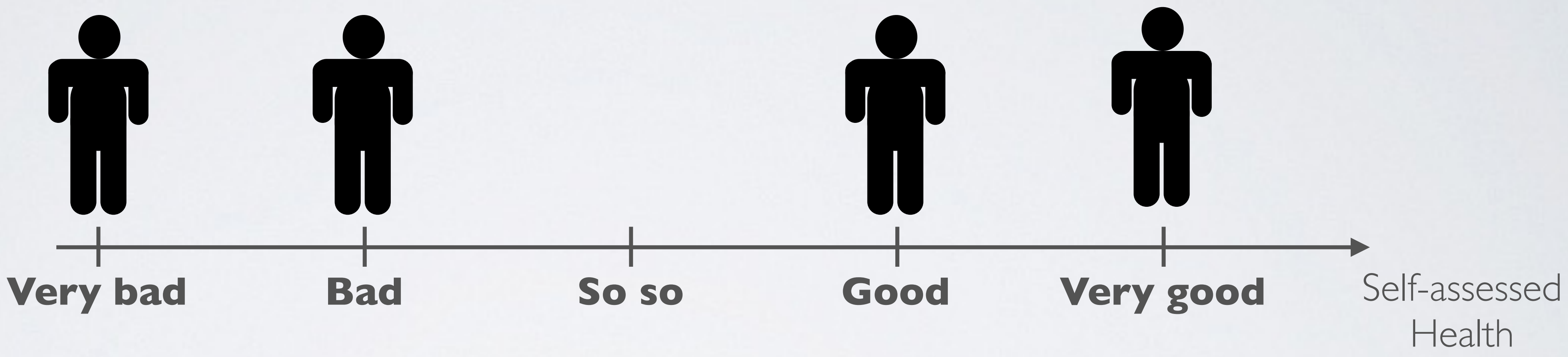


Numerical representations of the scale are allowed, but defined up to a strictly increasing transformation

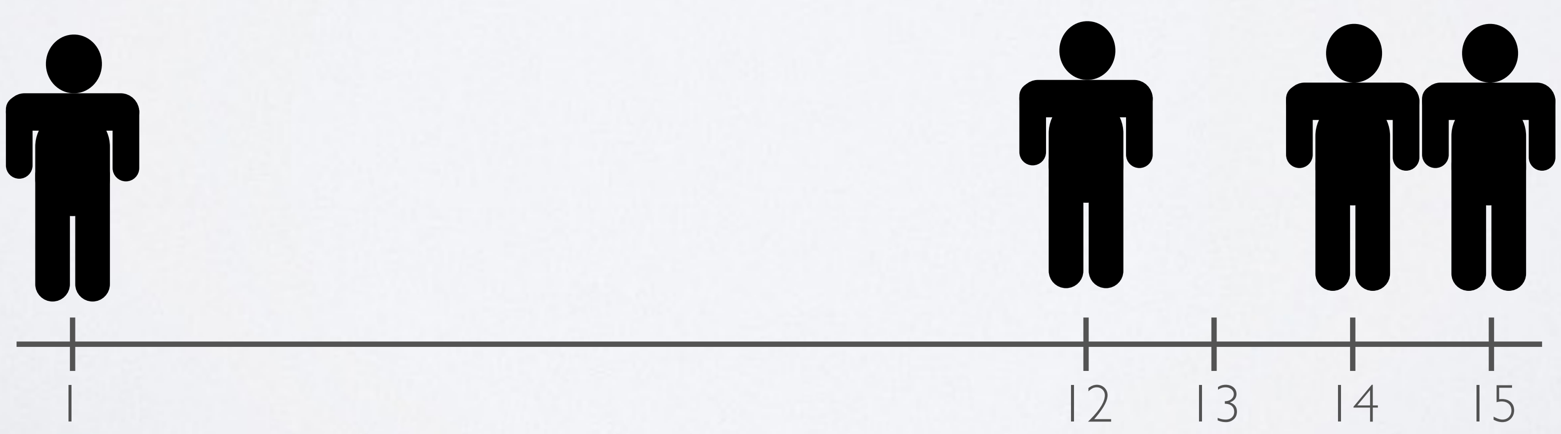


Specificities of ordinal measurement

A scale is a list of ordered categories

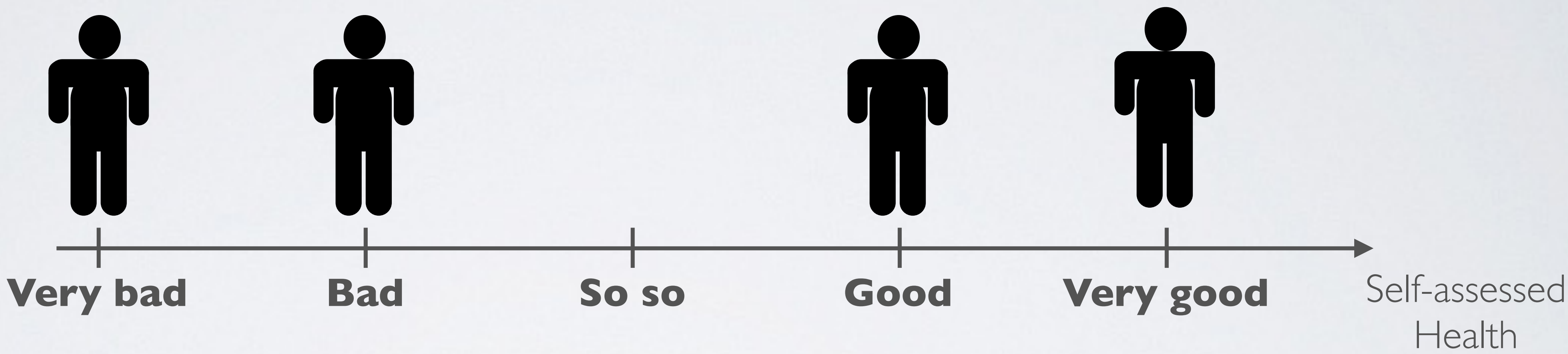


Numerical representations of the scale are allowed, but defined up to a strictly increasing transformation

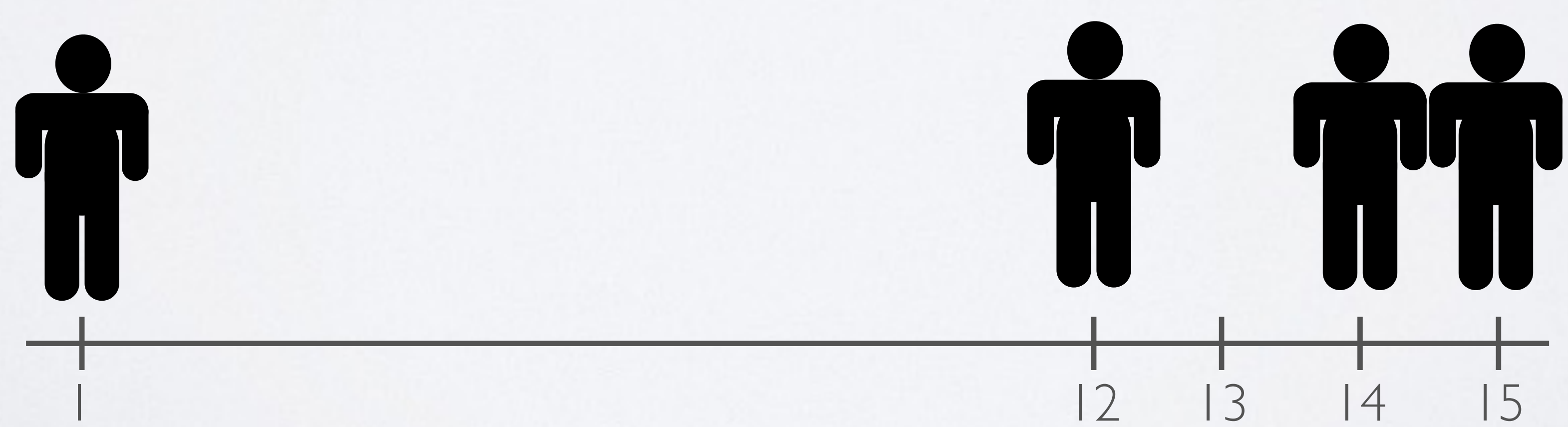


Specificities of ordinal measurement

A scale is a list of ordered categories



Numerical representations of the scale are allowed, but defined up to a strictly increasing transformation



the distance between two categories has no meaning ...

Specificities of ordinal measurement

How to define « the size of the cake » ?

- the ranking of the means of two distributions can be reversed by a transformation of the (numerical) scale
- Mendelson (1987): only the quantiles (ex: median) are invariant to such transformations
- efficiency considerations: captured by the notion of increment

What is the meaning of « inequality reduction » ?

- for a cardinal attribute : Pigou-Dalton transfer principle
- questionable for an ordinal attribute ... no alternative in the literature

Literature

Most of the literature : « cardinalization » of the scale

Only few papers recognize the specificities of ordinal measurement

Alison & Foster (J. Health Eco. 2004)

- applies to distributions with the same median
- single-crossing condition of the CDFs about the median

Abul-Naga & Yalcin (J. Health Eco. 2008)

- develop and apply indices consistent with inequality reduction in the sense of Alison & Foster

Cowell & Flachaire (WP 2014)

- definition of individual statuses, invariant to a transfo. of the scale
- inequality as distance from a reference point (mean, median, ...)

Framework and Definitions

Notation

We consider a population of n individuals

A scale is a set of ordered categories $\mathcal{C} = \{1, 2, \ldots, k\}$

A society *s* is a list $(n_1^s, n_2^s, \ldots, n_k^s)$, where n_j^s is the number of individuals in category *j*, with $\Sigma_{j=1}^k n_j^s = n$

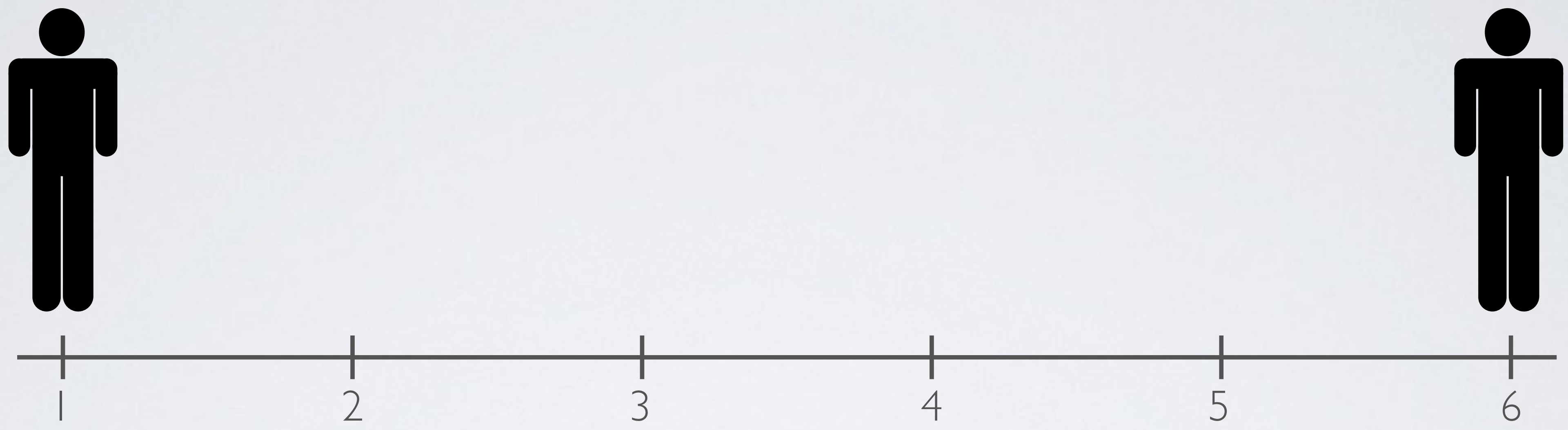
Hammond's transfers

We consider a transfer principle, due to Peter J. Hammond (Econometrica 1976), for capturing our intuition about meaning of inequality reduction in an ordinal setting

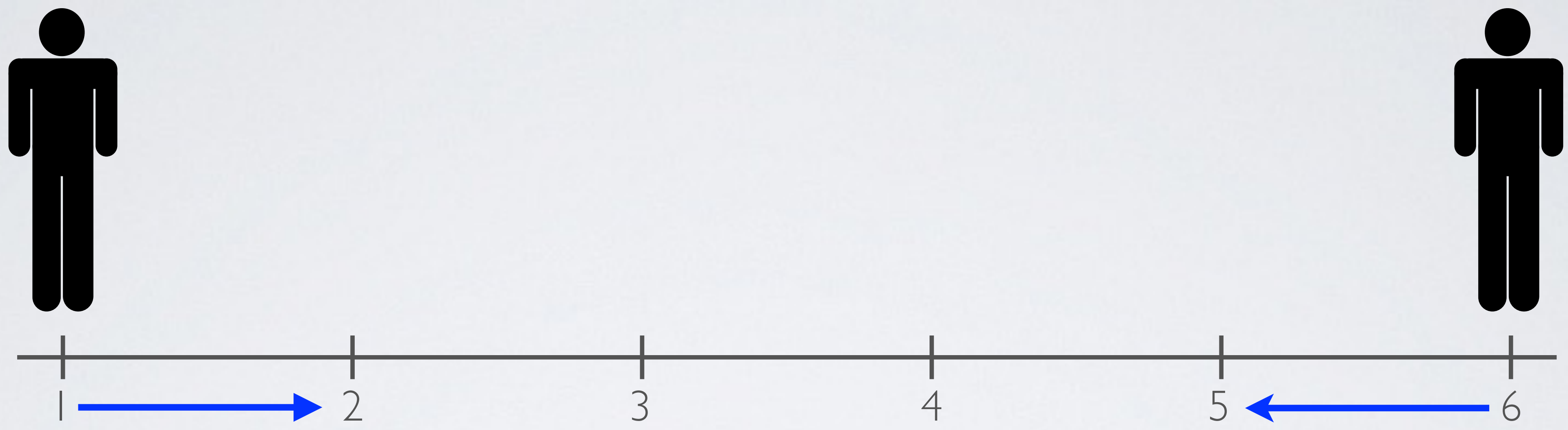
Hammond's (progressive) transfer :

- a transfer from a richer to a poorer individual, without reversing their positions on the ordinal scale, improves social welfare
- contrary to a PD transfer, which is a mean-preserving contraction in spread, nothing is preserved here

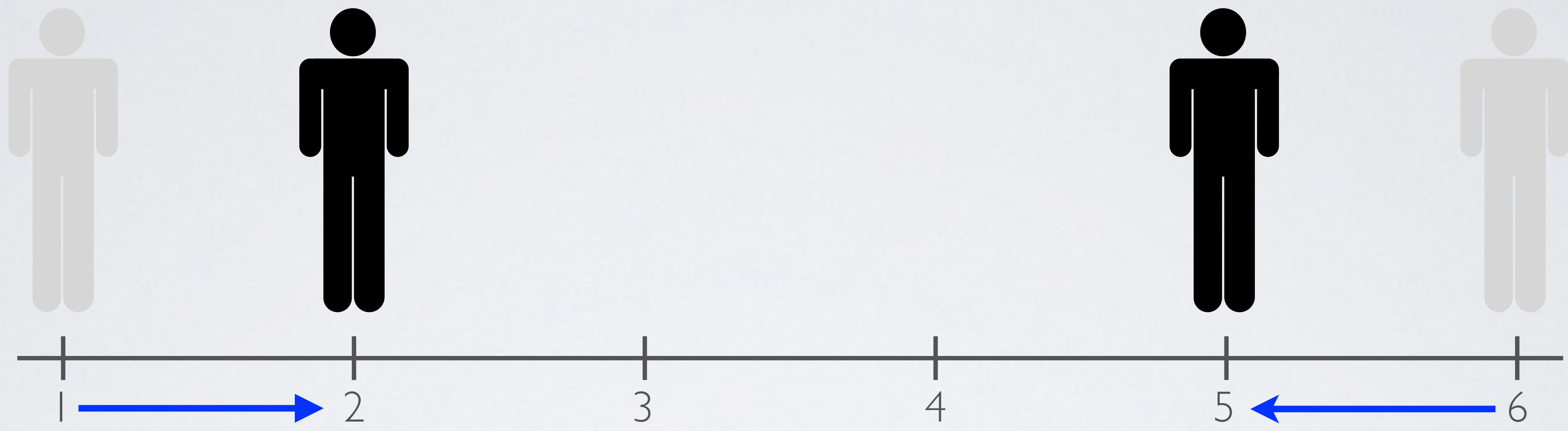
Hammond's transfers



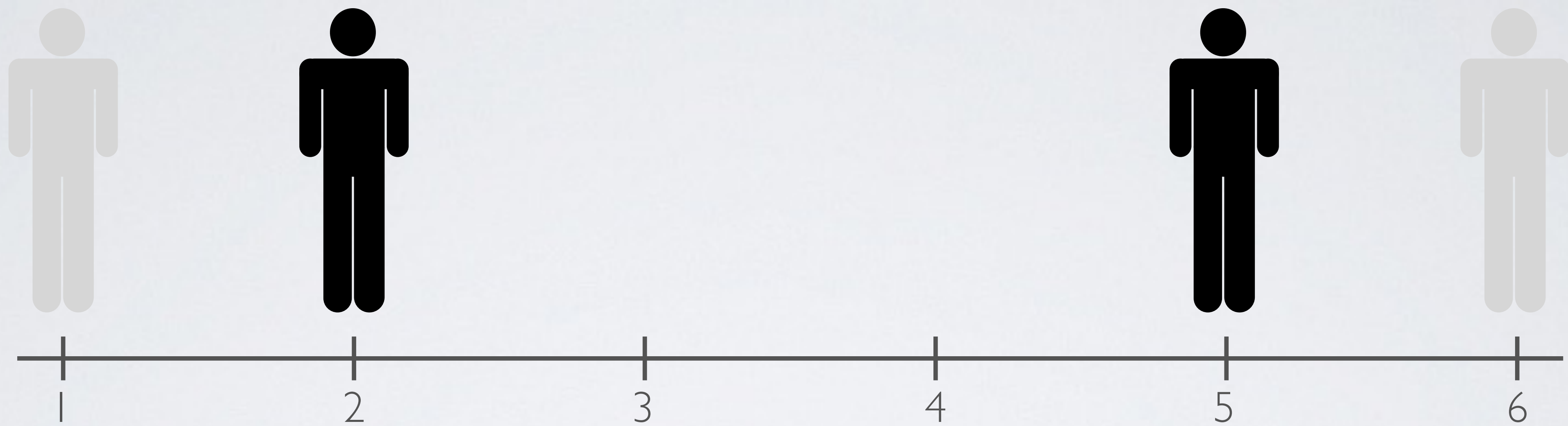
Hammond's transfers



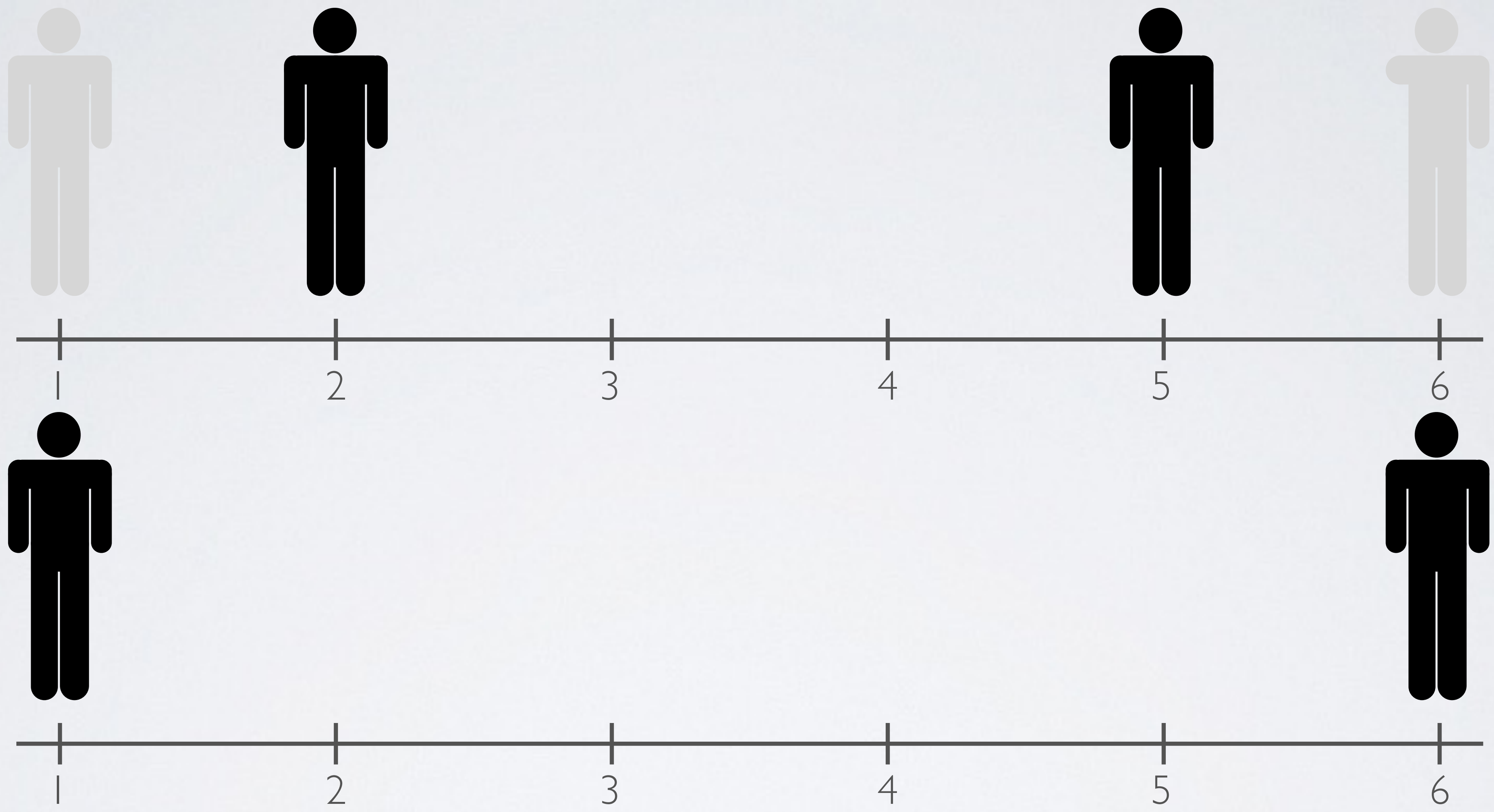
Hammond's transfers



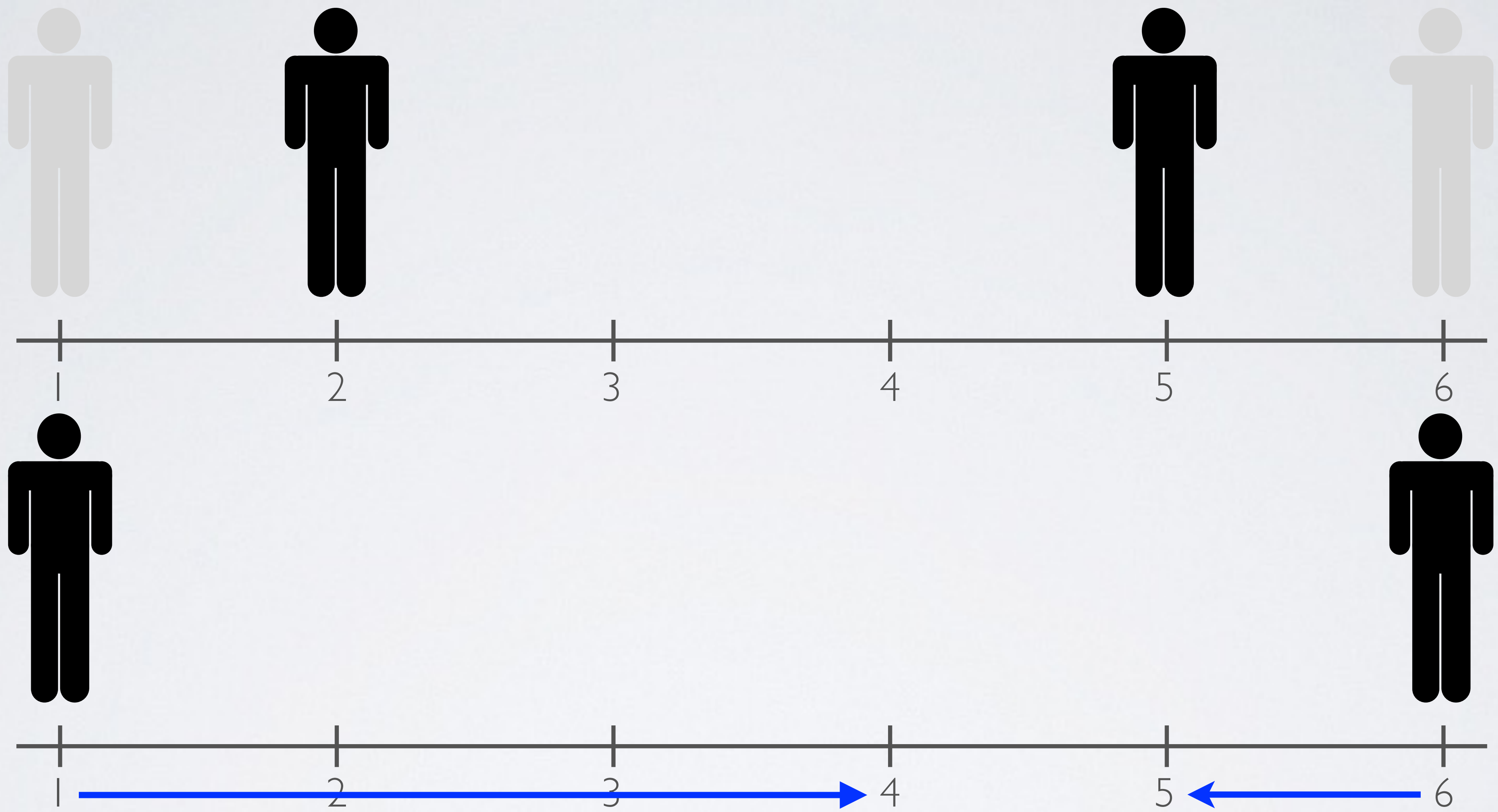
Hammond's transfers



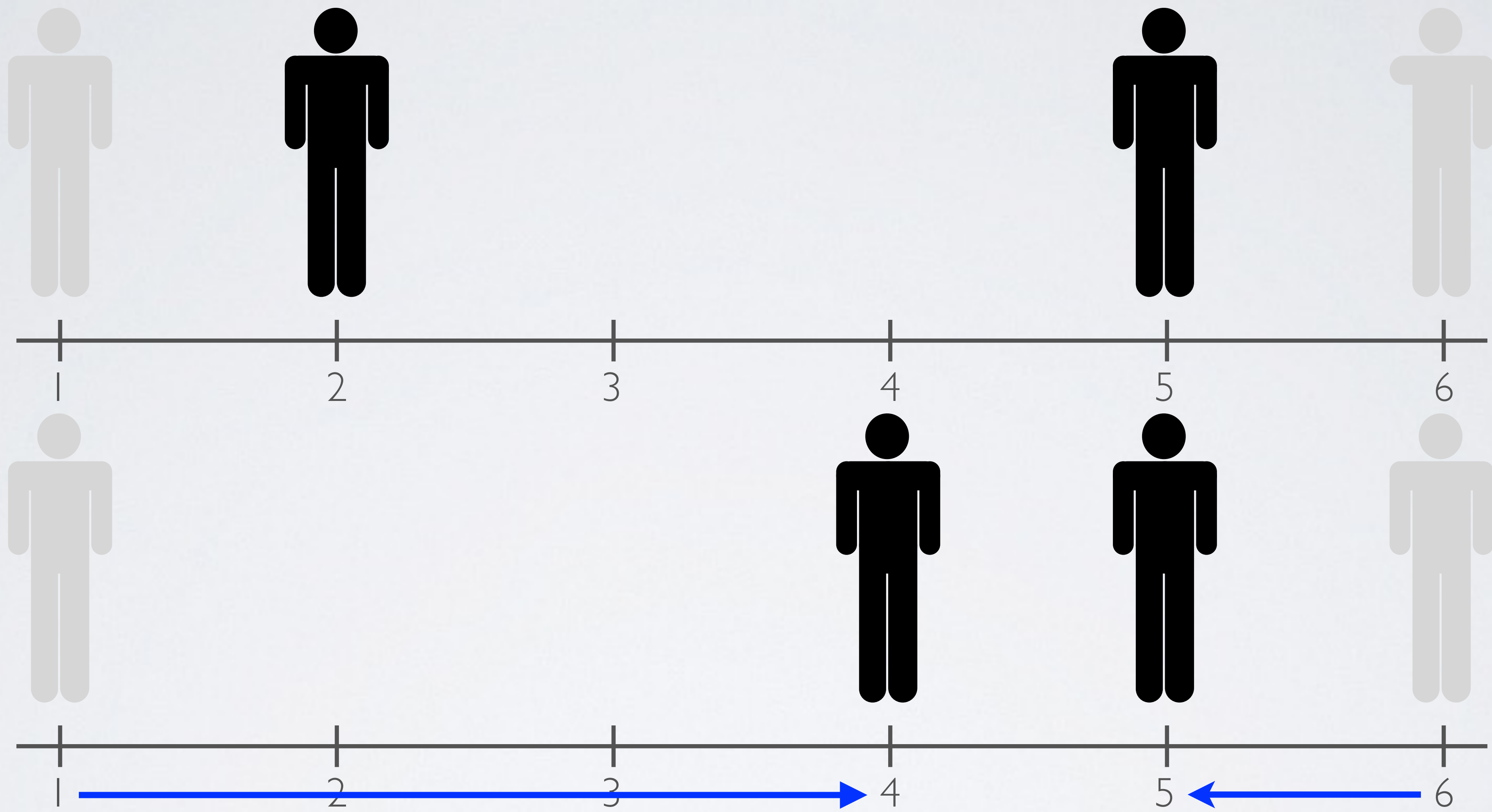
Hammond's transfers



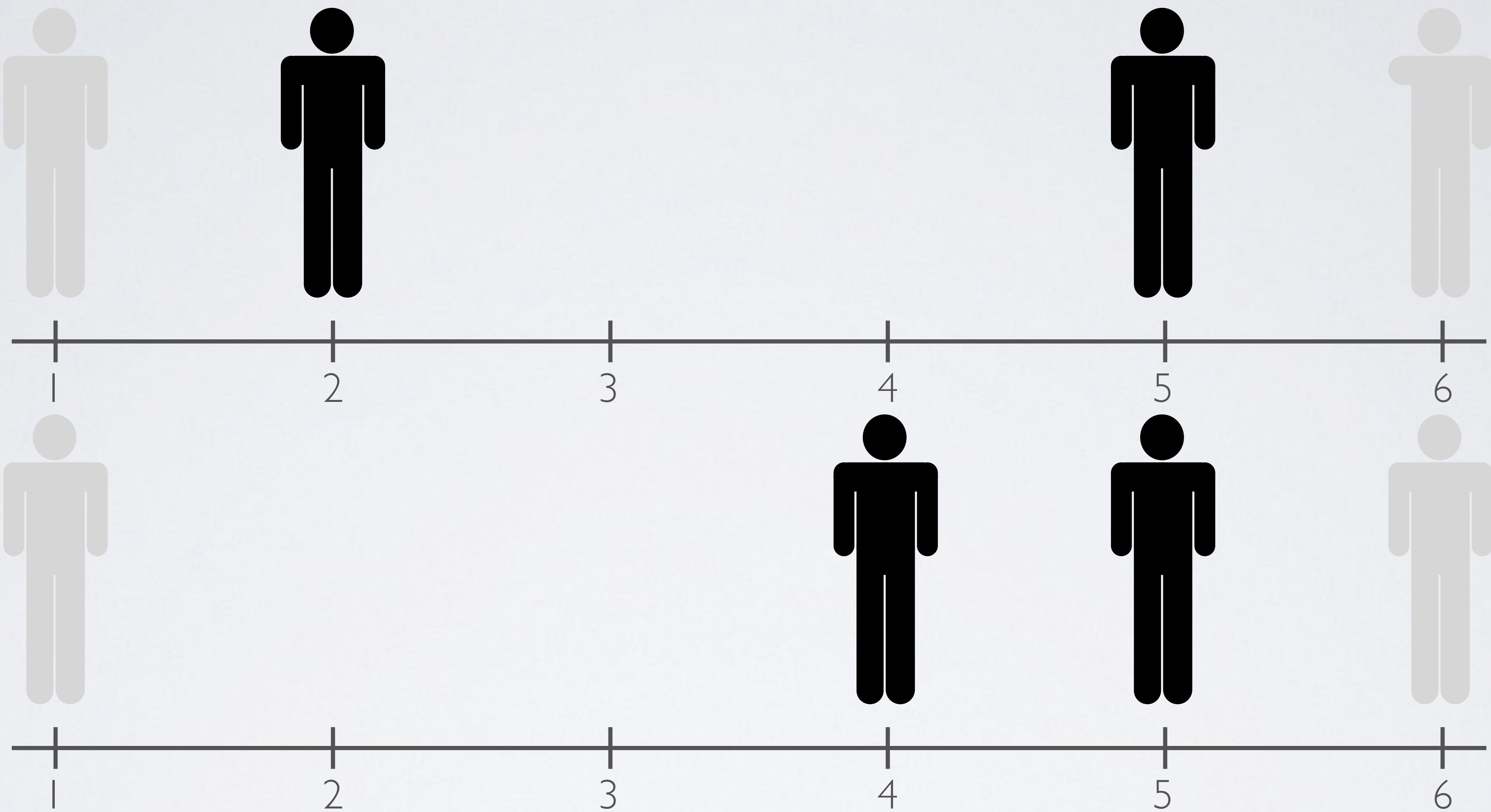
Hammond's transfers



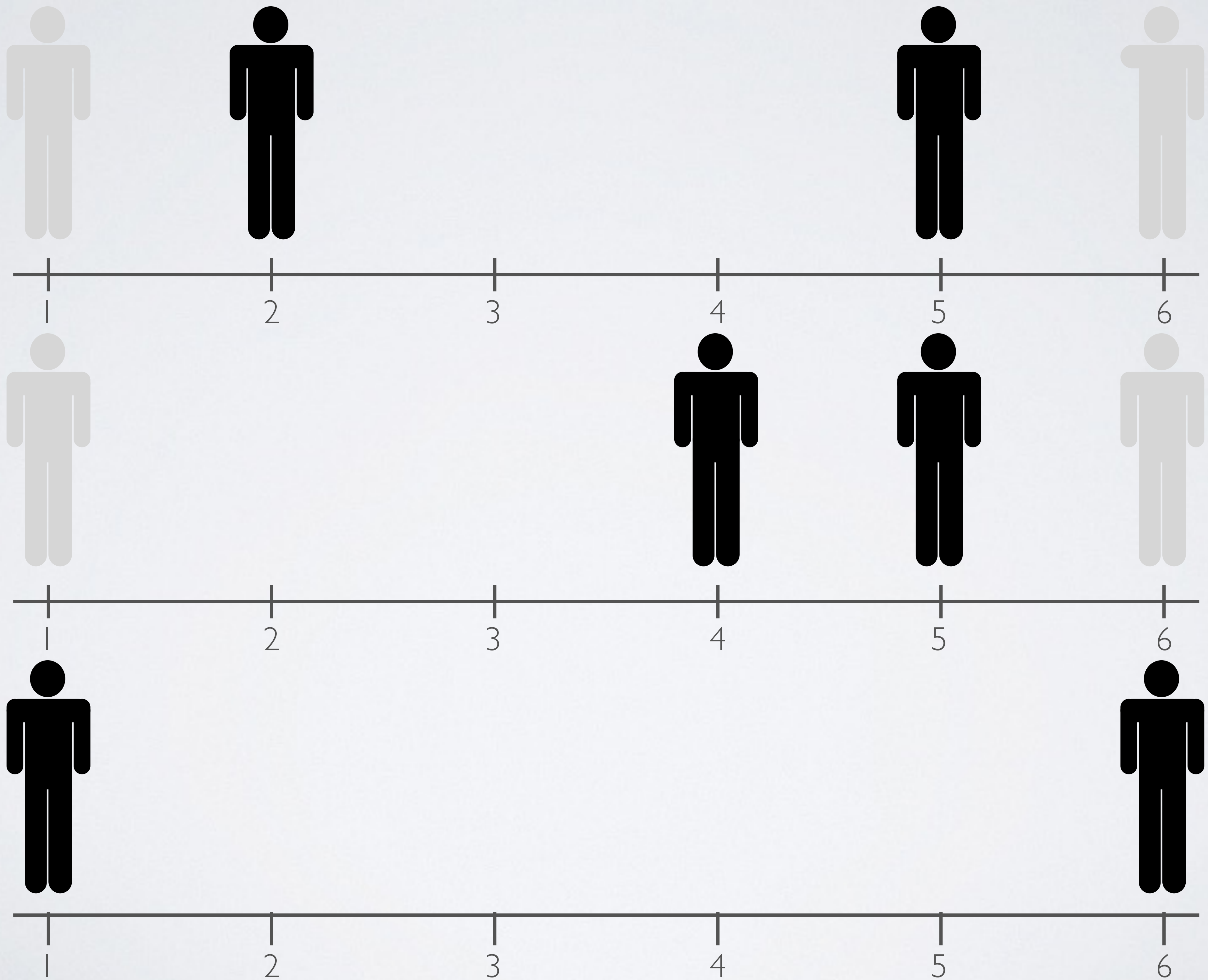
Hammond's transfers



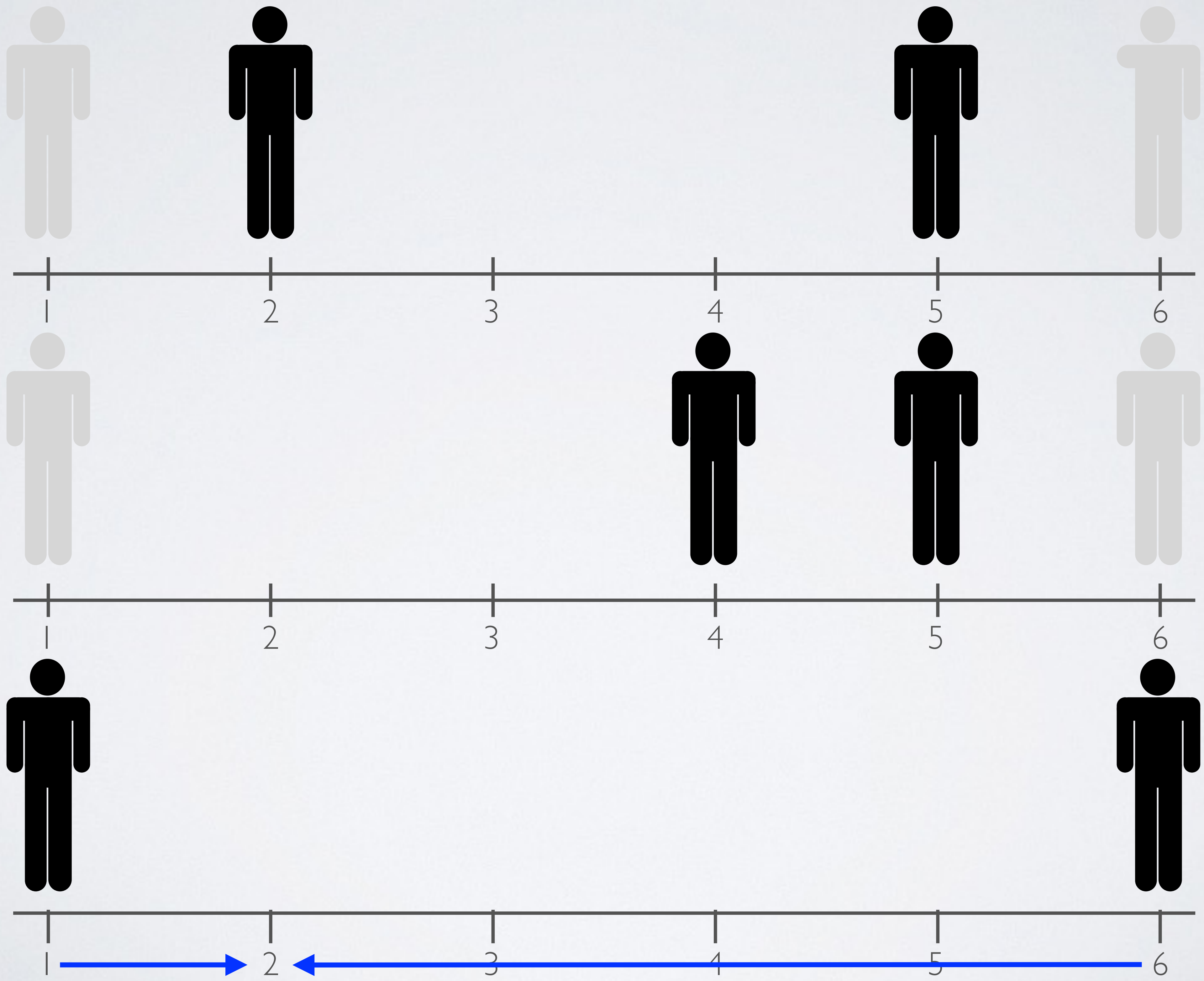
Hammond's transfers



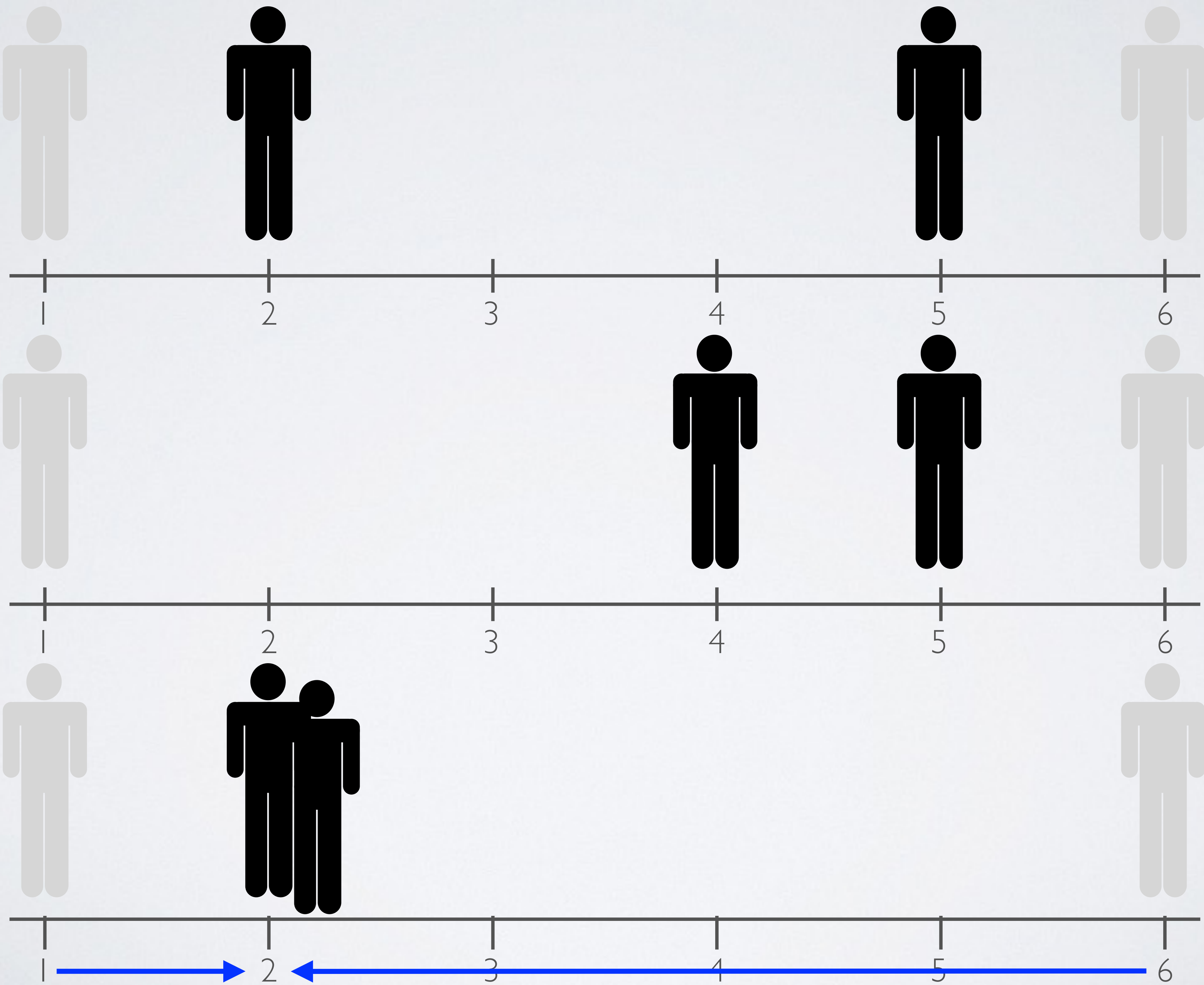
Hammond's transfers



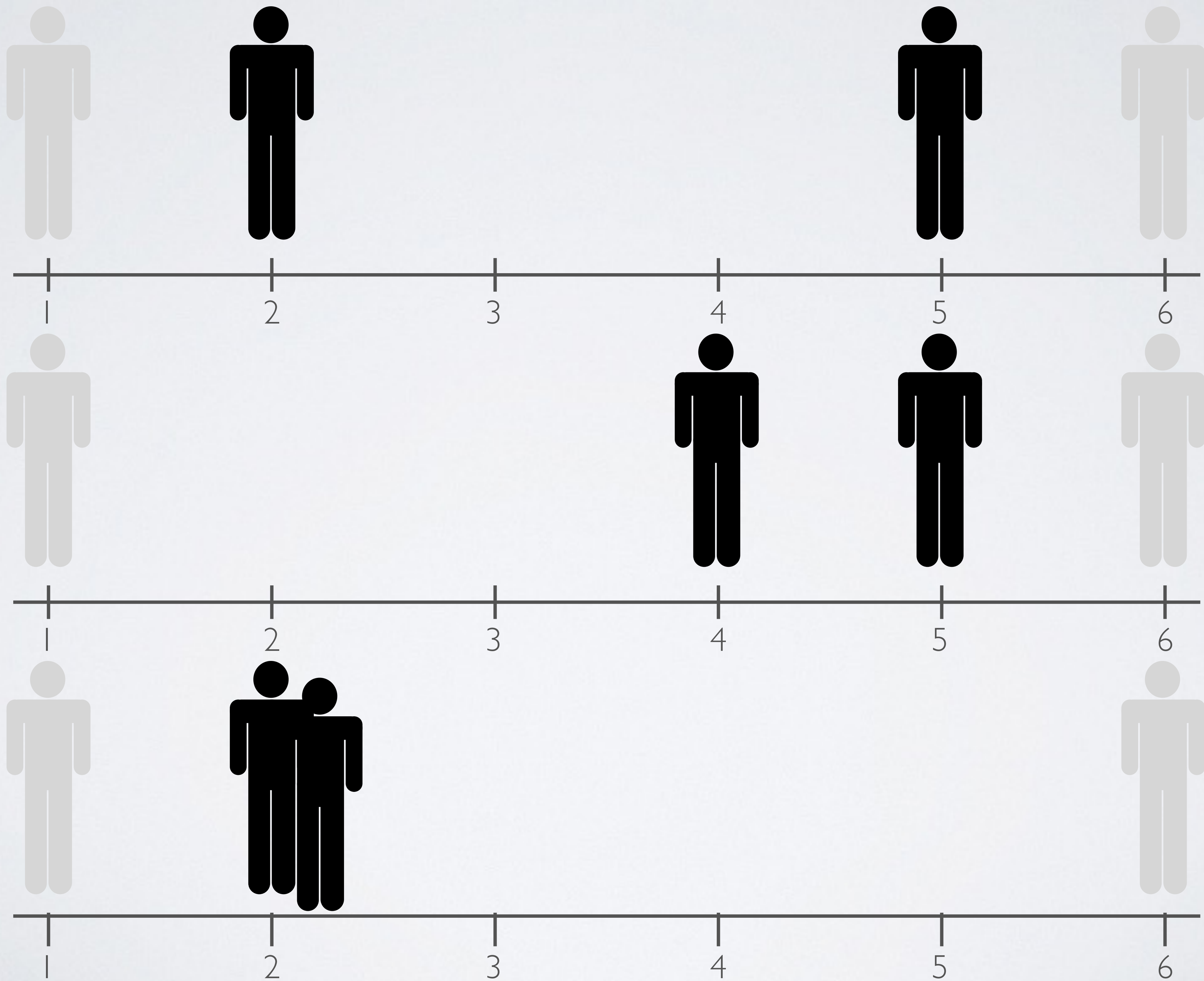
Hammond's transfers



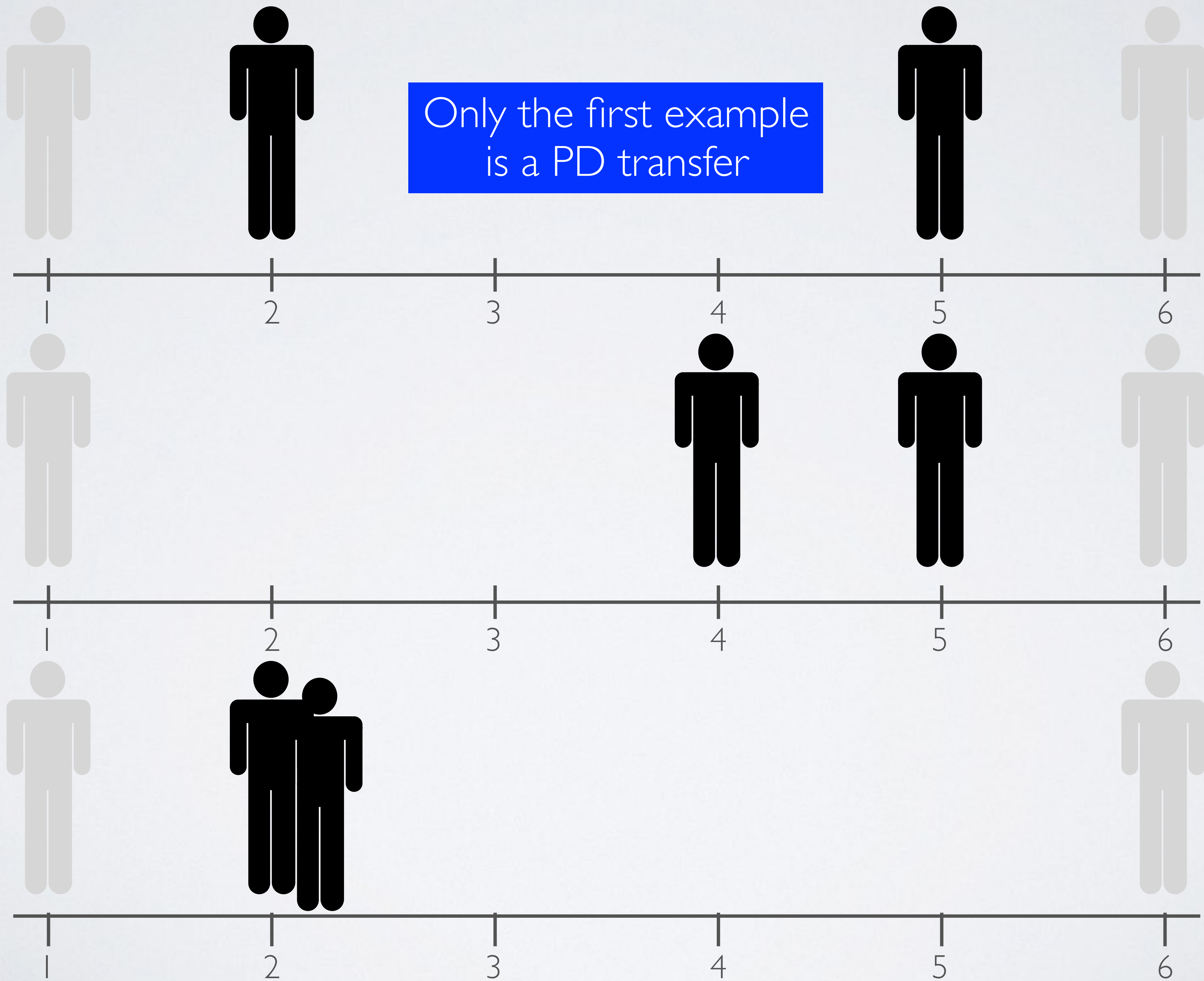
Hammond's transfers



Hammond's transfers



Hammond's transfers



Social welfare functions

We consider the following large class of social welfare functions :

Definition 1. *We say that a society s dominates a society s' for a family $\mathcal{A} \subset \mathbb{R}^k$ of evaluations of the k categories, denoted $s \succeq^{\mathcal{A}} s'$, if one has:*

$$\sum_{j=1}^{k} n_j^s \alpha_j \geq \sum_{j=1}^{k} n_j^{s'} \alpha_j\,, \quad \forall \alpha \in \mathcal{A}.$$

Normative foundations :

- (kind of) utilitarianism: weights interpreted as subjective utilities
- non-welfarist justification (Gravel, Marchand, Sen 2011)

number of individuals in category j, in society s

Social welfare functions

We consider the following large class of social welfare functions :

Definition 1. *We say that a society s dominates a society s' for a family $\mathcal{A} \subset \mathbb{R}^k$ of evaluations of the k categories, denoted $s \succeq^{\mathcal{A}} s'$, if one has:*

$$\sum_{j=1}^{k} n_j^s \alpha_j \geq \sum_{j=1}^{k} n_j^{s'} \alpha_j \,, \quad \forall \alpha \in \mathcal{A} .$$

Normative foundations :

- (kind of) utilitarianism: weights interpreted as subjective utilities
- non-welfarist justification (Gravel, Marchand, Sen 2011)

number of individuals in category j, in society s

Social welfare functions

weight associated to category j

We consider the following large class of social welfare functions :

Definition 1. *We say that a society s dominates a society s' for a family $\mathcal{A} \subset \mathbb{R}^k$ of evaluations of the k categories, denoted $s \succeq^{\mathcal{A}} s'$, if one has:*

$$\sum_{j=1}^{k} n_j^s \alpha_j \geq \sum_{j=1}^{k} n_j^{s'} \alpha_j \,, \quad \forall \alpha \in \mathcal{A}\,.$$

Normative foundations :

- (kind of) utilitarianism: weights interpreted as subjective utilities
- non-welfarist justification (Gravel, Marchand, Sen 2011)

Social welfare functions

We consider the following large class of social welfare functions :

Definition 1. *We say that a society s dominates a society s' for a family $\mathcal{A} \subset \mathbb{R}^k$ of evaluations of the k categories, denoted $s \succeq^{\mathcal{A}} s'$, if one has:*

$$\sum_{j=1}^{k} n_j^s \alpha_j \geq \sum_{j=1}^{k} n_j^{s'} \alpha_j \,, \quad \forall \alpha \in \mathcal{A} \,.$$

Normative foundations :

- (kind of) utilitarianism: weights interpreted as subjective utilities
- non-welfarist justification (Gravel, Marchand, Sen 2011)

Two particular subclasses will be of importance :

$$\mathcal{A}_1 = \{\alpha \in \mathcal{A} \,|\, \alpha_i \leq \alpha_j, \, 1 \leq i < j \leq k\}$$

$$\mathcal{A}_2 = \{\alpha \in \mathcal{A} \,|\, (\alpha_h - \alpha_g) \geq (\alpha_j - \alpha_i), \, 1 \leq g < h \leq i < j \leq k\}$$

The H-curve

The implementation criterion is based on the following curve :

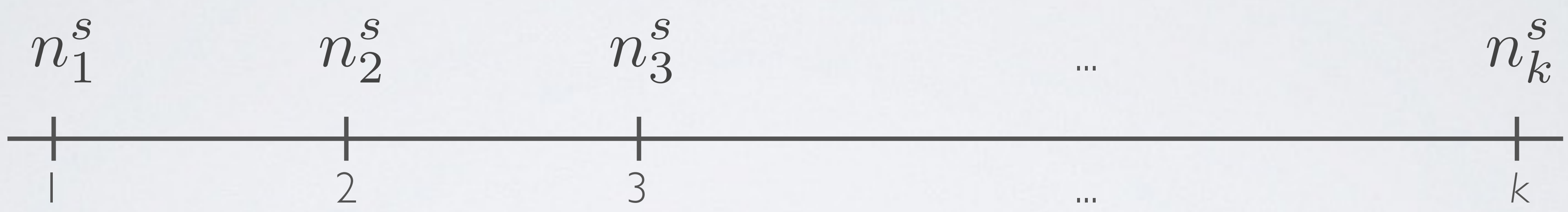
$$H(i;s) = \frac{1}{n} \sum_{h=1}^{i} \left(2^{i-h}\right) n_h\,, \ \forall i = 1, 2, \ldots, k$$

This curve is really easy to compute. We have :

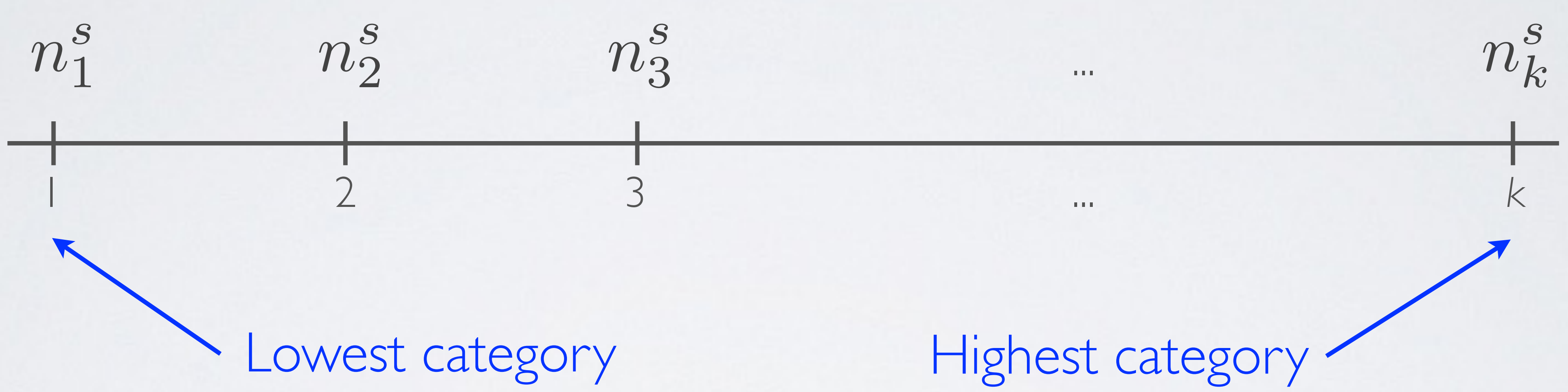
$$H(1;s) = F(1;s)$$ and $$H(i;s) = 2\,H(i-1;s) + \frac{n_i}{n}\,, \ \forall i = 2, 3, \ldots, k$$

Dominance: a society *s* dominates a society *s'* if the H-curve for society *s* lies nowhere above that of *s'*

The H-curve

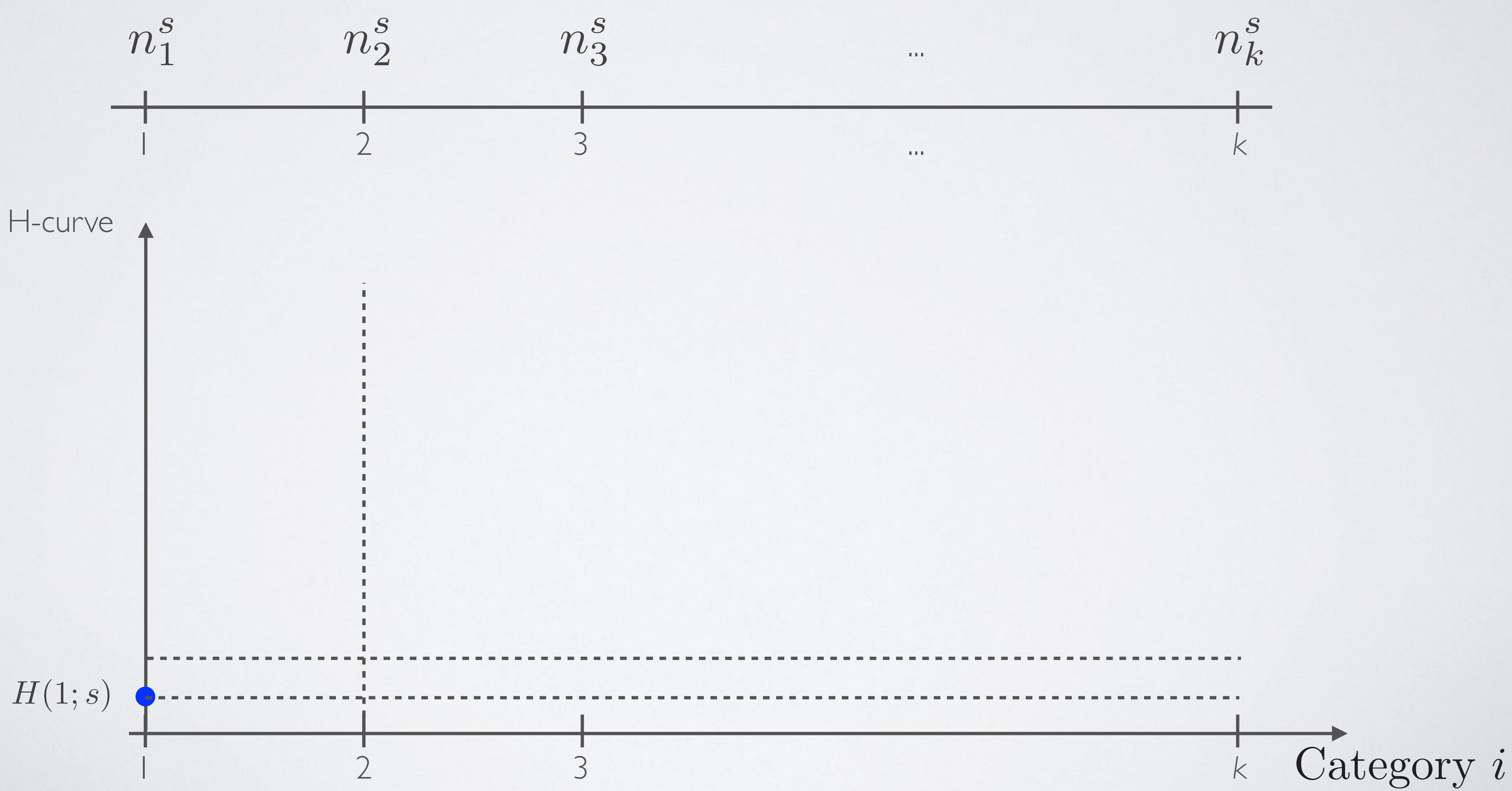


The H-curve

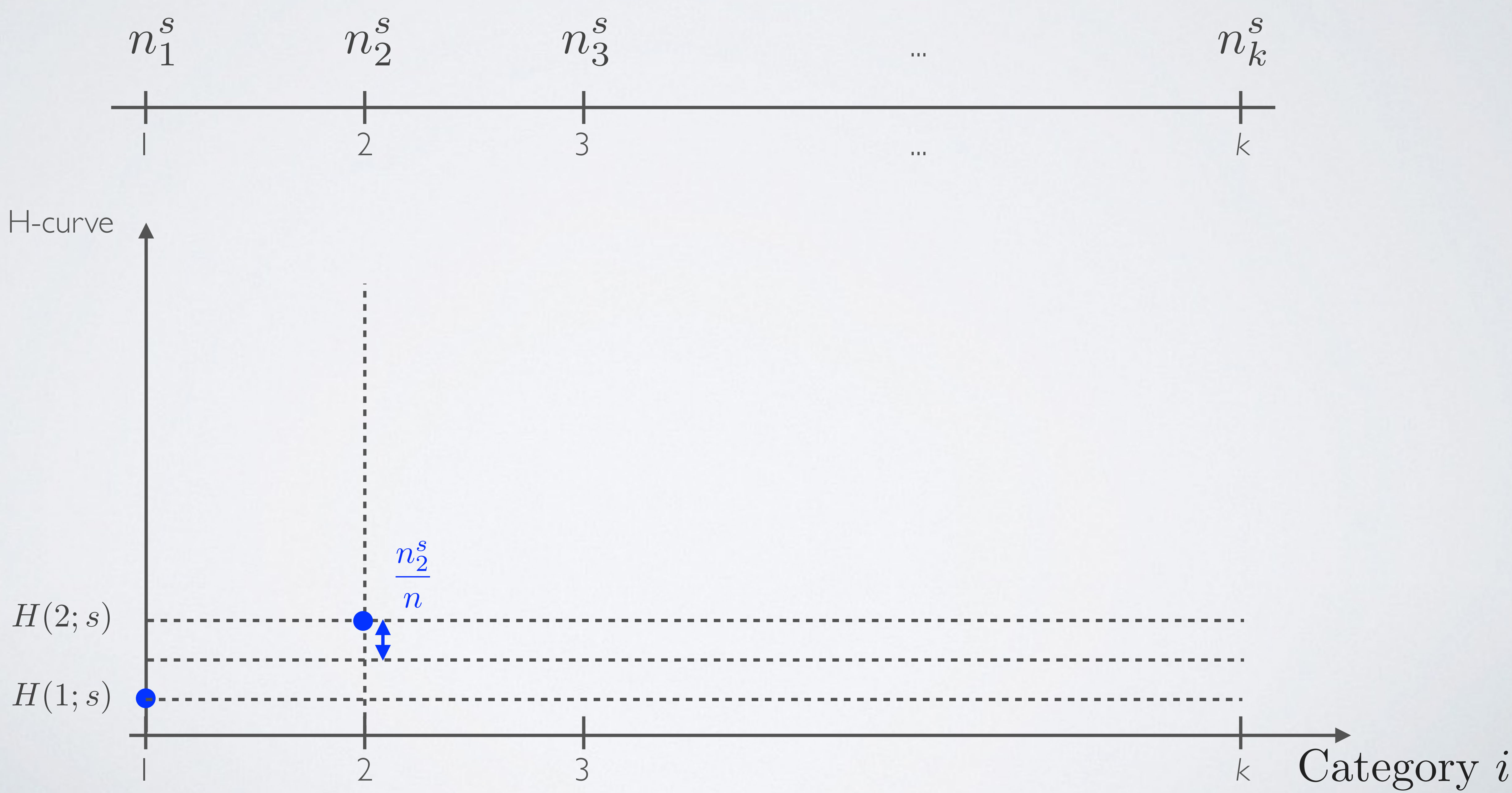


The H-curve

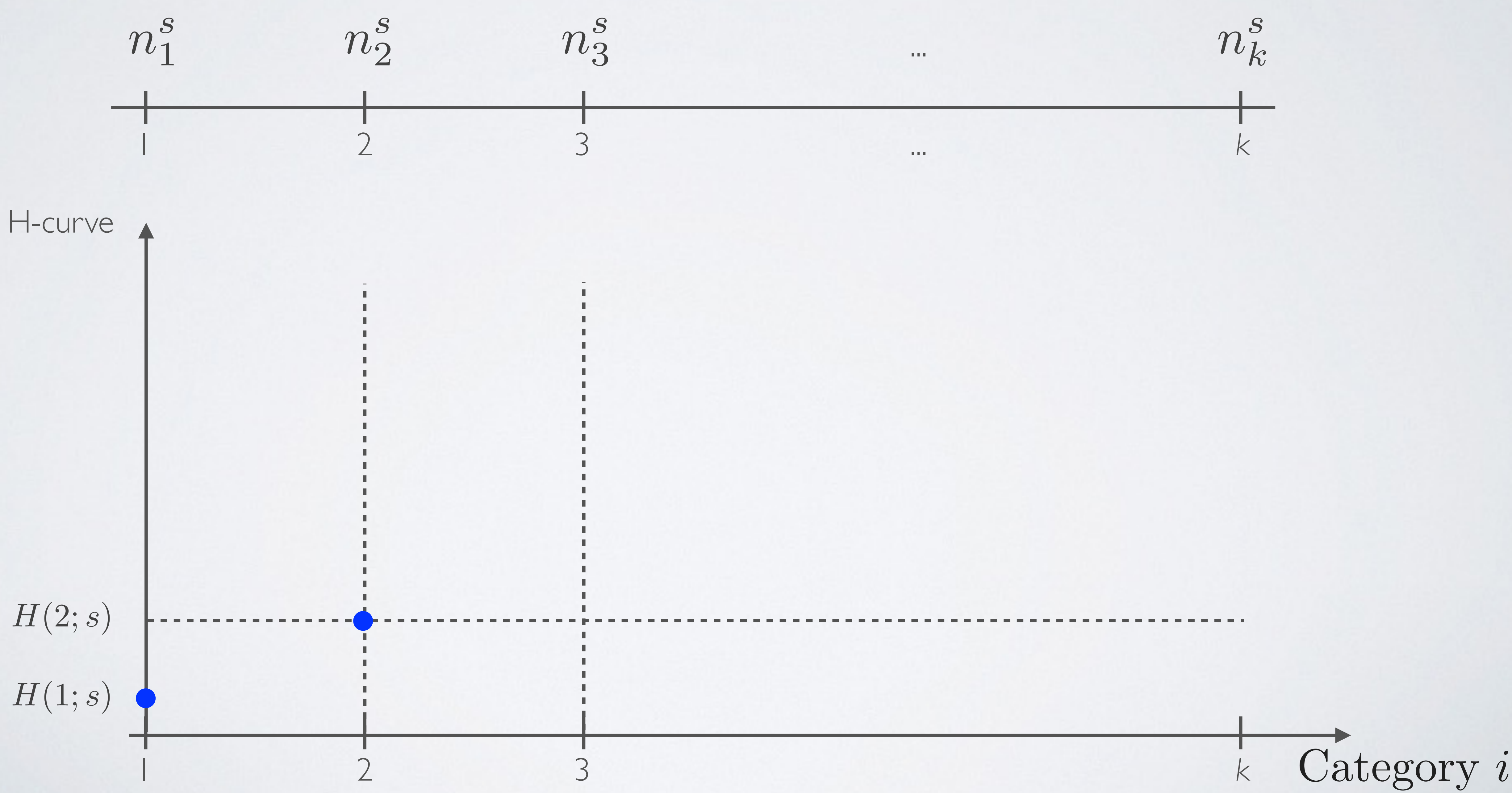
The H-curve



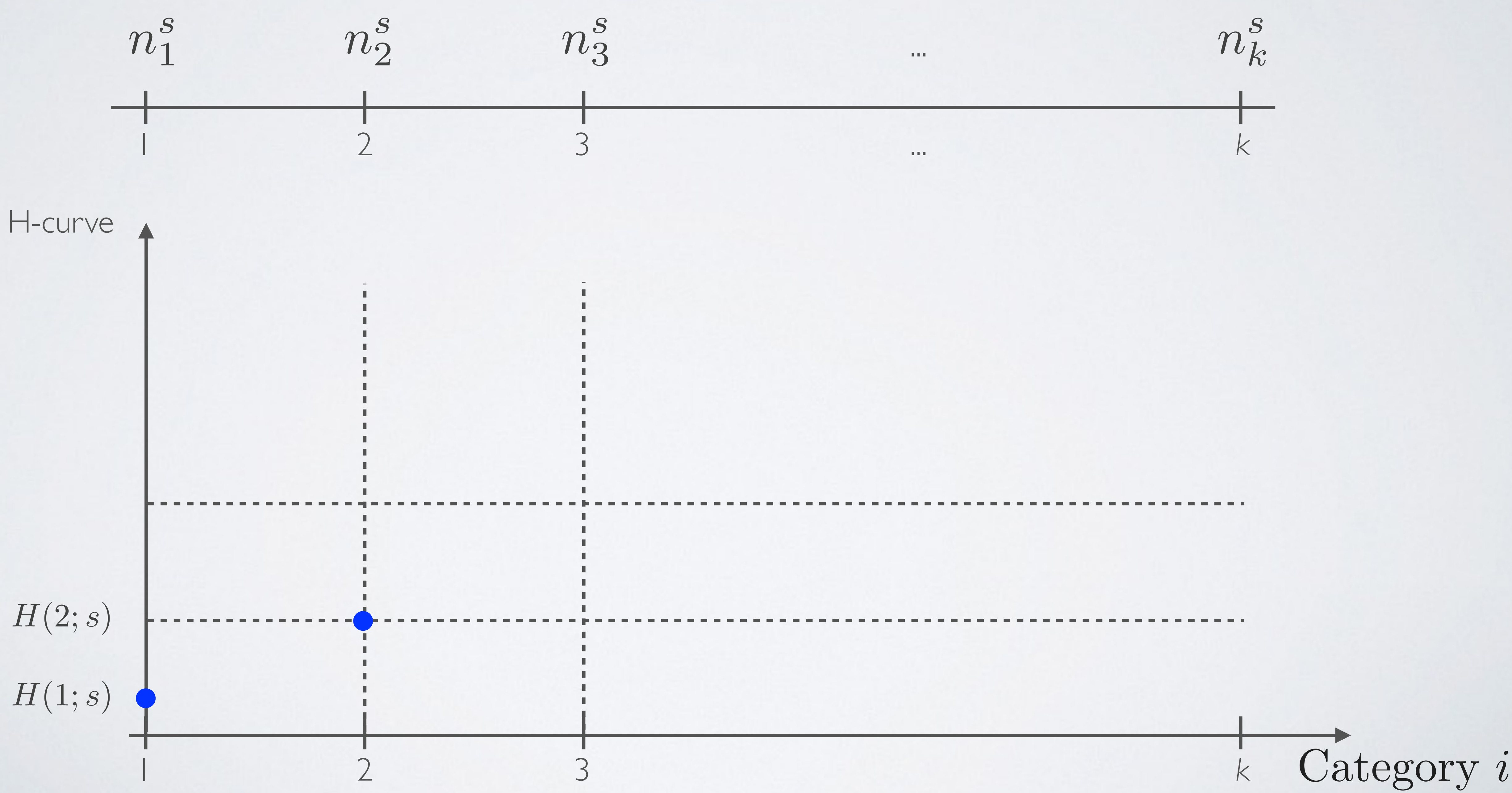
The H-curve



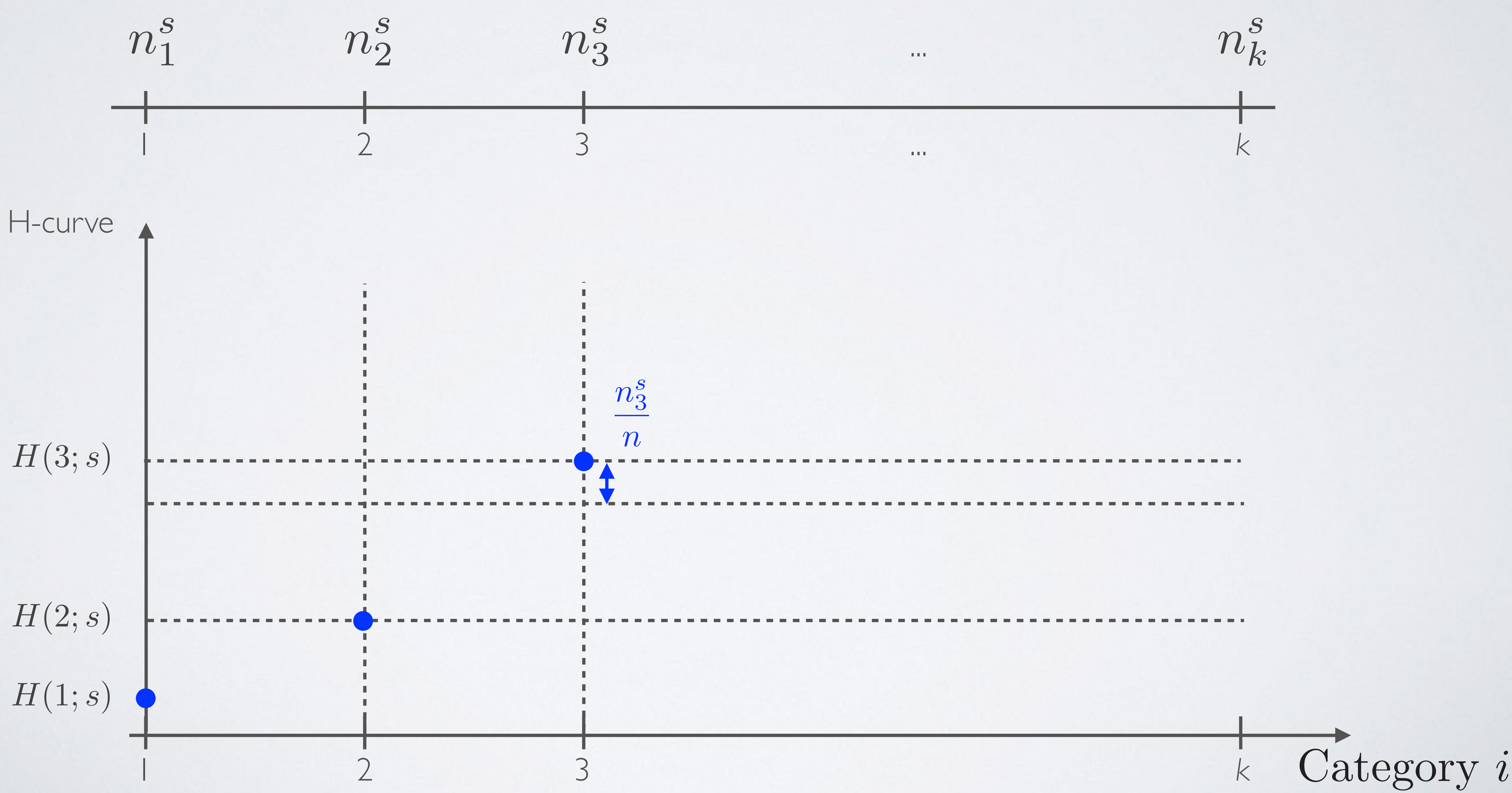
The H-curve



The H-curve

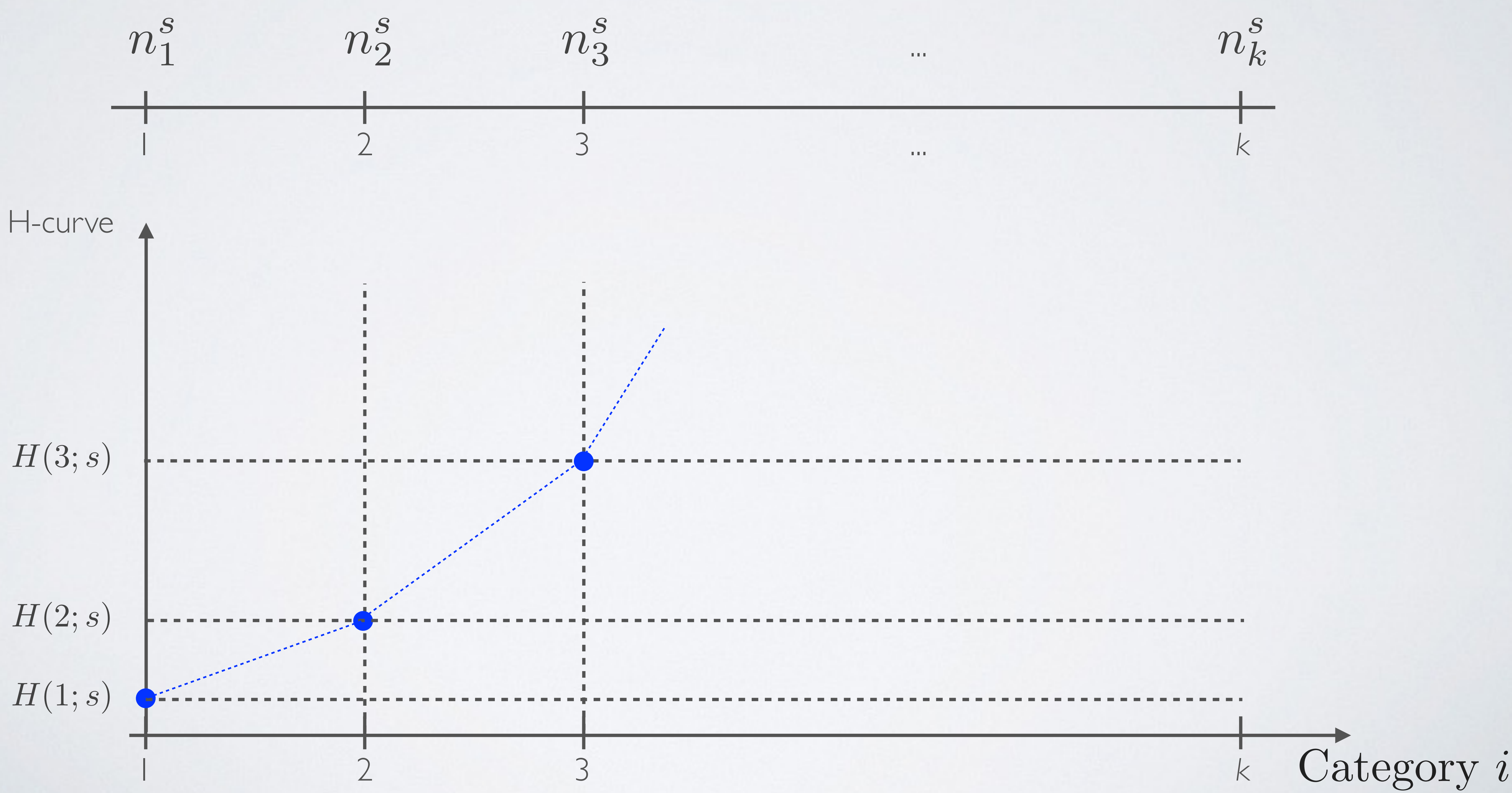


The H-curve



n_1^s
n_2^s
n_3^s
...
n_k^s
1
2
3
...
k
H-curve
$\frac{n_3^s}{n}$
$H(3; s)$
$H(2; s)$
$H(1; s)$
1
2
3
k
Category i

The H-curve



Results

Social welfare: efficiency and equity

Theorem 2. *For all distributions $s, s' \in C^n$, the following three statements are equivalent:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers and/or increments,*

(b) $\sum_{h=1}^{k} n_h^s \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \alpha_h$ *for all* $\alpha \in \mathcal{A}_1 \bigcap \mathcal{A}_2$,

(c) $H(i; s) \leq H(i; s')$ *for all* $i = 1, 2, \ldots, k-1$.

Social welfare: efficiency and equity

Theorem 2. *For all distributions $s, s' \in C^n$, the following three statements are equivalent:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers and/or increments,*

(b) $\sum_{h=1}^{k} n_h^s \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \alpha_h$ *for all* $\alpha \in \mathcal{A}_1 \bigcap \mathcal{A}_2$,

(c) $H(i; s) \leq H(i; s')$ *for all* $i = 1, 2, \ldots, k-1$.

transparent definition of efficiency and equity

Social welfare: efficiency and equity

Theorem 2. *For all distributions $s, s' \in \mathcal{C}^n$, the following three statements are equivalent:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers and/or increments,*

(b) $\sum_{h=1}^{k} n_h^s \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \alpha_h$ *for all* $\alpha \in \mathcal{A}_1 \bigcap \mathcal{A}_2$,

(c) $H(i; s) \leq H(i; s')$ *for all* $i = 1, 2, \ldots, k-1$.

transparent definition of efficiency and equity

robust ethical foundation for the SWF

Social welfare: efficiency and equity

Theorem 2. *For all distributions $s, s' \in C^n$, the following three statements are equivalent:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers and/or increments,*

(b) $\sum_{h=1}^{k} n_h^s \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \alpha_h$ *for all* $\alpha \in \mathcal{A}_1 \bigcap \mathcal{A}_2$,

(c) $H(i; s) \leq H(i; s')$ *for all* $i = 1, 2, \ldots, k-1$.

transparent definition of efficiency and equity

robust ethical foundation for the SWF

implementation criterion

Social welfare: efficiency and equity

Theorem 2. *For all distributions $s, s' \in C^n$, the following three statements are equivalent:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers and/or increments,*

(b) $\sum_{h=1}^{k} n_h^s \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \alpha_h$ *for all* $\alpha \in \mathcal{A}_1 \bigcap \mathcal{A}_2$,

(c) $H(i; s) \leq H(i; s')$ *for all* $i = 1, 2, \ldots, k-1$.

Social welfare: efficiency and equity

Theorem 2. *For all distributions $s, s' \in C^n$, the following three statements are equivalent:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers and/or increments,*

(b) $\sum_{h=1}^{k} n_h^s \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \alpha_h$ *for all* $\alpha \in \mathcal{A}_1 \bigcap \mathcal{A}_2$,

(c) $H(i; s) \leq H(i; s')$ *for all* $i = 1, 2, \ldots, k-1$.

increasing and « strongly » concave

Social welfare: efficiency and equity

Theorem 2. *For all distributions $s, s' \in C^n$, the following three statements are equivalent:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers and/or increments,*

(b) $\sum_{h=1}^{k} n_h^s \, \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \, \alpha_h$ *for all* $\alpha \in \mathcal{A}_1 \bigcap \mathcal{A}_2$,

(c) $H(i; s) \leq H(i; s')$ *for all* $i = 1, 2, \ldots, k-1$.

increasing and
« strongly » concave

Remark 1. *We have* $\alpha \in \mathcal{A}_1 \bigcap \mathcal{A}_2$ *iff* $(\alpha_{i+1} - \alpha_i) \geq (\alpha_k - \alpha_{i+1})$ *for all* $i = 1, 2, \ldots, k-1$.

Illustration of « strong » concavity

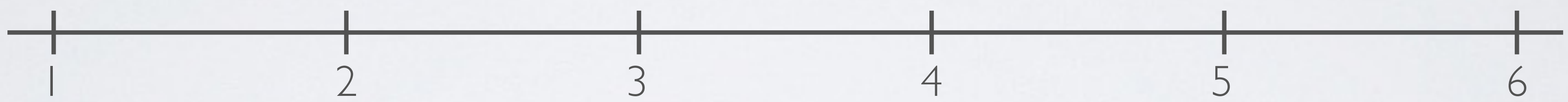


Illustration of « strong » concavity

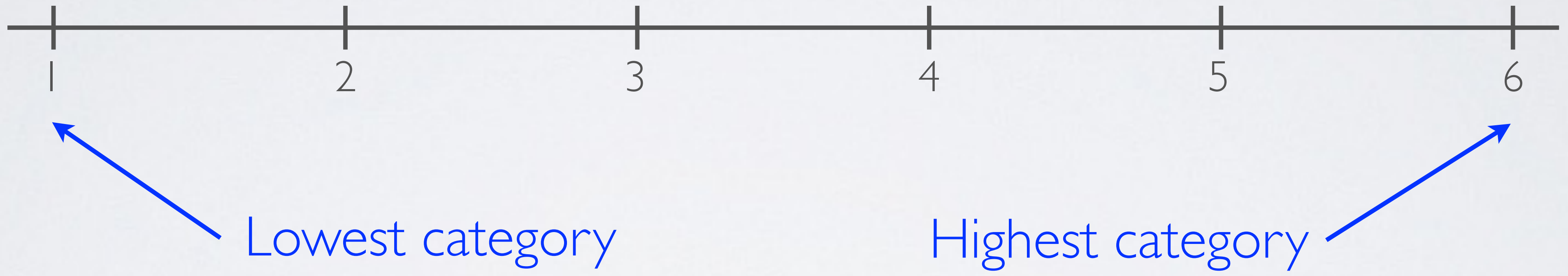


Illustration of « strong » concavity

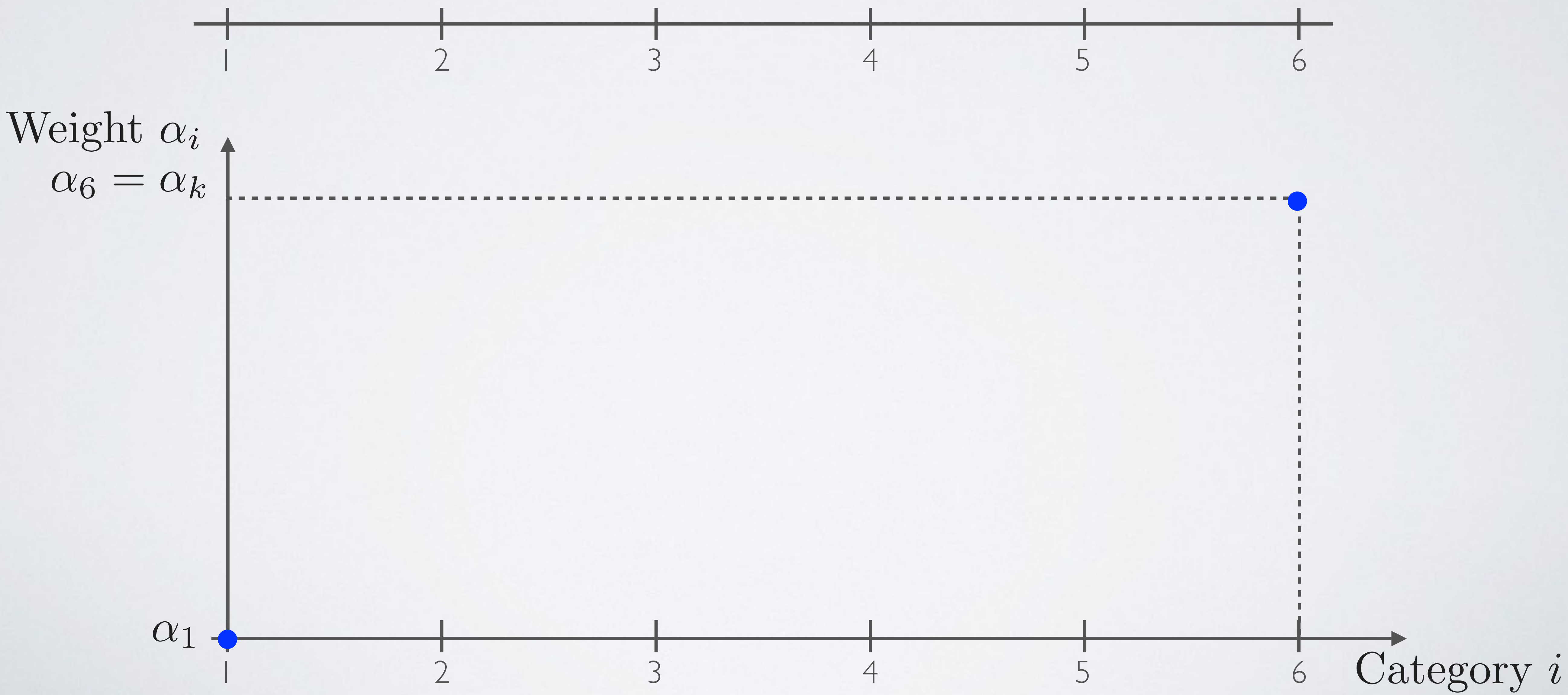


Illustration of « strong » concavity

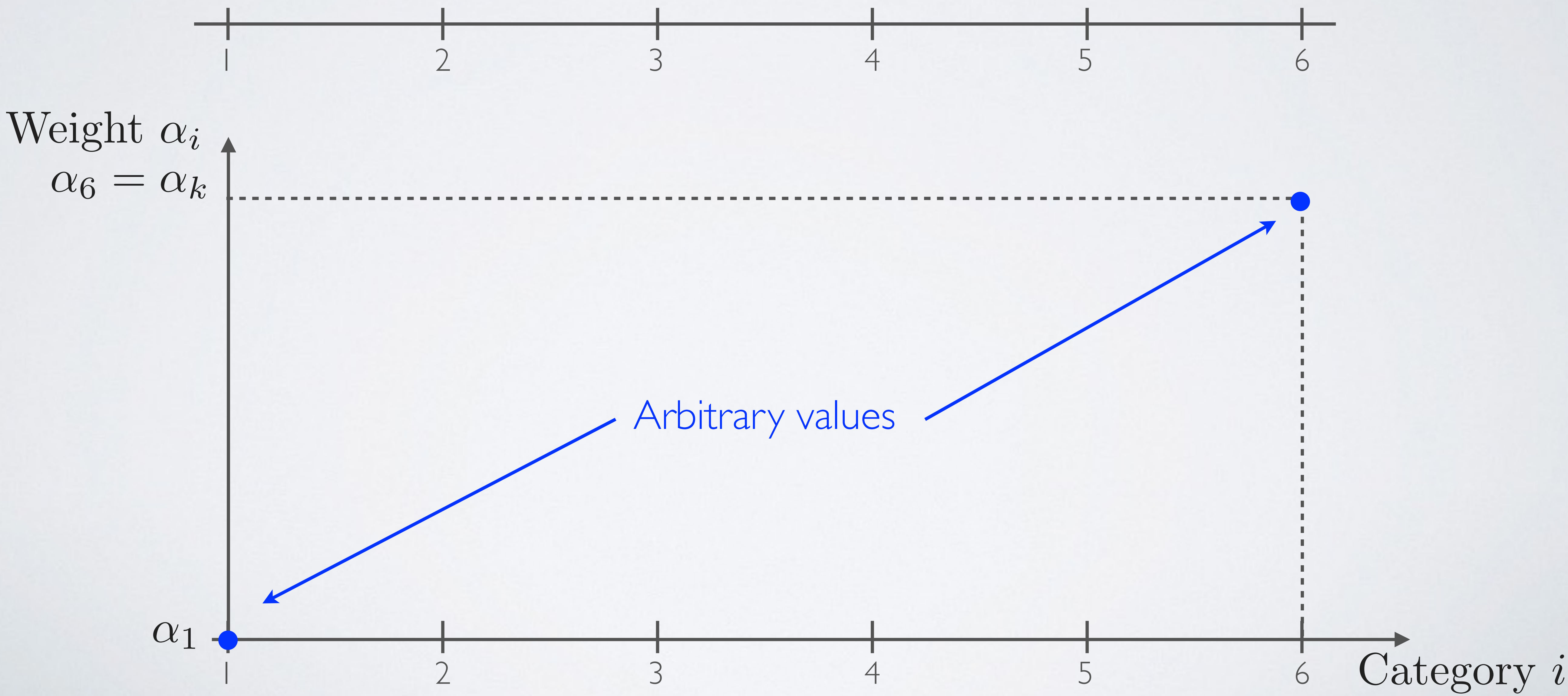
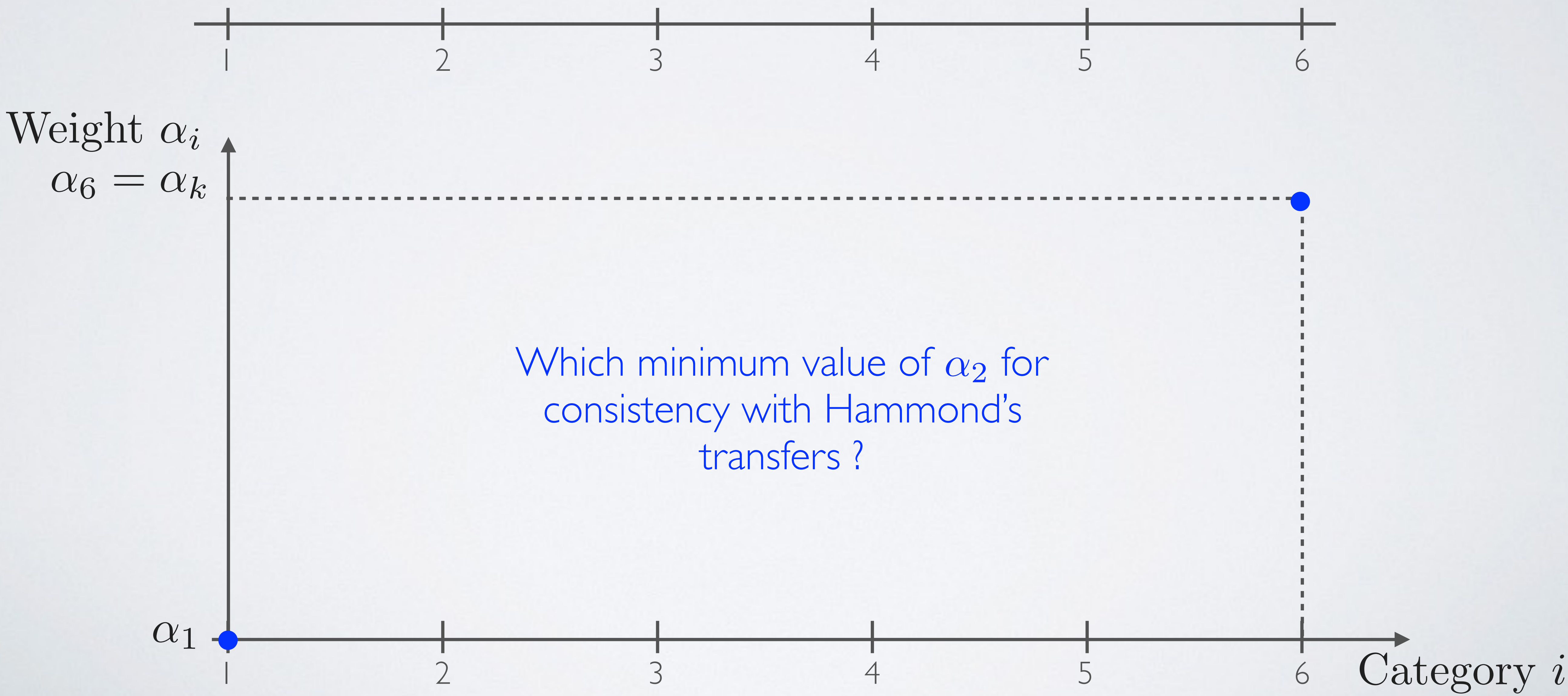


Illustration of « strong » concavity



Which minimum value of α_2 for consistency with Hammond's transfers ?

Illustration of « strong » concavity

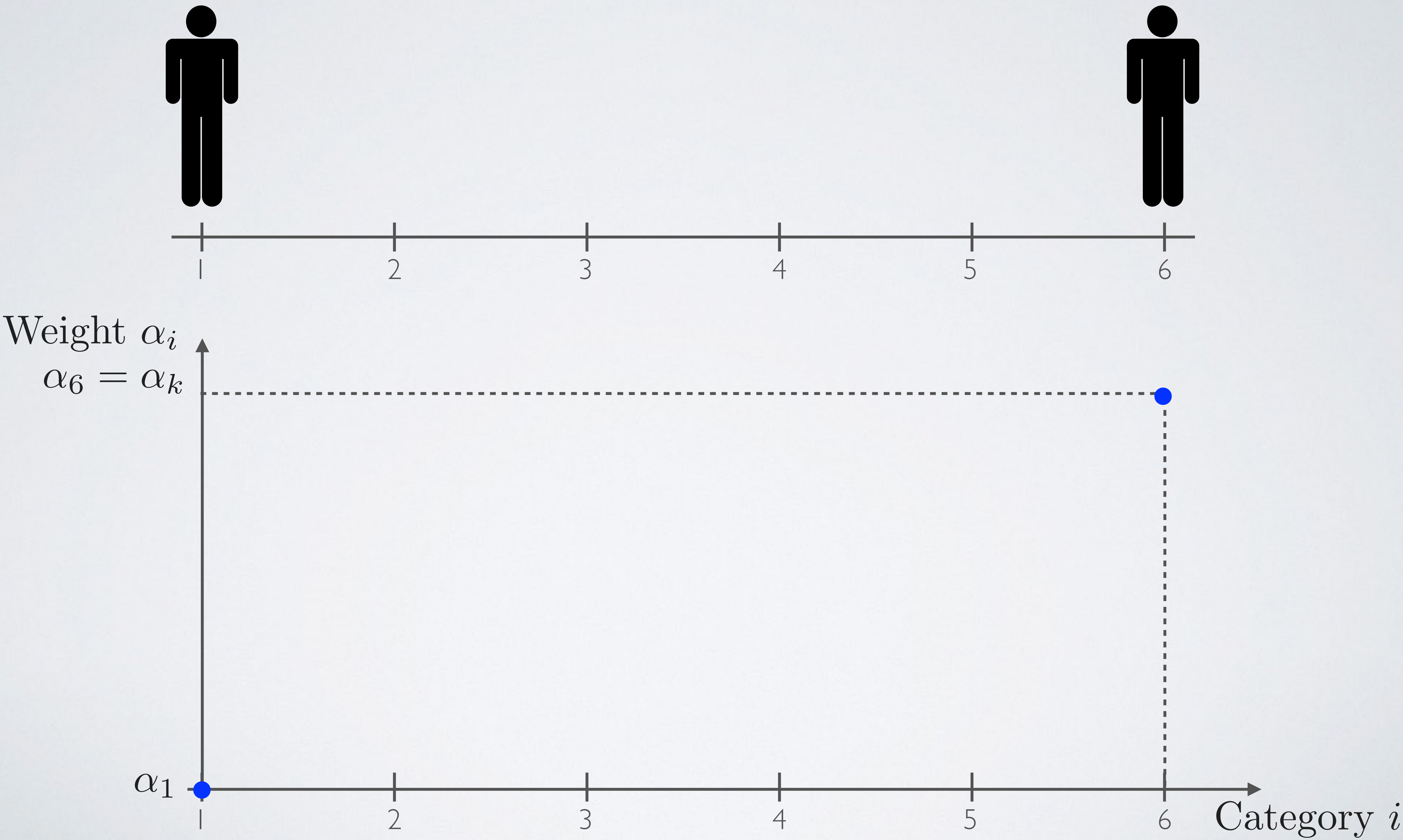


Illustration of « strong » concavity

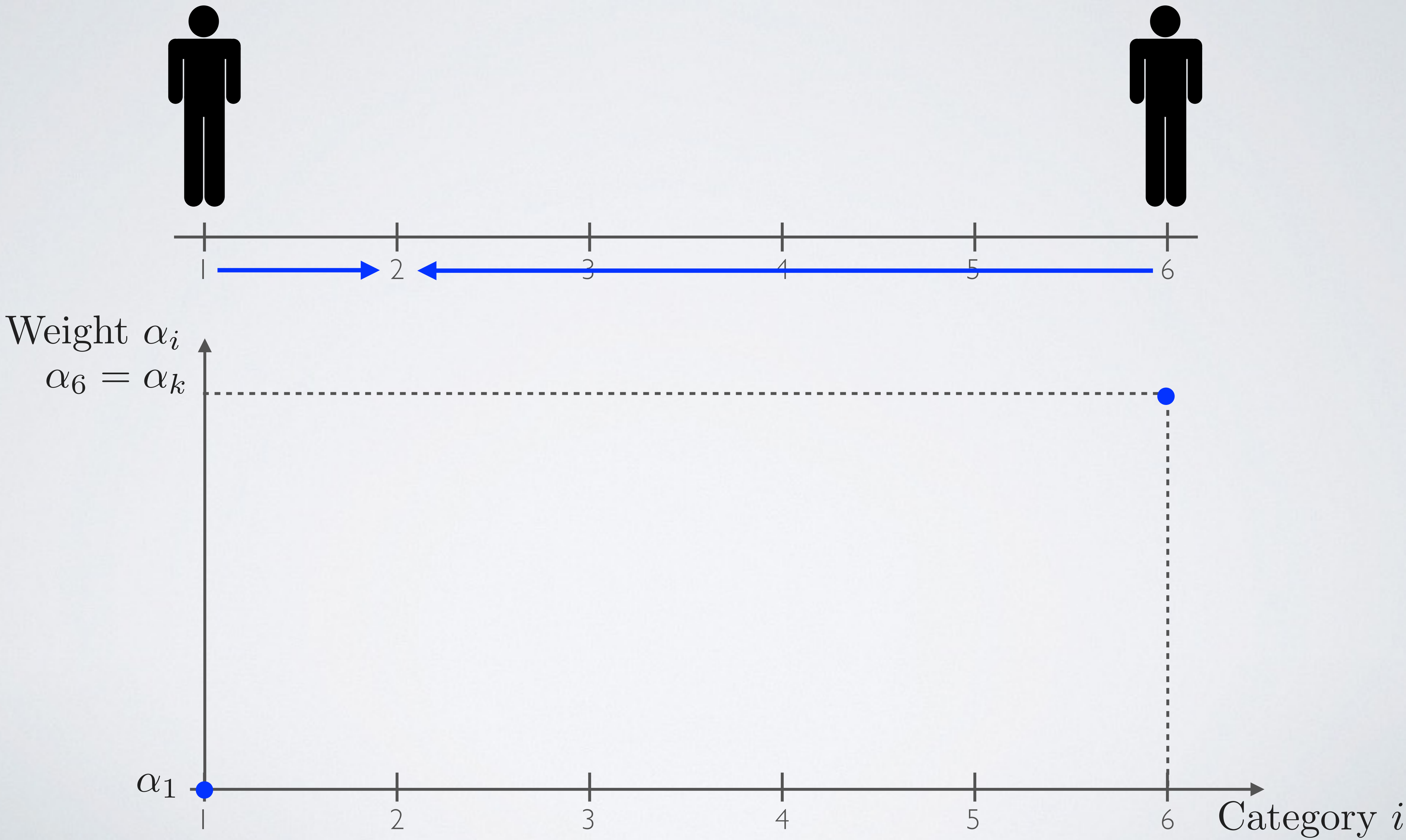


Illustration of « strong » concavity

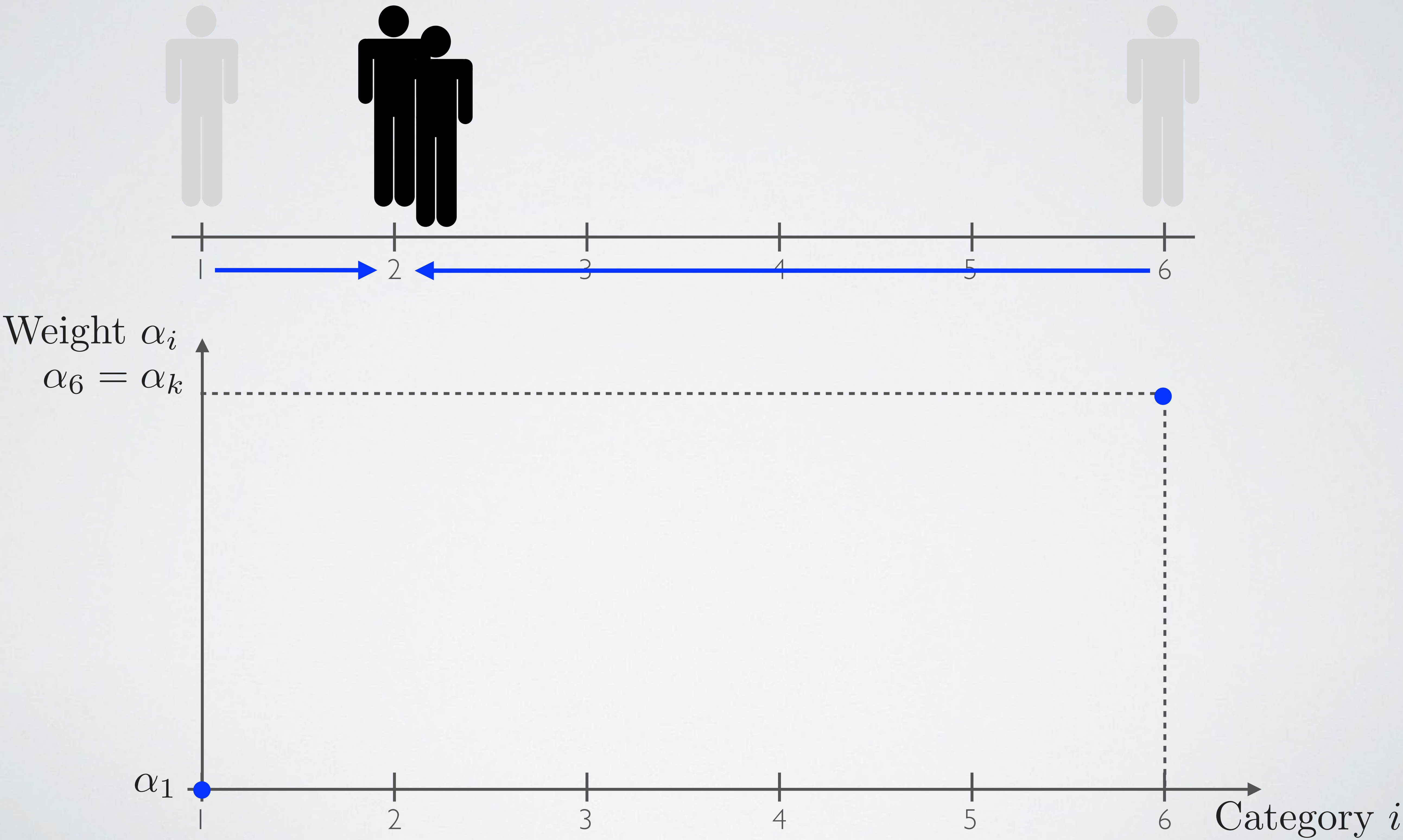


Illustration of « strong » concavity

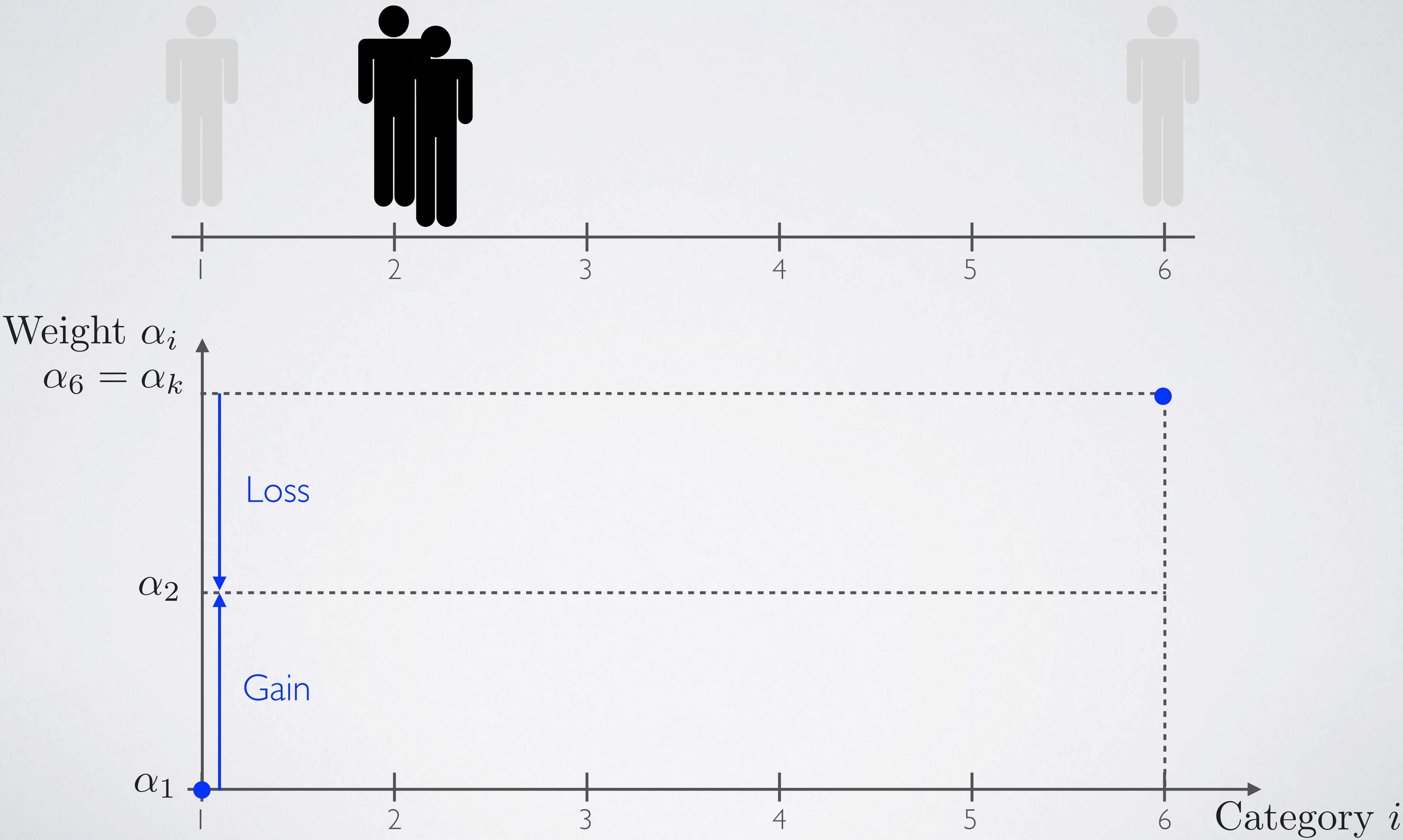


Illustration of « strong » concavity

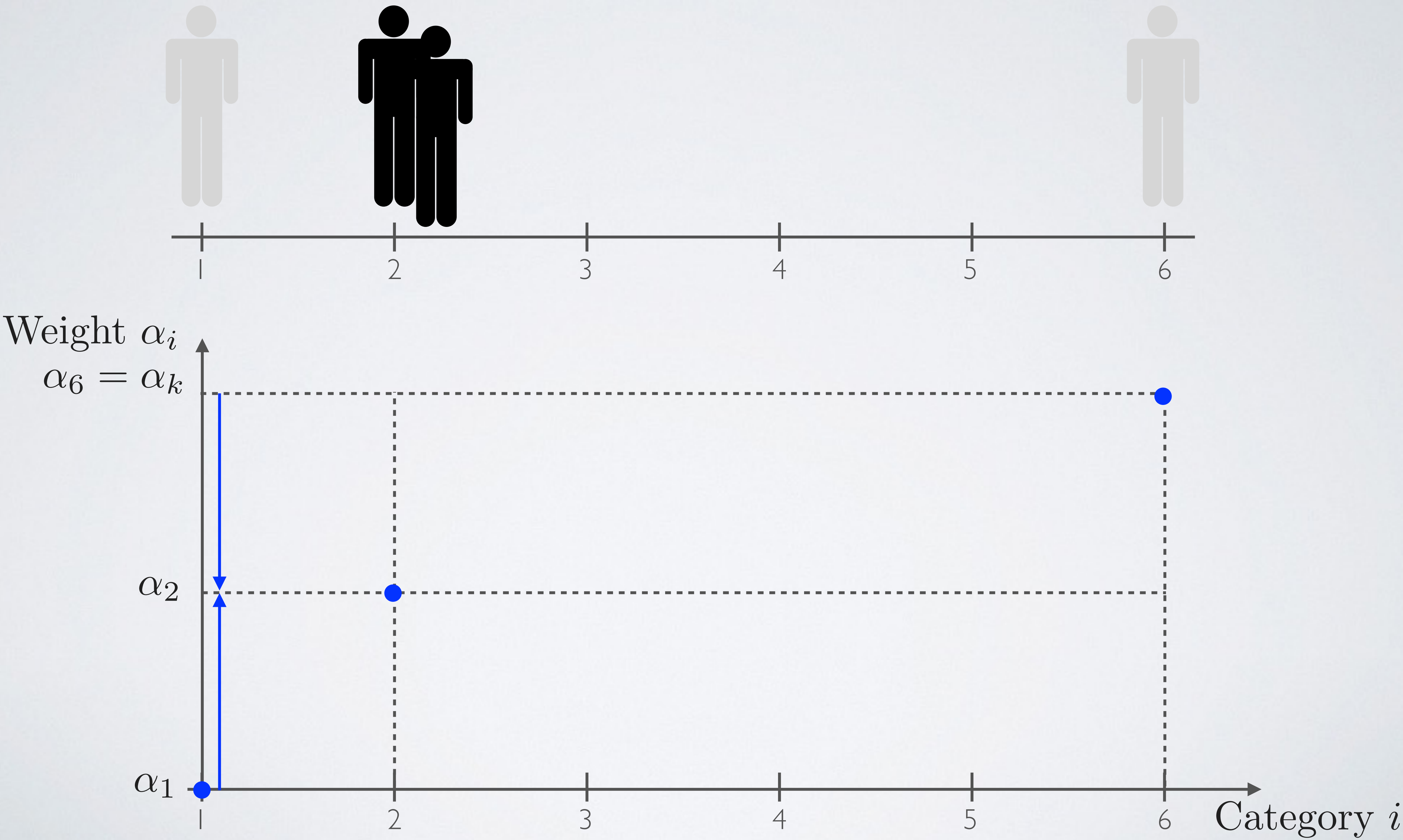


Illustration of « strong » concavity

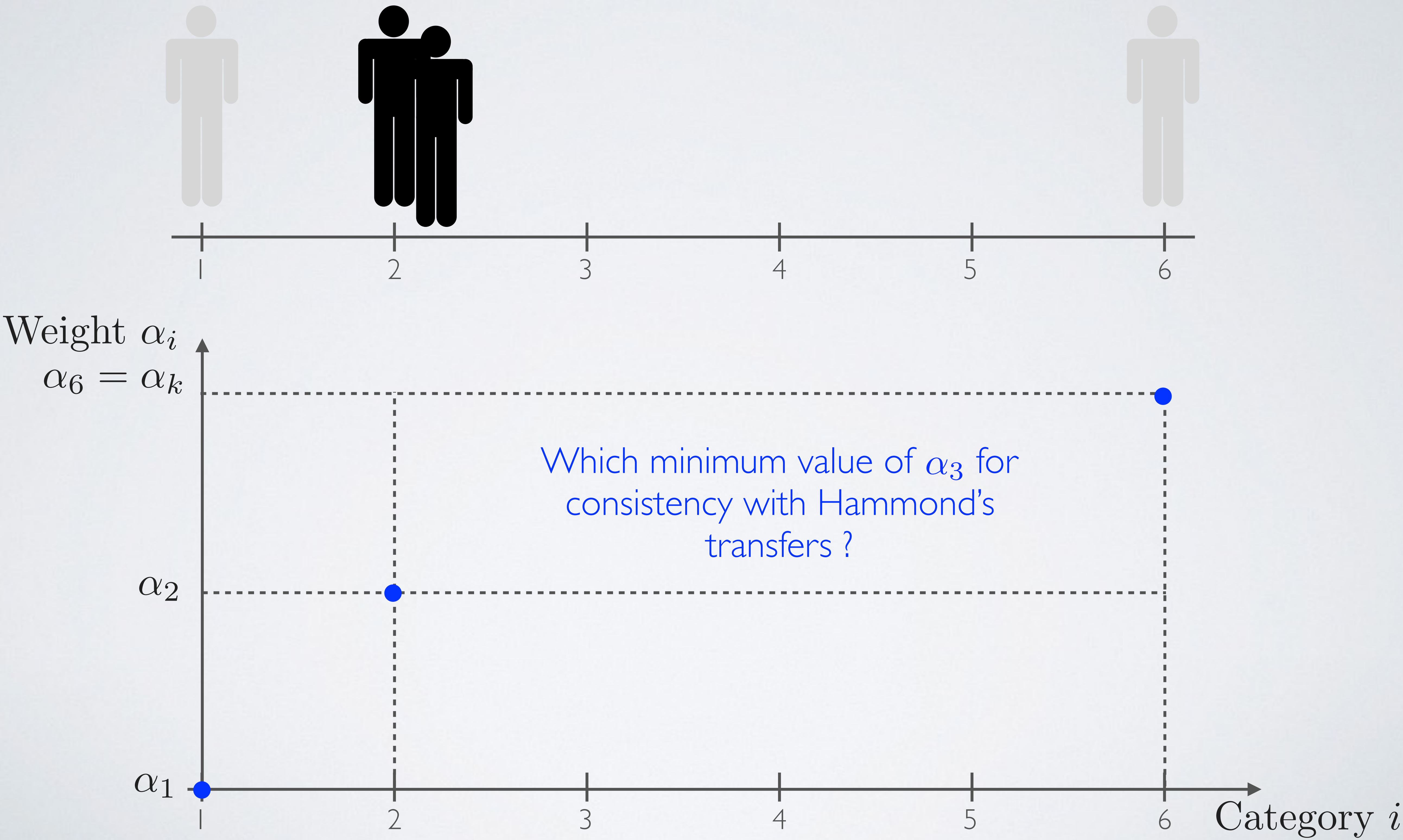


Illustration of « strong » concavity

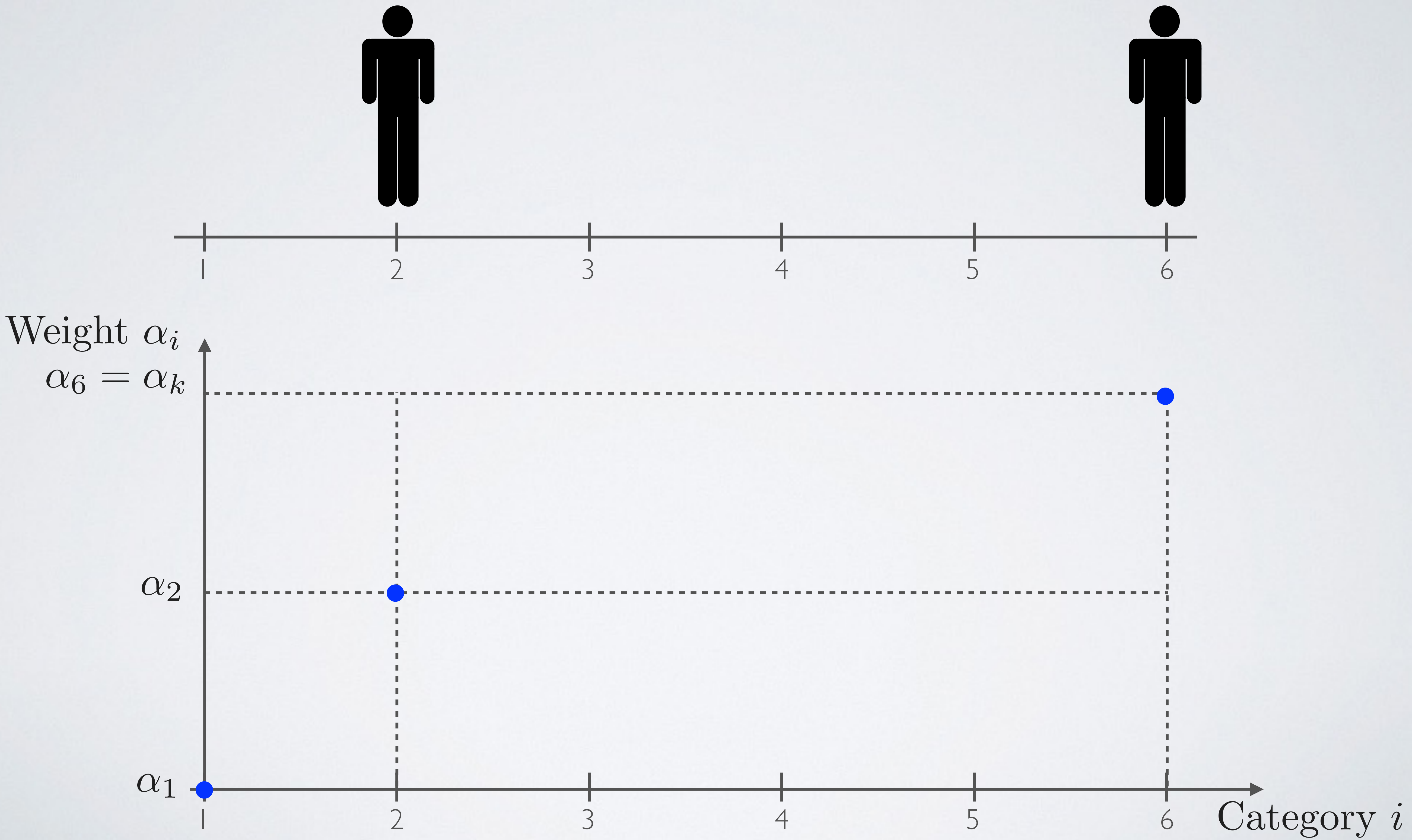


Illustration of « strong » concavity

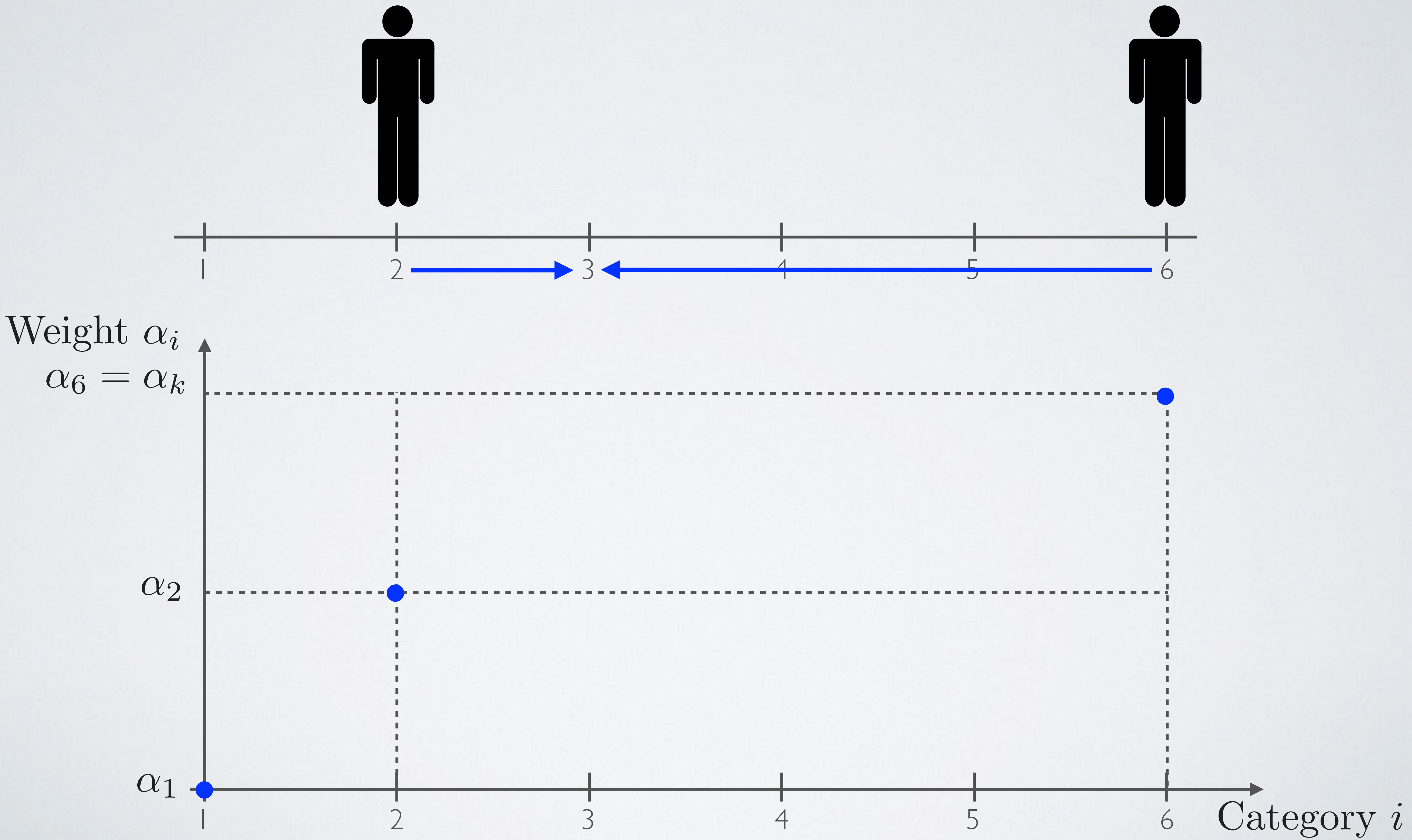


Illustration of « strong » concavity

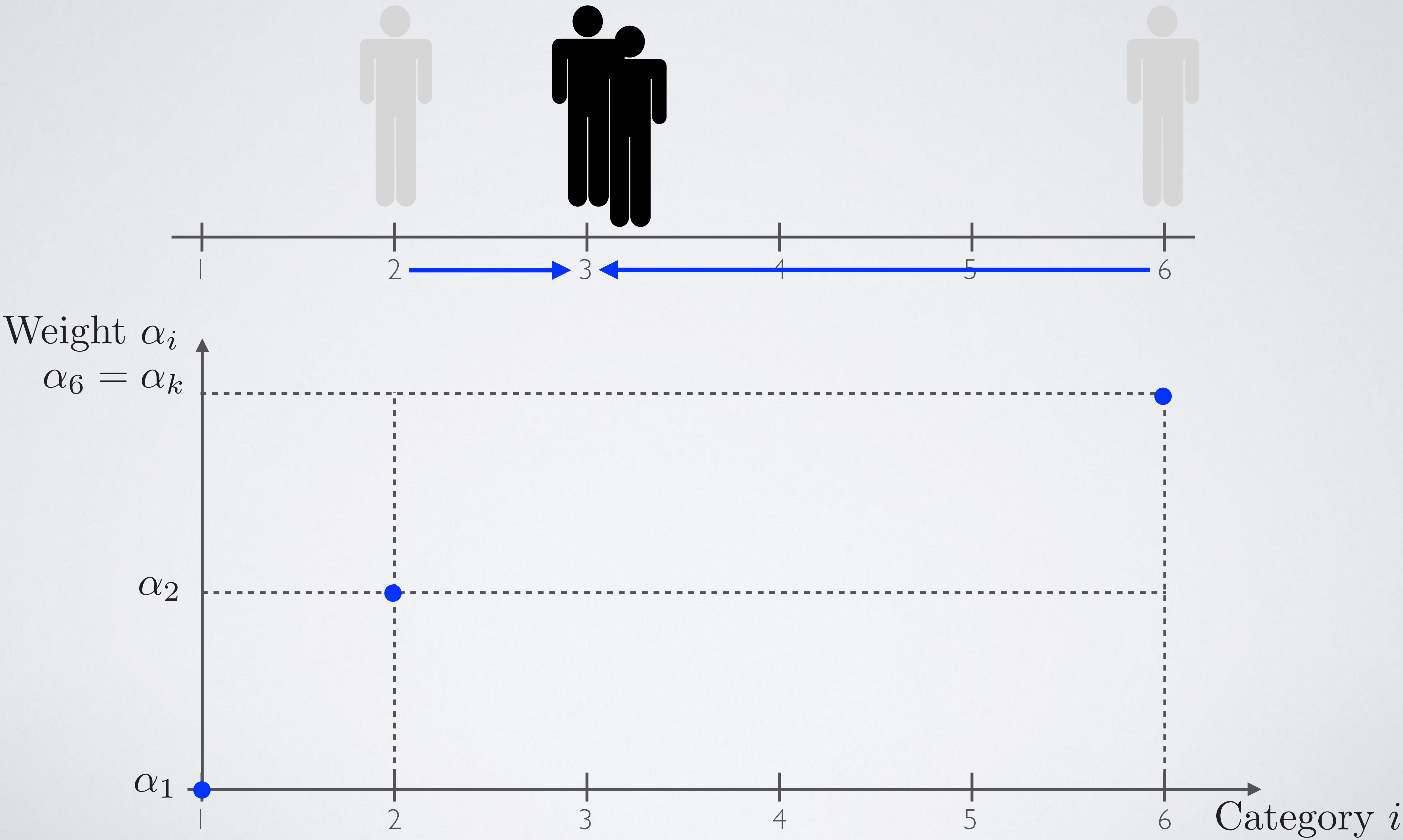


Illustration of « strong » concavity

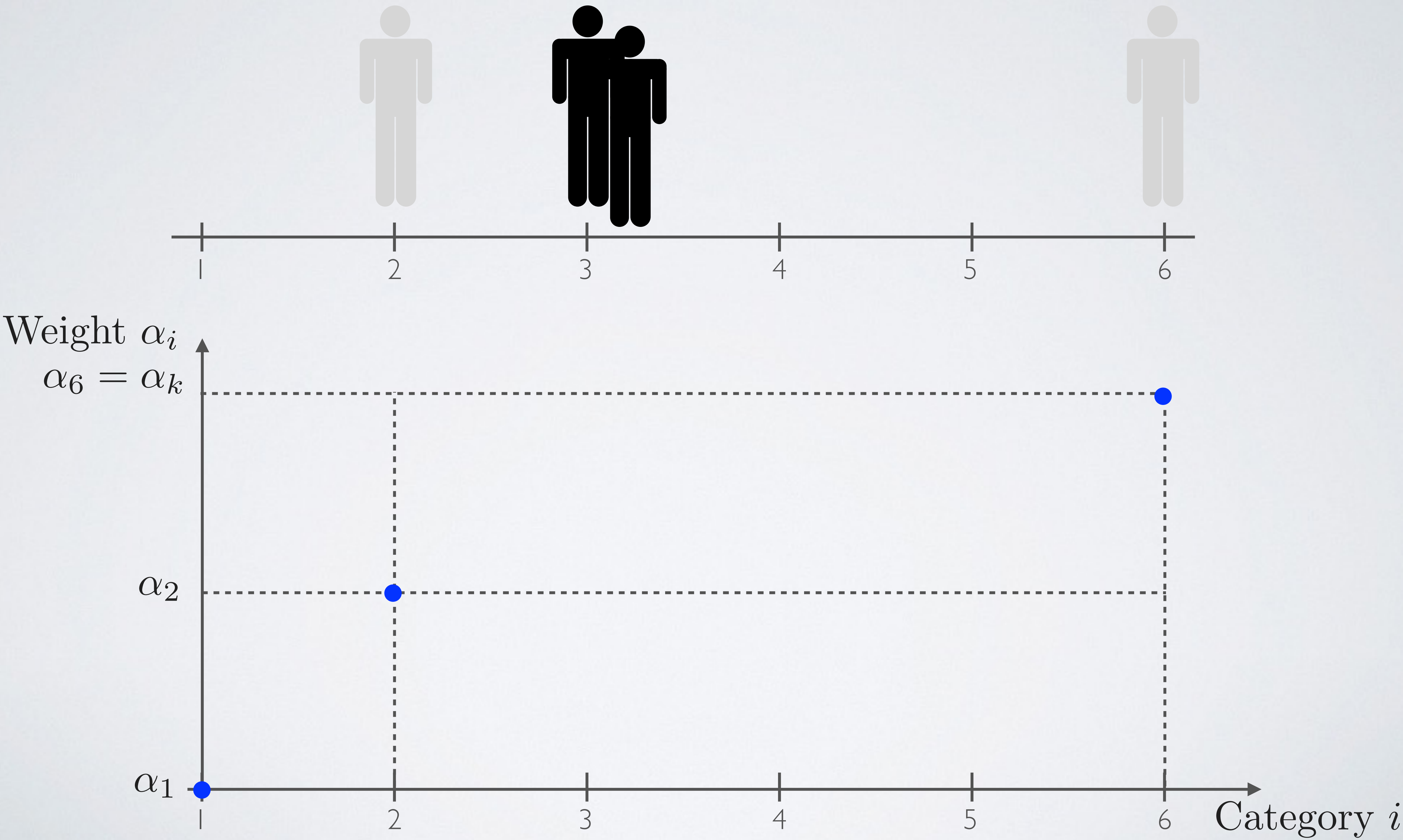


Illustration of « strong » concavity

Illustration of « strong » concavity

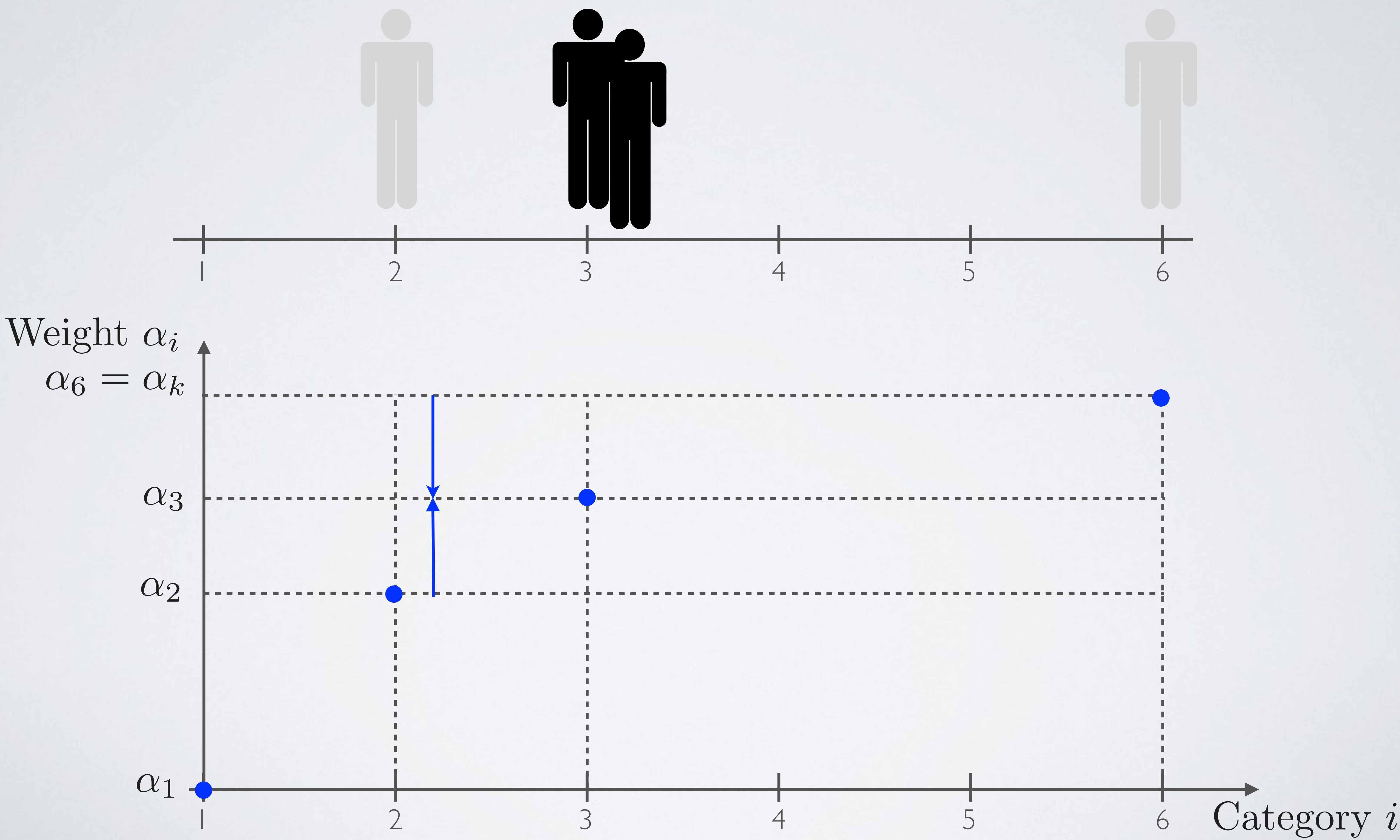


Illustration of « strong » concavity

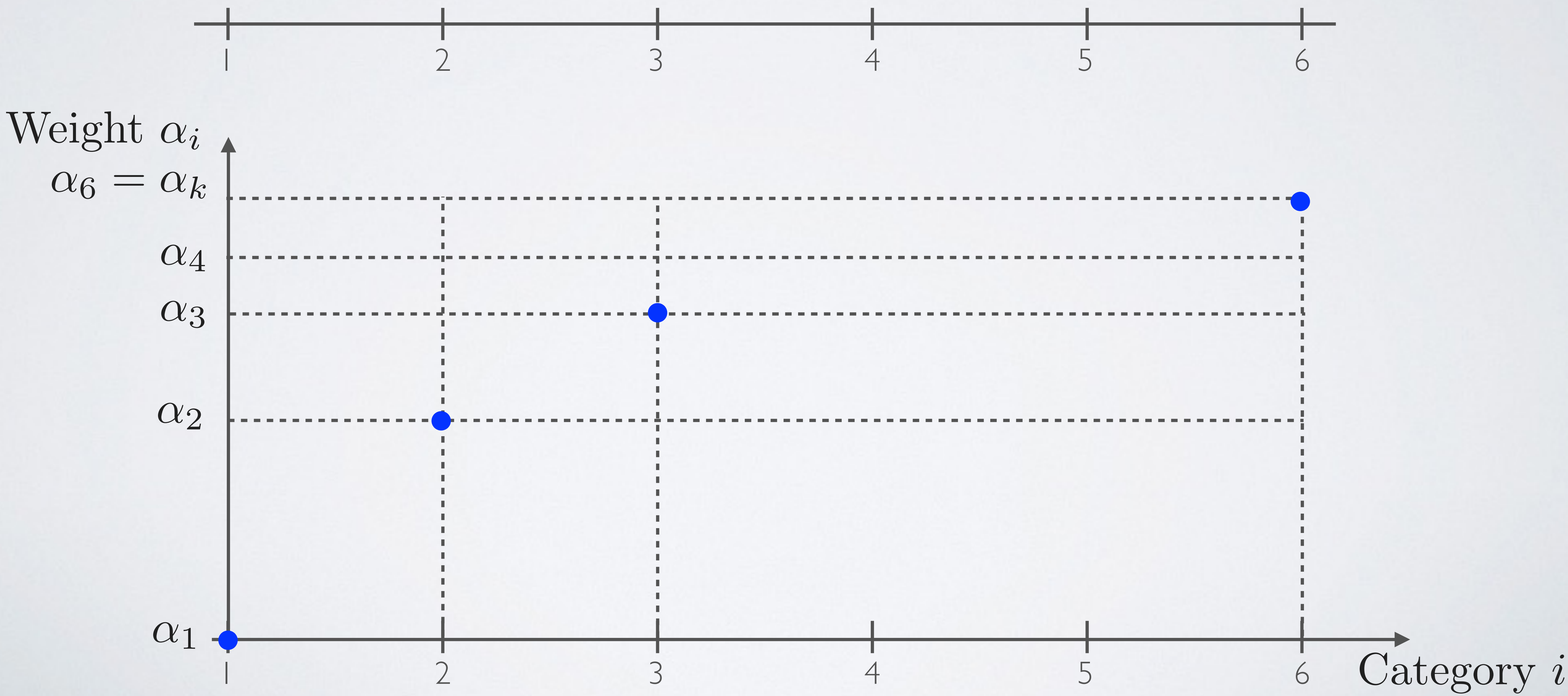


Illustration of « strong » concavity

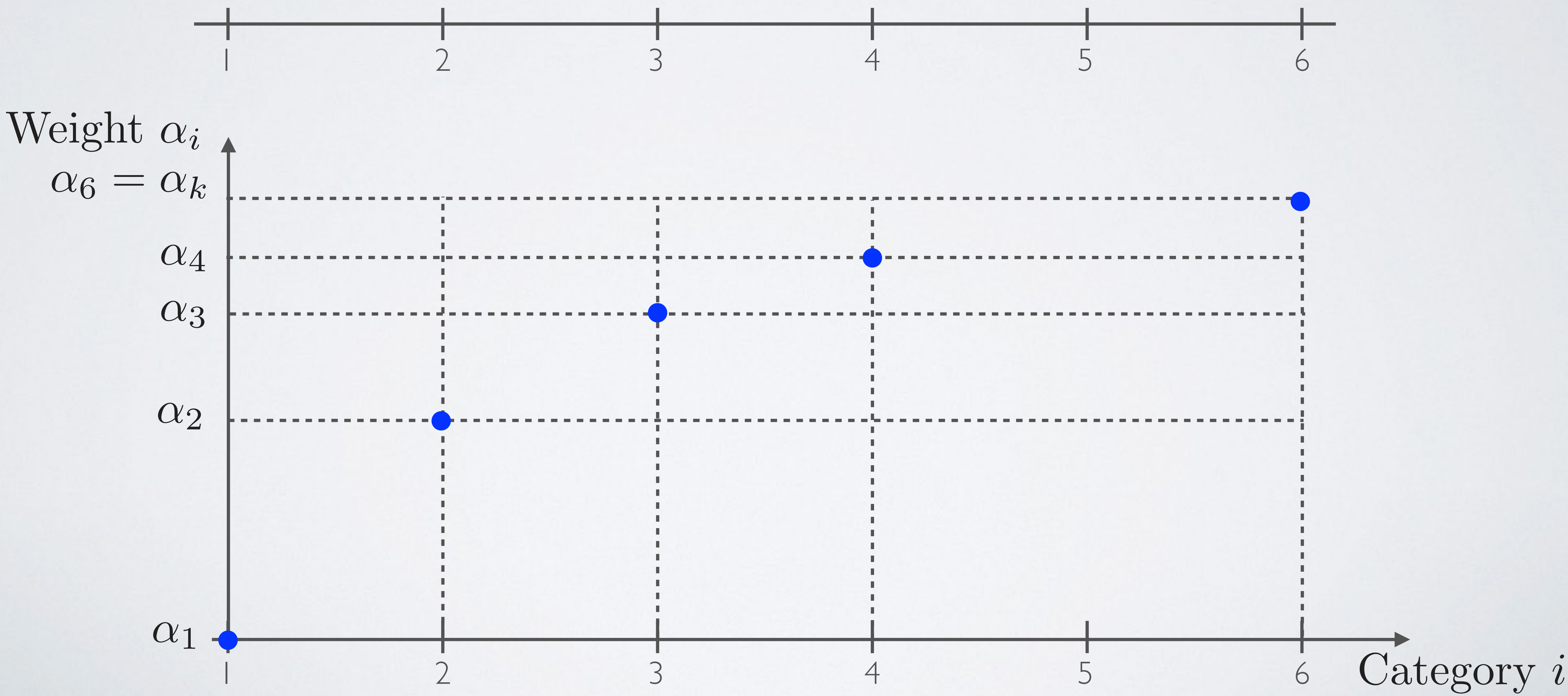


Illustration of « strong » concavity

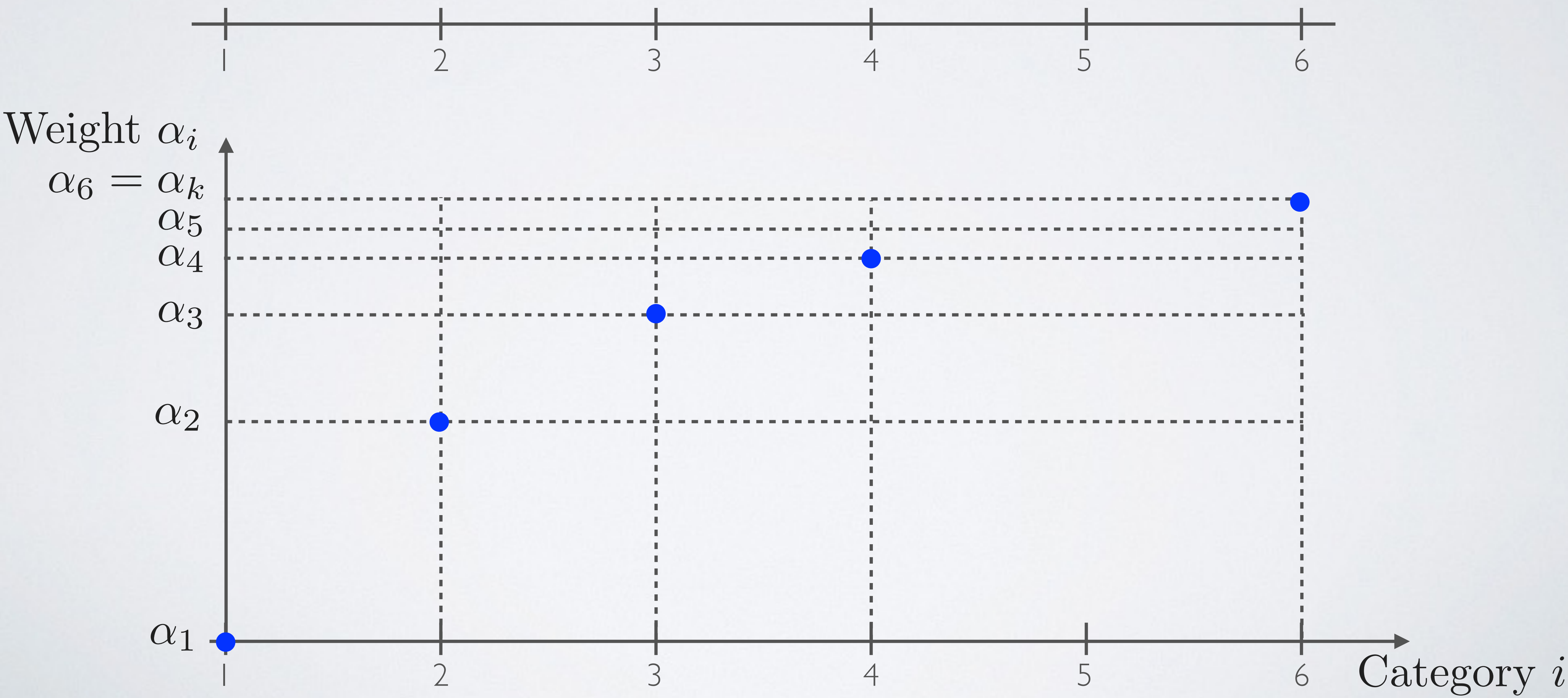


Illustration of « strong » concavity

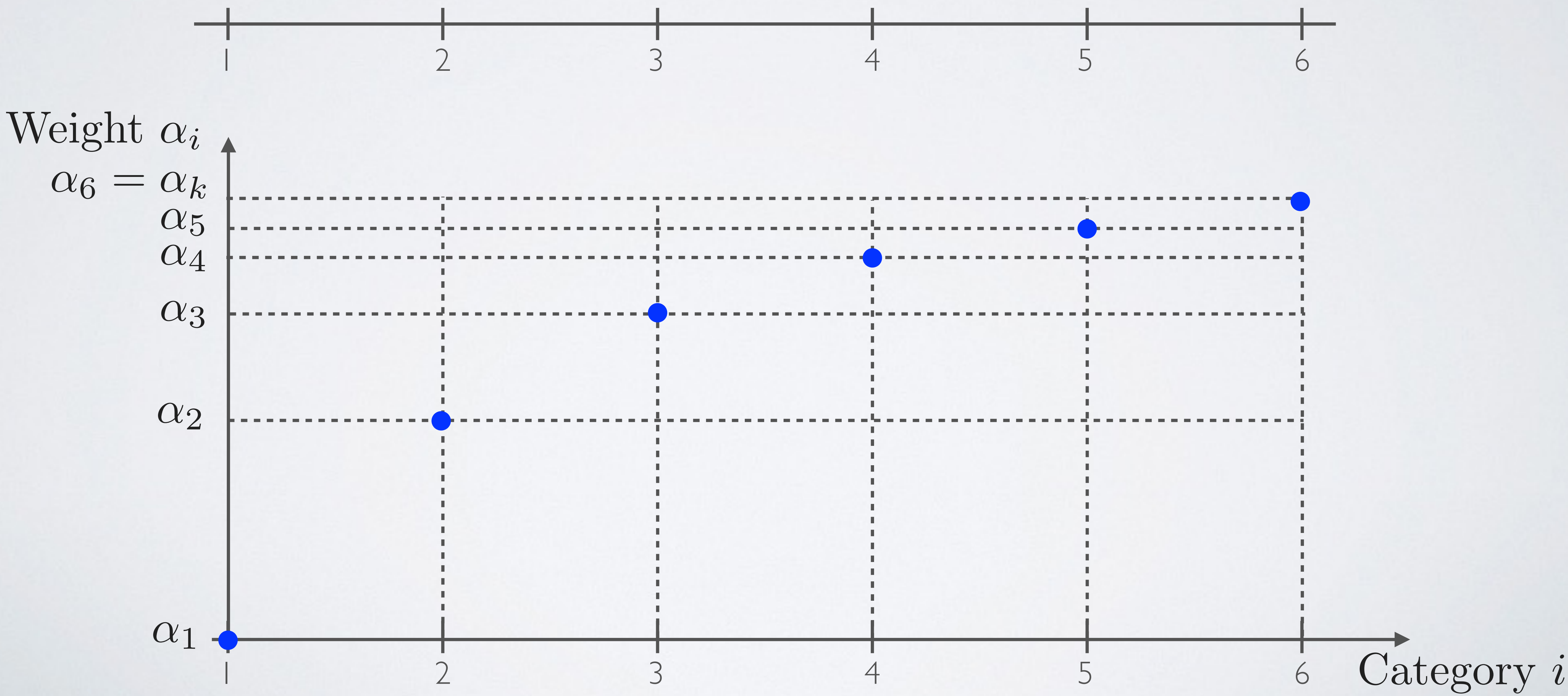
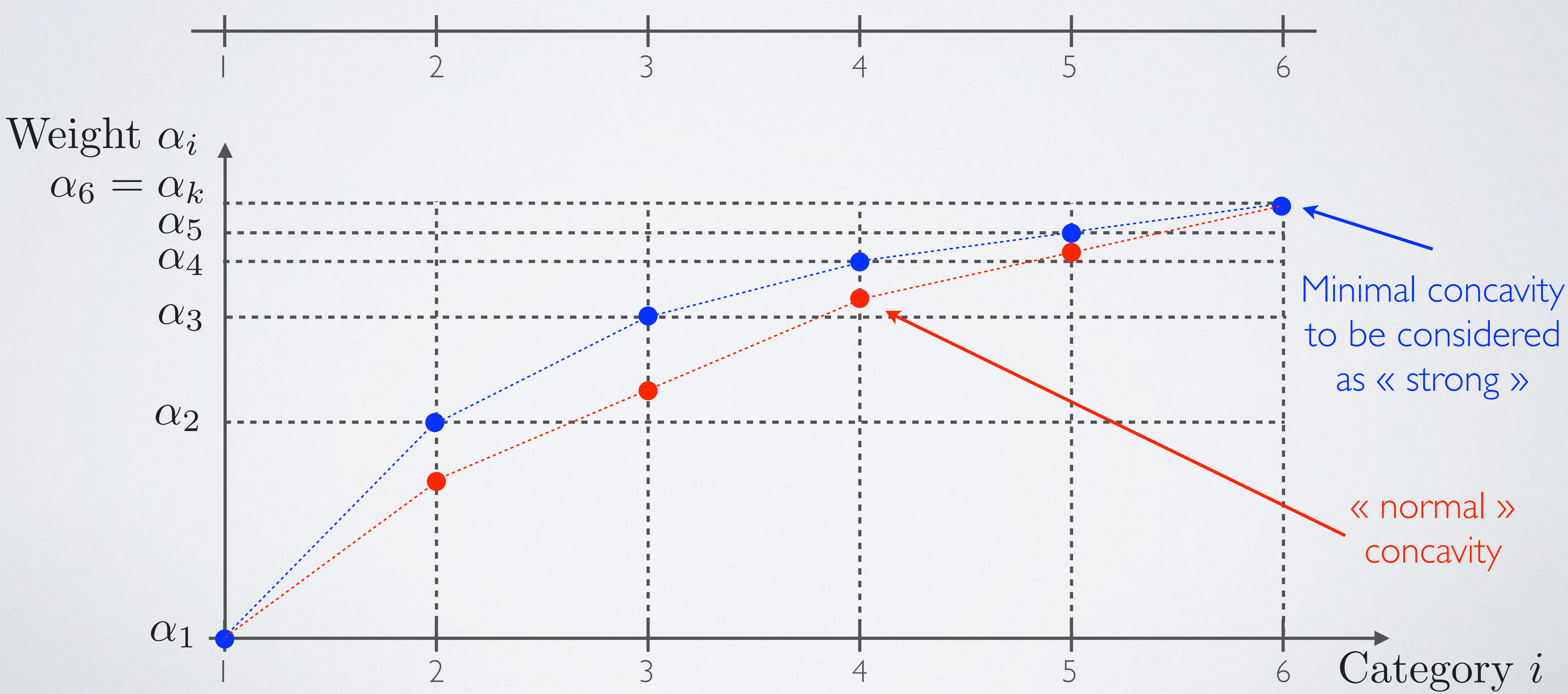


Illustration of « strong » concavity



Dual theorem: inefficiency and equity

Dual H-curve :

$$\bar{H}(i;s) = \frac{1}{n} \sum_{h=i+1}^{k} \left(2^{h-i-1}\right) n_h \,, \; \forall i = 1, 2, \ldots, k-1$$

Simple computation from the survival function :

$$H(k-1;s) = \bar{F}(k-1;s)$$ and $$\bar{H}(i;s) = 2\,\bar{H}(i+1;s) + \frac{n_{i+1}}{n} \,, \; \forall i = 1, 2, \ldots, k-2$$

Dual theorem :

Theorem 4. *For all distributions $s, s' \in C^n$, the following three statements are equivalent:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers and/or decrements,*

(b) $\sum_{h=1}^{k} n_h^s \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \alpha_h$ *for all* $\alpha \in \bar{\mathcal{A}}_1 \bigcap \mathcal{A}_2$,

(c) $\bar{H}(i;s) \leq \bar{H}(i;s')$, *for all* $i = 1, 2, \ldots, k-1$.

The last, but not the least, objective

Can we isolate equity considerations ?

- as compared to the cardinal framework, (in)efficiency cannot be «neutralised» but letting the means fixed
- stochastic dominance strategy : intersection of weak-super-majorization and weak-sub-majorization

Theorem 5. *For all distributions $s, s' \in \mathcal{C}^n$, consider the following three statements:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers,*

(b) $\sum_{h=1}^{k} n_h^{s} \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \alpha_h$ *for all* $\alpha \in \mathcal{A}_2$,

(c) $H(i; s) \leq H(i; s')$ *and* $\bar{H}(i; s) \leq \bar{H}(i; s')$, *for all* $i = 1, 2, \ldots, k-1$.

The last, but not the least, objective

Can we isolate equity considerations ?

- as compared to the cardinal framework, (in)efficiency cannot be «neutralised» but letting the means fixed
- stochastic dominance strategy : intersection of weak-super-majorization and weak-sub-majorization

We already have :

Theorem 5. *For all distributions $s, s' \in C^n$, consider the following three statements:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers,*

(b) $\sum_{h=1}^{k} n_h^s \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \alpha_h$ *for all* $\alpha \in \mathcal{A}_2$,

(c) $H(i;s) \leq H(i;s')$ *and* $\bar{H}(i;s) \leq \bar{H}(i;s')$, *for all* $i = 1, 2, \ldots, k-1$.

The last, but not the least, objective

Can we isolate equity considerations ?

- as compared to the cardinal framework, (in)efficiency cannot be «neutralised» but letting the means fixed
- stochastic dominance strategy : intersection of weak-super-majorization and weak-sub-majorization

We already have :

Theorem 5. *For all distributions $s, s' \in C^n$, consider the following three statements:*

(a) *s is obtained from s' by means of a finite sequence of Hammond's transfers,*

(b) $\sum_{h=1}^{k} n_h^s \alpha_h \geq \sum_{h=1}^{k} n_h^{s'} \alpha_h$ *for all* $\alpha \in \mathcal{A}_2$,

(c) $H(i;s) \leq H(i;s')$ *and* $\bar{H}(i;s) \leq \bar{H}(i;s')$, *for all* $i = 1, 2, \ldots, k-1$.

It remains to show that : $(c) \Rightarrow (a)$

Empirical Illustration

Distributions of body mass index

Body mass index as health indicator: mass(kg)/square-of-height(m)

- obesity and overweight are increasingly recognized as major problems (both for health and for self-esteem)
- so can be « underweight » (anorexia)
- often used as a diagnostic tool to identify pathologic weights

Six levels usually defined :

> 40 :	morbid obesity
[35-40] :	severe obesity
[30-35[:	mild obesity
[25-30[:	over-weight
[18-25[:	norm
< 18	underweight

Distributions of body mass index

Body mass index as health indicator: mass(kg)/square-of-height(m)

- obesity and overweight are increasingly recognized as major problems (both for health and for self-esteem)
- so can be « underweight » (anorexia)
- often used as a diagnostic tool to identify pathologic weights

Six levels usually defined :

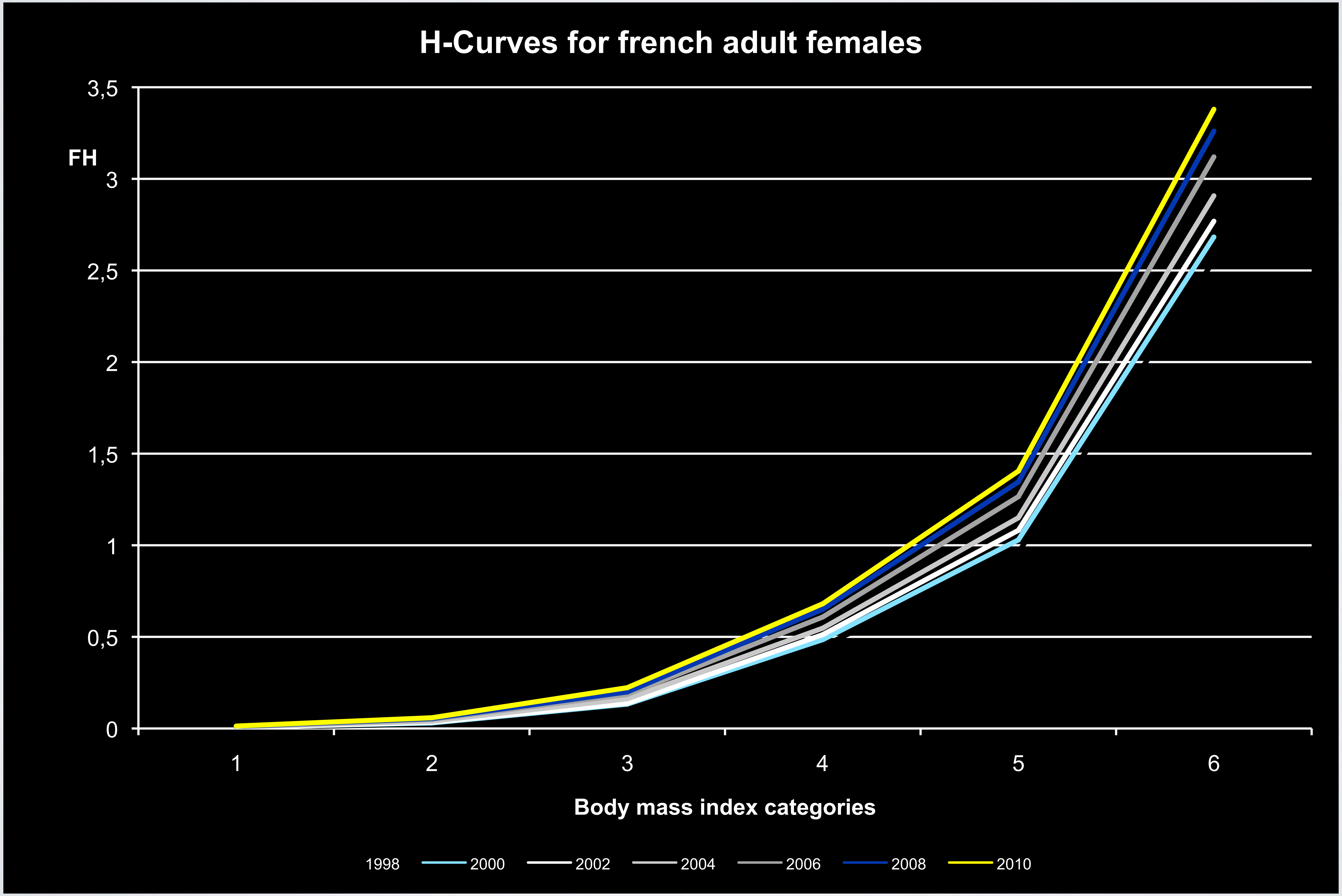
> 40 :	morbid obesity
[35-40] :	severe obesity
[30-35[:	mild obesity
[25-30[:	over-weight
[18-25[:	norm
< 18	underweight

> 40 :	morbid obesity
[35-40] :	severe obesity
[30-35[:	mild obesity
[25-30[:	over-weight
< 18 :	underweight
[18-25[:	norm

From the lowest
to the
highest category

Data source: survey ESPS (panel since 1998)

Distributions of body mass index



Conclusion

Conclusion

We have provided a « foundational » theorem, and some extensions, for normative evaluation dealing with distributions of a discrete ordinal attribute

The approach is easily workable : implemention criteria

Need to do :

- to develop ordinal inequality indices consistent with Hammond's transfers
- to make empirical applications (with statistical inference)
- multidimensional generalizations ?

Appendix

Elementary transformations

Definition 2 (Increment). *Given two distributions $s, s' \in C^n$, we will say that s is obtained from s' by means of an increment, if there exist $j \in \{1, \ldots, k-1\}$ such that:*

$$n_h^s = n_h^{s'}, \ \forall\, h \neq j, j+1\,;$$

$$n_j^s = n_j^{s'} - 1\ ;\ n_{j+1}^s = n_{j+1}^{s'} + 1\,.$$

Definition 3 (Hammond's transfer). *Given two distributions $s, s' \in C^n$, we will say that s is obtained from s' by means of a Hammond's (progressive) transfer, if there exist four categories $1 \leq g < h \leq i < j \leq k$ such that:*

$$n_l^s = n_l^{s'}, \ \forall\, l \neq g, h, i, j\,;$$

$$n_g^s = n_g^{s'} - 1\ ;\ n_h^s = n_h^{s'} + 1\,;$$

$$n_i^s = n_i^{s'} + 1\ ;\ n_j^s = n_j^{s'} - 1\,.$$

Elementary transformations

efficiency

Definition 2 (Increment). *Given two distributions $s, s' \in C^n$, we will say that s is obtained from s' by means of an increment, if there exist $j \in \{1, \ldots, k-1\}$ such that:*

$$n_h^s = n_h^{s'}, \ \forall\ h \neq j, j+1\,;$$

$$n_j^s = n_j^{s'} - 1\ ;\ n_{j+1}^s = n_{j+1}^{s'} + 1\,.$$

Definition 3 (Hammond's transfer). *Given two distributions $s, s' \in C^n$, we will say that s is obtained from s' by means of a Hammond's (progressive) transfer, if there exist four categories $1 \leq g < h \leq i < j \leq k$ such that:*

$$n_l^s = n_l^{s'}, \ \forall\ l \neq g, h, i, j\,;$$

$$n_g^s = n_g^{s'} - 1\ ;\ n_h^s = n_h^{s'} + 1\,;$$

$$n_i^s = n_i^{s'} + 1\ ;\ n_j^s = n_j^{s'} - 1\,.$$

Elementary transformations

efficiency

Definition 2 (Increment). *Given two distributions $s, s' \in C^n$, we will say that s is obtained from s' by means of an increment, if there exist $j \in \{1, \ldots, k-1\}$ such that:*

$$n_h^s = n_h^{s'},\ \forall\ h \neq j, j+1\,;$$

$$n_j^s = n_j^{s'} - 1\ ;\ n_{j+1}^s = n_{j+1}^{s'} + 1\,.$$

Definition 3 (Hammond's transfer). *Given two distributions $s, s' \in C^n$, we will say that s is obtained from s' by means of a Hammond's (progressive) transfer, if there exist four categories $1 \leq g < h \leq i < j \leq k$ such that:*

$$n_l^s = n_l^{s'},\ \forall\ l \neq g, h, i, j\,;$$

$$n_g^s = n_g^{s'} - 1\ ;\ n_h^s = n_h^{s'} + 1\,;$$

$$n_i^s = n_i^{s'} + 1\ ;\ n_j^s = n_j^{s'} - 1\,.$$

equity

Inequality of self-reported health status

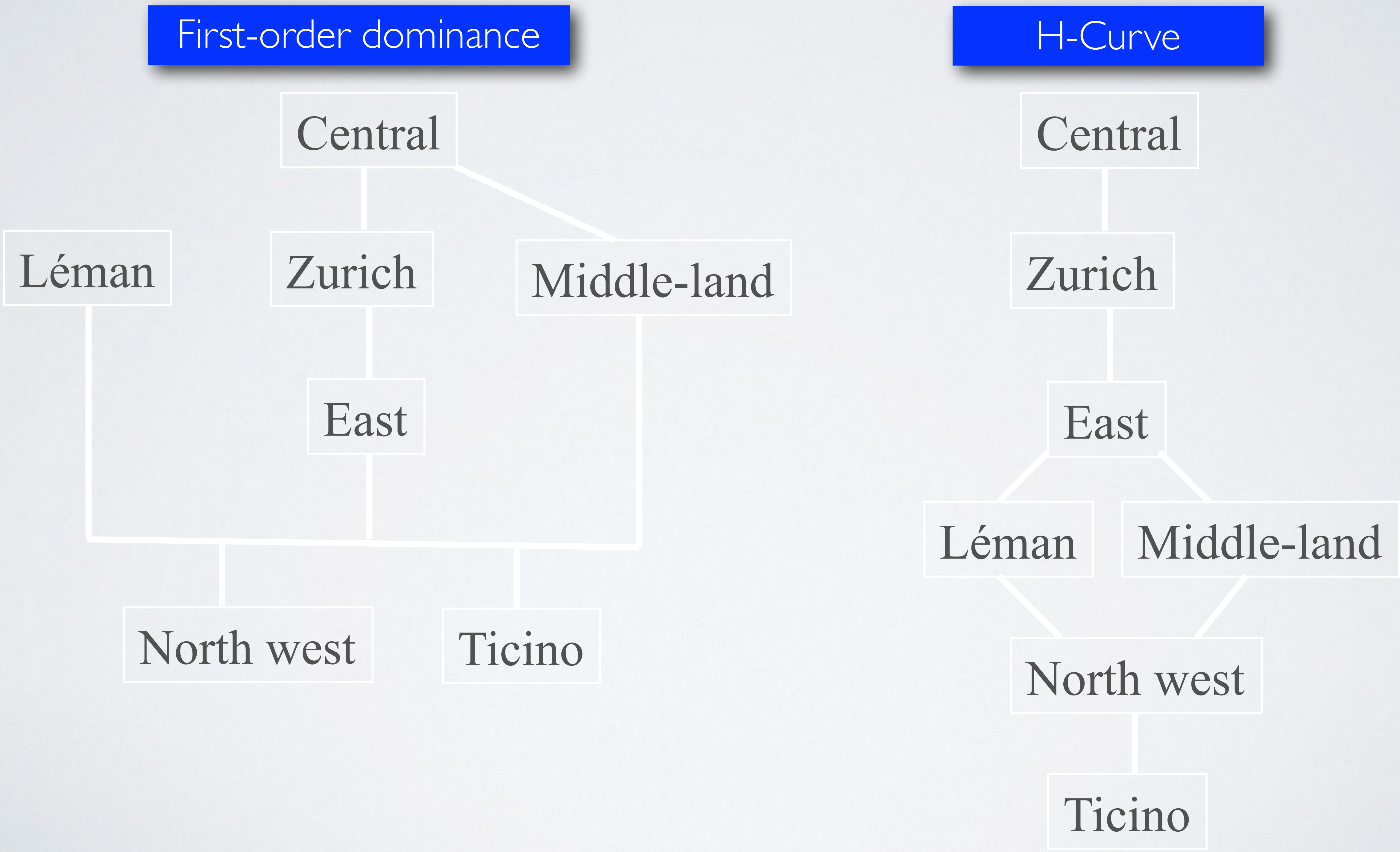
Data are taken from Abul-Naga & Yalcin (J. Health Eco. 2008)

- Swiss Health Survey (SHS), by Switzerland's Fed. Stat. Office in 2002
- 19.706 observations from 7 satistical areas
- 5 categories : very bad, bad, so so, good, very good

Table 2
Cumulative distribution of SRHS in the seven statistical areas of Switzerland

Area	SRHS distribution				
	Very bad	Bad	So so	Good	Very good
Leman	0.01	0.05	0.16	0.72	1.00
North-West	0.01	0.05	0.18	0.81	1.00
Central	0.00	0.02	0.13	0.76	1.00
Middle-Land	0.01	0.04	0.17	0.77	1.00
East	0.00	0.03	0.14	0.78	1.00
Ticino	0.01	0.06	0.17	0.87	1.00
Zurich	0.00	0.03	0.13	0.78	1.00

Inequality of self-reported health status



Extension: Refinement of the grid

In classical social choice theory, Hammond equity principle is tightly connected to the so-called leximin ordering

- Def : a society dominates another society if the poorer individual is strictly better in the first one. If equal situations, comparison of the second poorer individuals, and so on and so forth ...
- Leximin is a complete quasi-ordering

Bosmans & Ooghe (2013): the ony continuous, anonymous, Pareto-sensitive and Hammond-sensitive quasi-ordering is the maximin criterion

The first part of the paper : fixed scale / grid.

We show that dominance according to the H-curve (not a complete quasi-ordering) converges to leximin (complete)

Extension: Refinement of the grid

Initial scale / grid :

$$\mathcal{C} = \{1, 2, \ldots, k\}$$

Refinement of the grid :

$$\mathcal{C}(t) = \left\{ \frac{1}{2^t}, \frac{2}{2^t}, \ldots, \frac{(2^t)\, k}{2^t} \right\}, \quad t \in \mathbb{N}$$

The H-criterion depends upon the grid. We obtain :

Theorem 6. *For all societies $s, s' \in \mathcal{C}^n$, the following two statements are equivalent:*

(a) $\exists t \in \mathbb{N} \ : \ s \succeq_H^t s'$,

(b) $s \succeq_L s'$.