Stochastic dominance approaches to the measurement of health inequality

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Fourteenth Winter School on Inequality and Collective Welfare Theory

Overview

1. Socioeconomic health inequality measurement framework
2. Some issues with socioeconomic health inequality measures
3. A stochastic dominance approach for dealing with measurement scales
4. A stochastic dominance approach for dealing with the choice of indices

Socioeconomic health inequality measurement framework

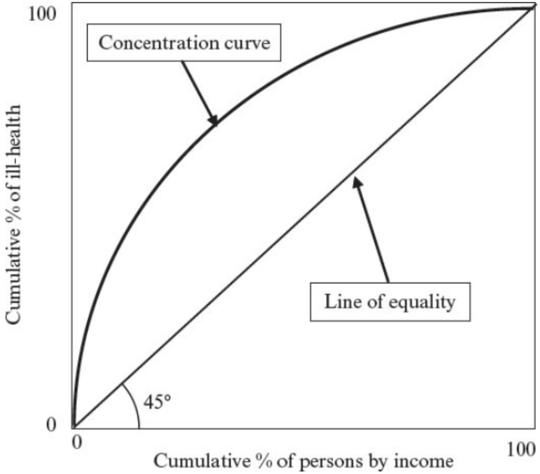
Early literature on health inequality measurement

- Le Grand and Rabin (1986) propose to use Lorenz curves and the Gini coefficient to depict inequalities in health.
- These are measures of pure health inequalities. They fail to capture socioeconomic inequalities in health.

The health concentration curve and the health concentration index

- The idea was introduced by Wagstaff, Paci and van Doorslaer (1989)
- The health concentration curve plots the cumulative proportions of the population, **ranked by their socioeconomic status**, against the cumulative proportion of health.
- The health concentration index, denoted by $C(F_{H,Y})$, is then defined as twice the area between the concentration curve and the diagonal.
- The concentration curve and the concentration index can be also used when inequality in *ill*-health is to be assessed.
- The health concentration curve will be identical to the Lorenz curve if and only if the ranking of units of analysis by health is the same as the ranking by socioeconomic status.

The concentration curve



Health achievement and health concentration indices.

- The mathematical expression of the canonical health concentration index:

$$C(F_{H,Y}) = \frac{1}{\mu_h}\int_0^1 [1-2(1-p)]\, h(p)\mathrm{d}p,$$

where μ_h is average health and $h(p) = \mathbb{E}\left[H \middle| Y = F_Y^{-1}(p)\right]$.

- Wagstaff (2002) explains that the health concentration index embodies a specific value of health inequality aversion. To allow for more flexibility, he proposes an extended class of health concentration indices:

$$C(F_{H,Y};\nu) = \frac{1}{\mu_h}\int_0^1 \left[1-\nu(1-p)^{\nu-1}\right] h(p)\mathrm{d}p.$$

- The canonical health concentration index is a specific case of this class for $\nu = 2$.

Health achievement and health concentration indices.

- Wagstaff (2002) also pointed to the fact that using the concentration index overlooks the average level of health in the populations under comparison. He proposes a class of health achievement indices defined as
$$A(F_{H,Y};\nu) = \mu_h(1 - C(F_{H,Y};\nu)) = \int_0^1 \nu(1-p)^{\nu-1} h(p)\mathrm{d}p.$$

Some issues with socioeconomic health inequality measures

Measurement issues

- There are many issues faced by the analyst.
- Today we will focus on two issues in the measurement of socioeconomic health inequality that can be addressed using a stochastic dominance approach:
  - **Measurement scale:** many health variables are ordinal in nature.
  - **Choice of the index obeying a set of normative principles:** this is linked with the standard use in the income inequality literature.

A stochastic dominance approach for dealing with measurement scales

Measurement scales

- **Ratio scale:** Variables in this category enjoy the richest mathematical structure. For any two values, say x_1 and x_2:
  - the ratio x_2/x_1 is a meaningful quantity,
  - the distance $x_2 - x_1$ is a meaningful quantity,
  - there exist a natural ordering or the values along the scale, i.e. comparisons such as $x_2 \geq x_1$ or $x_2 \leq x_1$ make sense.
- Income is a natural example. If we consider $10,000 and $20,000:
  - the ratio $20,000/10,000 = 2$ is a meaningful quantity,
  - the distance $20,000 - 10,000 = 10,000$ is a meaningful quantity,
  - we know that $20,000 > 10,000$.
- The Gini index and the Lorenz curve are well adapted to such variables.

Measurement scales

- **Interval scale:**
  - the distance $x_2 - x_1$ is a meaningful quantity,
  - there exist a natural ordering or the values along the scale, i.e. comparisons such as $x_2 \geq x_1$ or $x_2 \leq x_1$ make sense.

- Temperature is a good example. If we consider 10 Celsius (50 Farenheit), 20 Celsius (68 Farenheit), 30 Celcius (86 Farenheit), 40 Celcius (104 Farenheit):
  - the ratio $20/10 = 40/20$ is not a meaningful quantity since $68/50 \neq 104/68$
  - the distance $20 - 10 = 30 - 20$ is a meaningful quantity since $86 - 68 = 68 - 50$
  - we know that $40 > 30 > 20 > 10$.

Inequality in temperature in September 2011

Table: Gini Index estimates of temperature inequality for September 2011

	Ottawa	Québec	Montréal

Source: Environment Canada's Weather Office web site (own estimation)

Inequality in temperature in September 2011

Table: Gini Index estimates of temperature inequality for September 2011

	Ottawa	Québec	Montréal
Gini with °C	0.120032	0.115051	0.114458

Source: Environment Canada's Weather Office web site (own estimation)

Inequality in temperature in September 2011

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	Ottawa	Québec	Montréal
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$$^{\circ}F = \frac{9}{5} \times {}^{\circ}C + 32$$

Inequality in temperature in September 2011

Table: Gini Index estimates of temperature inequality for September 2011

	Ottawa	Québec	Montréal
Gini with °C	0.120032	0.115051	0.114458
Gini with °F	0.059112	0.053594	0.057163

Source: Environment Canada's Weather Office web site (own estimation)

$$^{\circ}F = \frac{9}{5} \times {}^{\circ}C + 32$$

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Gini with $^\circ$F	0.059112	0.053594	0.057163
Ranking with $^\circ$C			

Source: Environment Canada's Weather Office web site (own estimation)

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Ranking with $^\circ$C			**1**

Source: Environment Canada's Weather Office web site (own estimation)

$$^\circ F = \frac{9}{5} \times {}^\circ C + 32$$

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Gini with $^\circ$F	0.059112	0.053594	0.057163
Ranking with $^\circ$C		**2**	**1**

Source: Environment Canada's Weather Office web site (own estimation)

$$^\circ F = \frac{9}{5} \times {}^\circ C + 32$$

Inequality in temperature in September 2011

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	Ottawa	Québec	Montréal
Gini with °C	0.120032	0.115051	0.114458
Gini with °F	0.059112	0.053594	0.057163
Ranking with °C	**3**	**2**	**1**

Source: Environment Canada's Weather Office web site (own estimation)

$$^\circ F = \frac{9}{5} \times {}^\circ C + 32$$

Inequality in temperature in September 2011

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Ranking with $^\circ$C	**3**	**2**	**1**
Ranking with $^\circ$F			

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Ranking with °C	**3**	**2**	**1**
Ranking with °F		**1**	

Source: Environment Canada's Weather Office web site (own estimation)

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Ranking with °C	**3**	**2**	**1**
Ranking with °F		**1**	**2**

Source: Environment Canada's Weather Office web site (own estimation)

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Inequality in temperature in September 2011

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Ranking with °C	**3**	**2**	**1**
Ranking with °F	**3**	**1**	**2**

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Inequality in temperature in September 2011

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Ranking with °C	**3**	**2**	**1**
Ranking with °F	**3**	**1**	**2**

Source: Environment Canada's Weather Office web site (own estimation)

$$^{\circ}F = \frac{9}{5} \times {}^{\circ}C + 32$$

INEQUALITY RANKINGS ARE ARBITRARY WHEN THE MEASURE IS APPLIED TO A NON RATIO-SCALE VARIABLE

Measurement scales

- **Ordinal scale:**
  - there exist a natural ordering or the values along the scale, i.e. comparisons such as $x_2 \succeq x_1$ or $x_2 \preceq x_1$ make sense.
- Self-reported health status, happiness are natural examples:
  - we know that being in good health $\succeq$ being in poor health
- **Nominal scale:** Categories that do not have a natural orderings
- Region of residence, hair color are good examples.

Related literature: existing solutions when facing ordinal variables

Table: Alternative Solutions

	Pure Ineq.	Socioeco Ineq.	Advantage	Costs
Allison & Foster(2004) F.O.D.	Yes	No	Depth	Partial order & SES
Abul Naga & Yalcin(2008) Index	Yes	No	Depth Complete order	SES
Zheng (2011) Transitions	No	Yes	Depth SES	Heterogeneity Within SES classes
Makdissi & Yazbeck (2014) Count	No	Yes	SES Complete order Heterogeneity	Depth
Cowell & Flachaire (2017) Index	Yes	No	Depth Complete order	SES
Makdissi & Yazbeck (2017) S.D. on a distorted version of $F(H)$	No	Yes	SES Depth Heterogeneity	Partial

Allison and Foster (2004)

- **Reference:** Allison, R. A. and J.E. Foster (2004), Measuring health inequality using qualitative data, *Journal of Health Economics*, 23, 505-524.
- Assume the health variable has K health categories such that $h_i \in \{1,2,...,K\}$ is the health status of individual $i \in \{1,2,\ldots,n\}$
- Let $\eta(h)$ is a numerical scale that assigns a numerical value to each category h of health.

Allison and Foster (2004)

- The average health status is given by:

$$\mu_h = \frac{1}{n}\sum_{i=1}^{n}\eta(h_i)$$

- In the risk and social welfare literature, first order stochastic dominance rank distributions for any increasing utility or social evaluation function. It is natural to apply it in this context to rank health distribution with respect to average health status for any increasing numerical scale.

Allison and Foster (2004)

Theorem

Let F_H^0 and F_H^1 be two distribution of health. $\mu\left(F_H^1\right) \geq \mu\left(F_H^0\right)$ *for all increasing numerical scale* $\eta(h)$ *if*

$$F_H^1(h) \leq F_H^0(h) \ \text{ for all } \ h \in \{1, 2, \ldots, K-1\}$$

Allison and Foster (2004)

- The *Average Absolute Deviation about the Median*, D_{med}, is given by:

$$D_{\text{med}}(F_H) = \mathbb{E}|\eta(h) - \text{median}_\eta(F_H)| = \frac{1}{n}\sum_{i=1}^{n}|\eta(h_i) - \text{median}_\eta(F_H)|$$

Theorem

Let F_H^0 and F_H^1 be two distribution of health. $D_{\text{med}}(F_H^1) \leq D_{\text{med}}(F_H^0)$ for all increasing numerical scale $\eta(h)$ if

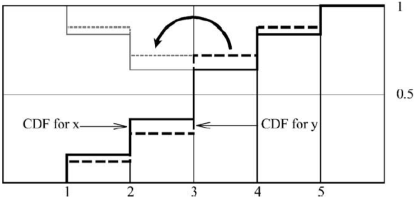
$$F_H^1(h) \leq F_H^0(h) \text{ for all } h \in \{1, 2, \ldots, \text{median}_\eta(F_H) - 1\}$$

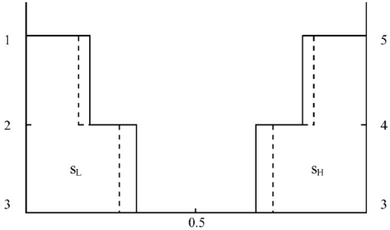
and

$$F_H^1(h) \geq F_H^0(h) \text{ for all } h \in \{\text{median}_\eta(F_H), \text{median}_\eta(F_H) + 1, \ldots, K - 1\}$$

Allison and Foster (2004)

Figure: Allison and Foster (2011) S-curves





Makdissi and Yazbeck (2017)

- **Reference:** Makdissi, P. and M. Yazbeck (2017), Robust Rankings of Socioeconomic Health Inequality Using a Categorical Variable, *Health Economics*, 26, 1132-1145.

- Population of N individuals

- Information on the joint distribution of health and socioeconomic statuses is given by $\{(h_i, r_i)\}_{i=1}^{N}$, where
  - h_i represents health status
  - r_i the rank in the distribution of living standards (income, total expenditures, occupational categories, education level, etc), starting from the lowest level to the highest level of living standards.

Theoretical framework

- A rank dependent health achievement or socioeconomic health inequality index can always be rewritten in a general form:

$$I = \sum_{i=1}^{N} \omega(r_i)\eta(h_i).$$

- When $\omega(r_i) = \frac{1}{N} - \frac{(N-r_i+1)^\nu - (N-r_i)^\nu}{N^\nu}$, $\nu > 1$, the index is the generalized extended health concentration index, $GC(\nu)$.
- When $\omega(r_i) = \frac{(N-r_i+1)^\nu - (N-r_i)^\nu}{N^\nu}$, $\nu \geq 1$, the index is the health achievement index, $A(\nu)$.

Theoretical framework: Using categorical variables

- Note that all these indices have been developed and discussed under the assumption that the researcher is using a ratio scale variable.
- Wagstaff's achievement indices and the generalized extended concentration indices are sensitive to scaling:

Table: Health distribution by socioeconomic status.

Socioeco. rank.	SAH A	SAH B
1	poor	poor
2	fair	fair
3	good	good
4	good	very good
5	very good	very good
6	excellent	excellent
7	very good	excellent
8	fair	poor
9	excellent	excellent
10	poor	poor

Table: Alternative scaling functions

	$\eta_1(h)$	$\eta_2(h)$	$\eta_3(h)$
Poor	1	1	1
Fair	2	10	2
Good	3	11	3
Very good	4	12	4
Excellent	5	13	10
	$A_1(2)$	$A_2(2)$	$A_3(2)$
A	2.80	9.20	3.40
B	2.95	8.95	3.90
	$GC_1(2)$	$GC_2(2)$	$GC_3(2)$
A	0.2000	0.2000	0.6000
B	0.1500	-0.2500	0.7000

Theoretical framework

Some notations:

- let $\mathscr{P}_k := \{i : h_i = k\}$: set of individuals with health status in the kth category.
- $\phi(k) = \sum_{i \in \mathscr{P}_k} \omega(r_i)$: proportion of total social weight of individuals in the health category k
- $\Phi^1(k) = \sum_{l=1}^{k} \phi(k)$: total social weight for individual with health status $\leq k$
- $\Phi^1(k)$ will play the same role as the cumulative distribution for F.O.D.

Theoretical framework: Theorem 1

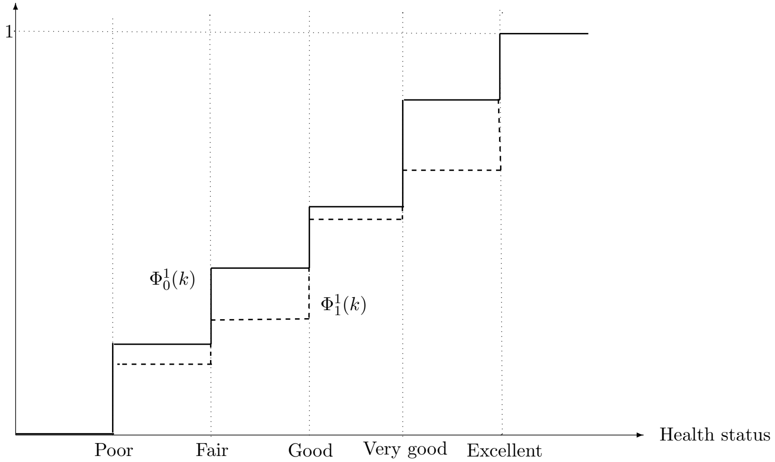
Theorem

$I_1 \geq I_0$ for all scaling functions $\eta(h)$ if and only if:

$$\Phi_0^1(k) \geq \Phi_1^1(k), \text{ for all } k \in \{1, 2, ..., K-1\}.$$

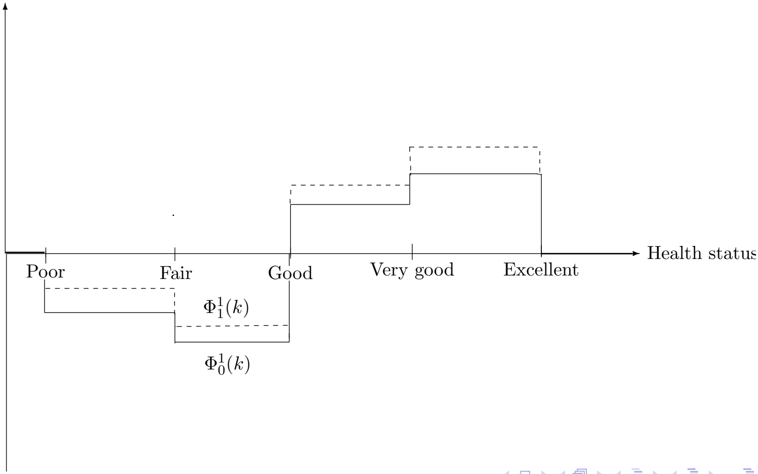
Theorem 1 applied on achievement indices

Figure: Theorem 1



Theorem 1 applied on inequality indices

Figure: Theorem 1



Theoretical Framework: Concave Scale Functions

- Concavity is a reasonable assumption if the analyst has a strong belief that differences between adjacent categories become less important as one moves towards the highest category.
- Let $\Phi^{2+}(k) = \sum_{j=1}^{k} \Phi^{1}(j)$.

Theorem

$I_1 \geq I_0$ for all concave scaling functions $\eta(h)$ if and only if:

$$\Phi_0^{2+}(k) \geq \Phi_1^{2+}(k), \text{ for all } k \in \{1, 2, \ldots, K-1\}.$$

Theoretical Framework: Convex Scale Functions

- Convexity is a reasonable assumption if the analyst has a strong belief that differences between adjacent categories become more important as one moves towards the highest category.
- Let $\Phi^{2-}(k) = \sum_{j=k}^{K-1} \Phi^{1}(j)$.

Theorem

$I_1 \geq I_0$ for all convex numerical scales $\eta(h)$ if and only if:

$$\Phi_0^{2-}(k) \geq \Phi_1^{2-}(k), \textit{ for all } k \in \{1, 2, \ldots, K-1\}.$$

Theoretical Framework: Relative Index of Health Inequality

- The relative index of socioeconomic health inequality, *RI*, can be expressed as:

$$RI = AI/\mu_\eta,$$

where *AI* is the absolute index of socioeconomic health inequality and μ_η and the average health status.

- If *RI* is $C(\nu)$ then *AI* is $GC(\nu)$.

Corollary

$RI_1 \le RI_0$ *for all* $\eta(h)$ *under consideration if*

$$AI_1 \le AI_0 \text{ for all } \eta(h) \text{ under consideration}$$

and,

$$\mu_{\eta 1} \ge \mu_{\eta 0} \text{ for all } \eta(h) \text{ under consideration.}$$

National Health Interview Survey for 2012

- The NHIS has monitored the health of the United States of America since 1957.
- The NHIS is a cross-sectional household interview survey that is representative of households and noninstitutional group quarters.
- We use information on household income to infer the socioeconomic rank of the individual.
- The surveys has includes a self-reported health status variable and a self-reported sadness variable, both with 5 categories.
- We compare health achievement and socioeconomic health inequality in 4 regions: Northeast, Midwest, South and West.

National Health Interview Survey for 2012

Table: Description of the two categorical variables

Would you say that your health in general is ...	During the past 30 days, how often did you feel so sad that nothing could cheer you up?
Poor	NONE of the time
Fair	A LITTLE of the time
Good	SOME of the time
Very good	MOST of the time
Excellent	ALL of the time

Theorem 1 Self-Reported Health Status

$A(2)$

	Northeast	Midwest	South	West
Northeast		D	D	ND
Midwest			ND	
South				
West		D	D	

$GC(2)$

	Northeast	Midwest	South	West
Northeast		ND	ND	
Midwest				
South		D		
West	D	D	D	

Theorem 2 Self-Reported Health Status

	Northeast	Midwest	South	West
		A(2)		
Northeast		D	D	ND
Midwest			D	
South				
West		D	D	
	Northeast	**Midwest**	**South**	**West**
		GC(2)		
Northeast		D	D	
Midwest				
South		D		
West	D	D	D	

Theorem 1 Self-Reported Sadness

	Northeast	Midwest	South	West
		A(2)		
Northeast		ND	D	ND
Midwest			ND	ND
South				ND
West				
	Northeast	Midwest	South	West
		GC(2)		
Northeast			ND	
Midwest	D		D	ND
South				
West	D		D	

Theorem 3 Self-Reported Sadness

	Northeast	Midwest	South	West
		A(2)		
Northeast			D	
Midwest	D		D	D
South				
West	D		D	
	Northeast	Midwest	South	West
		GC(2)		
Northeast			ND	
Midwest	D		D	ND
South				
West	D		D	

Theorem 1 and Corollary 1 Self-Reported Health Status

	Northeast	Midwest	South	West
		$C(2)$		
Northeast		ND	ND	ND
Midwest			ND	ND
South				
West			D	

Theorem 2 and Corollary 1 Self-Reported Health Status

	Northeast	Midwest	South	West
		$C(2)$		
Northeast		D	D	ND
Midwest			ND	
South				
West		D	D	

A stochastic dominance approach for dealing with the choice of indices

Normative foundation of socioeconomic health inequality measurement

- Inequality and social welfare indices are usually constructed on transfer principles.
- Bleichrodt and van Doorslaer (2004) show that the concentration indices are based on the *Principle of income-related health transfer*.

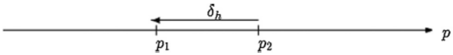


Fig. 1. Second order ethical principle.

Normative foundation of socioeconomic health inequality measurement

- Erreygers, Clarke and Van Ourti (2012) explain that Wagstaff's extended concentration indices incorporate an implicit *pro-poor* view of higher order aversion to socioeconomic health inequality.

- Makdissi and Yazbeck (2014) describe the transfer principles associated with Wagstaff's extended concentration indices and coin them *Pro-poor transfer sensitivity* of order $s \in \{3, 4, \dots\}$

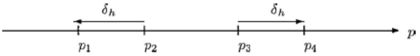


Fig. 2. Third order ethical principle.

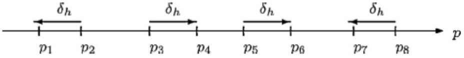


Fig. 3. Fourth order ethical principle.

Normative foundation of socioeconomic health inequality measurement

- Erreygers, Clarke and Van Ourti (2012) argues that a *pro-poor* view of higher inequality aversion may not be appropriate in the bi-variate context of socioeconomic health inequality measurement.

- They argue that a socioeconomic health inequality measure should pass the *upside-down test* and show that only the canonical health concentration index passes this test in Wagstaff's class of extended concentration indices and propose an alternative class of symmetric socioeconomic health inequality indices.

Normative foundation of socioeconomic health inequality measurement

- Khaled, Makdissi and Yazbeck (2018) describe the transfer principles associated with Erreygers, Clarke and Van Ourti symmetric indices and coin them *pro-extreme ranks* transfer sensitivity..

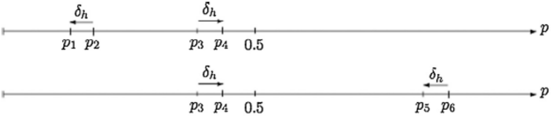


Fig. 2. Third order pro-extreme rank transfer sensitivity.

Stochastic dominance and health inequality

Table: Specific Equity Index vs. Distributional Dominance:

	Equity Indices	Distributional Dominance
Assumption SWF	Exact Specification	Wide class of SWF
Characteristics	Single number	No single number
	Cardinal ranking	Ordinal ranking
Type of Ordering	Complete	Partial
Robustness of outcome	Weak	Strong

Khaled, Makdissi and Yazbeck (2018)

- **Reference:** Khaled, M., P. Makdissi and M. Yazbeck (2018), Income-Related Health Transfers Principles and Orderings of Joint Distributions of Income and Health, *Journal of Health Economics*, 57, 315-331.

Dominance tests in health inequality literature

	Topic
Duclos & Échevin (2011)	Bi-dimensional social welfare
Garcia-Diaz & Sosa Rubi (2011)	Out-of-pocket health expenditures
Zheng (2011)	Health opportunities
Makdissi & Yazbeck (2014)	“Pro-poor” ethical principles
This paper	“Pro-poor” and “pro-extreme ranks” ethical principles Estimators of empirical curves and statistical tests

Paper in brief

1. Dominance for “pro-extreme-ranks” indices:
   - We formalize “pro-extreme-ranks” ethical principles.
   - We introduce new graphical concepts:
     - *Health range* curves
     - *Generalized health range* curves.
   - We show how health range curves and their generalized versions can be used to identify robust rankings of relative socioeconomic health inequality and health achievement for “pro-extreme ranks” ethical principles.
2. We derive natural estimators of all orders of:
   - Health concentration curves
   - Generalized health concentration curves
   - Health range curves
   - Generalized health range curves

Paper in brief

3. We derive consistent one sided statistical tests (KS type) for health achievement and socioeconomic health inequality dominance.

4. We offer an empirical illustration using information on cigarette smoking in the National Health Interview Surveys (US) for 1997 and 2014.

The measurement framework

- Assume we have two random variables health, H, and income, Y, with joint density $f_{H,Y}$
- The health status function is given by:

$$h(p) = \mathbb{E}[H|Y = F_Y^{-1}(p)]$$

- Health achievement index: $A = \int_0^1 \omega(p)h(p)\mathrm{d}p$
- Relative socioeconomic health inequality index: $I = \frac{1}{\mu_h}\int_0^1 \nu(p)h(p)\mathrm{d}p$
- where
  - $\mu_h = \int_0^1 h(p)\mathrm{d}p$ is the average health status
  - $\omega(p)$ and $\nu(p) = 1 - \omega(p)$ are weight functions
  - The functional forms of $\omega(p)$ and $\nu(p)$ embody the desired ethical principles.

Set of indices obeying the principle of income-related health transfer

- Sets of health achievement indices

$$\Omega^2 := \left\{ A(h) \;\middle|\; \begin{array}{c} \omega(p) \text{ is continuous and differentiable almost} \\ \text{everywhere over } [0,1], \int_0^1 \omega(p)\mathrm{d}p = 1, \\ \omega(1) = 0, \omega^{(1)}(p) \leq 0, \ \forall p \in [0,1] \end{array} \right\}$$

- Sets of socioeconomic health inequality indices

$$\Lambda^2 := \left\{ I(h) \;\middle|\; \begin{array}{c} \nu(p) \text{ is continuous and differentiable almost} \\ \text{everywhere over } [0,1], \int_0^1 \nu(p)\mathrm{d}p = 0, \\ \nu(1) = 1, \nu^{(1)}(p) > 0, \ \forall p \in [0,1] \end{array} \right\}$$

"Pro-poor" ethical principle

- Sets of "pro-poor" achievement indices

$$\Omega_\pi^s := \left\{ A(h) \in \Omega^2 \;\middle|\; \begin{array}{c} \omega(p) \text{ is continuous and } (s-1)\text{-time differentiable almost} \\ \text{everywhere over } [0,1]\,, \omega^{(i)}(1) = 0, (-1)^i \, \omega^{(i)}(p) \geq 0, \ \forall p \in [0,1], \\ \forall i = 1, 2, \ldots, s-1 \end{array} \right\}$$

- Sets of "pro-poor" socioeconomic health inequality indices

$$\Lambda_\pi^s := \left\{ I(h) \in \Lambda^2 \;\middle|\; \begin{array}{c} \nu(p) \text{ is continuous and } (s-1)\text{-time differentiable almost} \\ \text{everywhere over } [0,1]\,, \nu^{(i)}(1) = 0, (-1)^{i+1} \, \nu^{(i)}(p) \geq 0, \ \forall p \in [0,1], \\ \forall i = 1, 2, \ldots, s-1 \end{array} \right\}$$

“Upside-down test”

- Sets of socioeconomic health inequality indices passing the “Upside-down test”

$$\Lambda^2_\rho := \left\{ I(h) \in \Lambda^2 \mid \nu(1-p) = -\nu(p)\ \forall p \in [0,1] \right\}.$$

- Sets of achievement indices passing the “Upside-down test”

$$\Omega^2_\rho := \left\{ A(h) \in \Omega^2 \mid \omega(1-p) = 2-\omega(p)\ \forall p \in [0,1] \right\}.$$

“Pro-extreme ranks” ethical principle

- Sets of “pro-extreme ranks” achievement indices

$$\Omega_\rho^s := \left\{ A(h) \in \Omega_\rho^2 \,\middle|\, \begin{array}{c} \omega(p)\text{is continuous and } (s-1)\text{-time differentiable almost} \\ \text{everywhere over } [0,1], \omega^{(i)}(0.5) = 0, (-1)^i \omega^{(i)}(p) \geq 0, \\ \forall p \in [0, 0.5],\ \forall i = 1, 2, \ldots, s-1 \end{array} \right\}.$$

- Sets of “pro-extreme ranks” socioeconomic health inequality indices

$$\Lambda_\rho^s := \left\{ I(h) \in \Lambda_\rho^2 \,\middle|\, \begin{array}{c} \nu(p)\text{is continuous and } (s-1)\text{-time differentiable almost} \\ \text{everywhere over } [0,1], \nu^{(i)}(0.5) = 0, (-1)^{i+1} \nu^{(i)}(p) \geq 0, \\ \forall p \in [0, 0.5],\ \forall i = 1, 2, \ldots, s-1 \end{array} \right\}$$

Health concentration and health range curves

- Health concentration curves are defined over $[0, 1]$ as:

$$C^s(p) = \begin{cases} C(p) = \frac{1}{\mu_h} \int_0^p h(u)\mathrm{d}u & \text{if } s = 2 \\ \int_0^p C^{s-1}(p)\mathrm{d}p & \text{if } s \in \{3, 4, \dots\} \end{cases}$$

- Generalized health concentration curves: $GC(p) = \mu_h C(p)$ and $GC^s(p) = \mu_h C^s(p)$.

- Health range curves are defined over $[0, 0.5]$:

$$R^s(p) = \begin{cases} \frac{1}{\mu_h} \int_0^p r(u)\mathrm{d}u & \text{if } s = 2 \\ \int_0^p R^{s-1}(p)\mathrm{d}p & \text{if } s \in \{3, 4, \dots\} \end{cases}$$

where $r(p) = h(1 - p) - h(p)$.

- Generalized health range curves: $GR^s(p) = \mu_h R^s(p)$.

Orderings for income-related health transfer principle

Theorem

Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $I(h_1) \leq I(h_2)$ for all $I(h) \in \Lambda^2$ if and only if

$$C_1^2(p) \geq C_2^2(p) \text{ for all } p \in [0,1].$$

Theorem

Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $A(h_1) \geq A(h_2)$ for all $A(h) \in \Omega^2$ if and only if

$$GC_1^2(p) \geq GC_2^2(p) \text{ for all } p \in [0,1].$$

Orderings for pro-poor ethical principles

Theorem

Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health.
$I(h_1) \leq I(h_2)$ for all $I(h) \in \Lambda^s_\pi$ if and only if

$$C^s_1(p) \geq C^s_2(p) \text{ for all } p \in [0, 1].$$

Theorem

Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health.
$A(h_1) \geq A(h_2)$ for all $A(h) \in \Omega^s_\pi$ if and only if

$$GC^s_1(p) \geq GC^s_2(p) \text{ for all } p \in [0, 1].$$

Orderings for pro-extreme ranks ethical principles

Theorem

Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $I(h_1) \leq I(h_2)$ for all $I(h) \in \Lambda^s_\rho$ if and only if

$$R^s_2(p) \geq R^s_1(p) \text{ for all } p \in [0, 0.5].$$

Theorem

Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $A(h_1) \geq A(h_2)$ for all $A(h) \in \Omega^s_\rho$ if and only if

$$GR^s_2(p) \geq GR^s_1(p) \text{ for all } p \in [0, 0.5].$$

and,

$$\mu_{h1} \geq \mu_{h2}$$

Estimation

- Health concentration curves (on $p \in [0, 01]$):

$$\widehat{C}^s(p) = \frac{1}{(s-1)!n\overline{h}} \sum_{i=1}^{N} h_i(p - \widehat{F}^{-1}(y_i))^{s-2} \mathbb{1}(y_i \leq \widehat{F}_Y^{-1}(p))$$

- Generalized health concentration curves: $\widehat{GC}^s(p) = \overline{h}\widehat{C}^s(p)$
- Health range curves (on $p \in [0, 0.5]$):

$$\begin{aligned} \widehat{R}^s(p) &= \frac{1}{(s-1)!n\overline{h}} \sum_{i=1}^{N} h_i \Big[(\widehat{F}^{-1}(y_i) - (1-p))^{s-2} \mathbb{1}(y_i > \widehat{F}_Y^{-1}(1-p)) \\ &\qquad - (p - \widehat{F}^{-1}(y_i))^{s-2} \mathbb{1}(y_i \leq \widehat{F}_Y^{-1}(p)) \Big] \end{aligned}$$

- Generalized health range curves (on $p \in [0, 0.5]$): $\widehat{GR}^s(p) = \overline{h}\widehat{R}^s(p)$.

Consistent tests

- Denote by D one of the 4 curves.
- Define the new function $D_{12}(p) := D_1(p) - D_2(p)$ for $p \in \mathscr{S}$, where $\mathscr{S} := [0,1]$ for $C^s(p)$ and $GC^s(p)$ and $\mathscr{S} := [0,0.5]$ for $R^s(p)$ and $GR^s(p)$
- The test for the first three theorems is:

$$
\begin{aligned}
H_0 &: D_{12}(p) \geqslant 0, \forall p \in \mathscr{S} \\
H_1 &: D_{12}(p) < 0 \text{ for some } p \in \mathscr{S}
\end{aligned}
$$

- Let $\tau = \sup_p D_{12}(p)$ and $\hat{\tau}$, a non-parametric estimator of τ.
- Since the asymptotic distribution of $\hat{\tau}$ will be that of a functional of two-dimensional Gaussian process that is very complicated to construct, we use a bootstrap method for these tests.

Consistent tests

- The test for the last theorem is an intersection-union test:

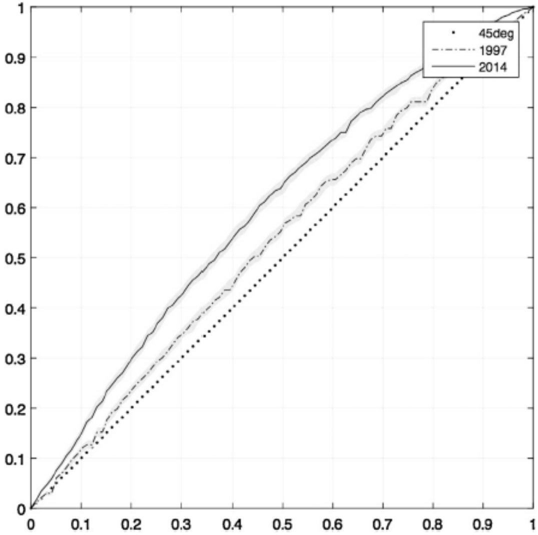
$$
\begin{aligned}
H_0 &: D_{12}(p) \leq 0, \forall p \in \mathscr{S} \text{ and } \overline{h}_1 \geq \overline{h}_1 \\
H_1 &: D_{12}(p) > 0 \text{ for some } p \in \mathscr{S} \text{ or } \overline{h}_1 < \overline{h}_1
\end{aligned}
$$

- We also use a bootstrap method for this test.

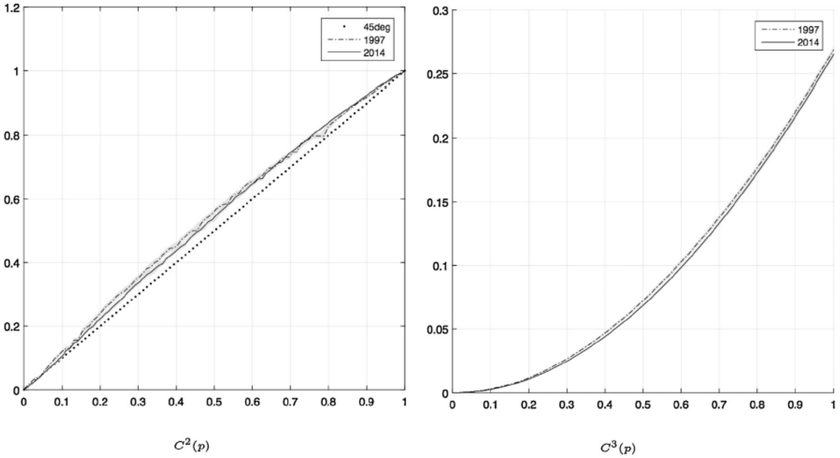
National Health Interview Survey

- The NHIS has monitored the health of the United States of America since 1957.
- The NHIS is a cross-sectional household interview survey that is representative of households and noninstitutional group quarters.
- We use information on household income, cigarette consumption and BMI between 1997 and 2014.

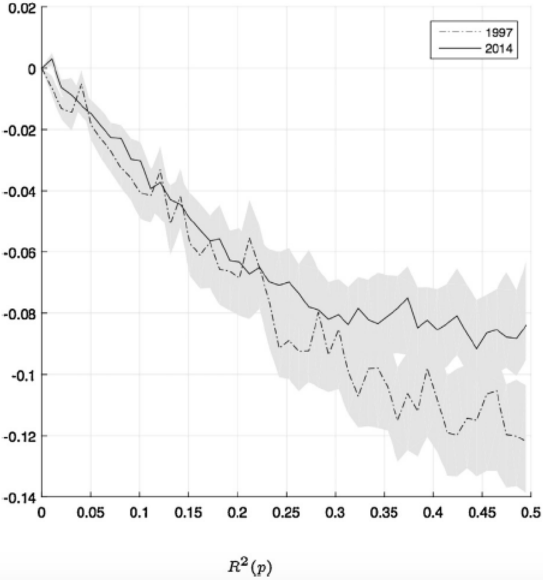
Concentration curves for cigarette consumption



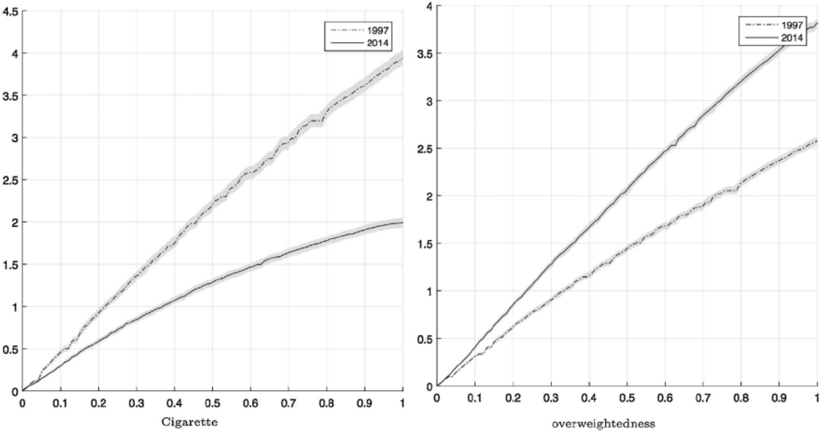
Concentration curves for overweightedness



Range curves for overweightedness



Generalized concentration curves



Khaled, Makdissi and Yazbeck on absolute socioeconomic health inequality comparisons

- **Reference:** Khaled, M., P. Makdissi and M. Yazbeck (2018), On the importance of the upside down test in absolute socioeconomic health inequality comparisons, Working Paper No. 1800003, Canadian Center for Health Economics.

Socioeconomic Health Inequalities: Absolute vs. Relative

- While the WHO suggested that both measures should be reported, empirically the reality is different.
- King, Harper and Young (2012) show that:
  - only 7% of research produced reports both measures.
  - only 18% of research produced reports absolute measures.

The Paper in Brief

- **Objective**: Propose methods that allow researchers to identify robust rankings of absolute socioeconomic health inequality comparisons.
- **Contributions**:
  - Derive dominance conditions for robust orderings of absolute socioeconomic health inequality comparisons.
  - Show that the canonical assumptions made in the literature do not allow for robust rankings of absolute socioeconomic measure of health inequality (at the second order).
  - Shed light on the importance of the role of the upside down principle in providing robust rankings of absolute socioeconomic health inequalities.
  - Shed light on a possible reason why the canonical absolute measure of socioeconomic health inequalities are rarely reported by researchers.

Identification of Robust Orderings: Theorem 1

- Absolute socioeconomic health inequality under the principle of income related health transfer

Theorem

Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health.
$I_A(h_1(p)) \leq I_A(h_2(p))$ for all $I_A(h(p)) \in \Lambda^2_A$ if and only if

$$GC_1(p) \geq GC_2(p) \text{ for all } p \in [0,1],$$

and,

$$\mu_{h_2} \geq \mu_{h_1},$$

- Note there is one additional constraint.

Identification of Robust Orderings: Corollary 1

Corollary

Let $f^1_{Y,H}$ and $f^2_{Y,H}$ represent two joint densities of income and health. $I_A(h_1(p)) \leq I_A(h_2(p))$ for all $I(h(p)) \in \Lambda^2_A$ if and only if $\mu_{h_1} = \mu_{h_2}$ and $I_R(h_1(p)) \leq I_R(h_2(p))$ for all $I_R(h(p)) \in \Lambda^2_R$.

- Under the principle of income related health transfer. One can identify robust orderings only if $\mu_{h_1} = \mu_{h_2}$.
  - Ranking are equivalent to ranking $I_R(.)$ if $\mu_{h_1} = \mu_{h_2}$.
  - $I_A(.)$ rankings are arbitrary if $\mu_{h_1} \neq \mu_{h_2}$.

Identification of Robust Orderings: Corollary 1

- Under the principle of income related health transfer, the assumptions made on $\nu(p)$ requires that the weight function has a non negative slope.
- There is no anchor point at which the weight function turns from negative to positive.
- Threshold at which $\nu(p)$ turns from negative to positive is anywhere on $(0, 1)$.
- But the upside down principle proposed by Erreygers, Clarke and van Ourti (2012) offers an ethical principle that naturally accounts for this issue: the symmetry around the median principle.

Identification of Robust Orderings: Theorem 2

Theorem

Let $f^1_{Y,H}$ *and* $f^2_{Y,H}$ *represent two joint densities of income and health.* $I_A(h_1(p)) \leq I_A(h_2(p))$ *for all* $I_A(h(p)) \in \Lambda^2_{A\rho}$ *if and only if*

$$GR_2(p) \geq GR_1(p) \text{ for all } p \in [0, 0.5].$$

Identification of Robust Orderings: Higher Order

- We have pointed that at the second order under the principle of income related health transfer, it is not possible to obtain a robust ranking for absolute socioeconomic health inequality unless average health in the distributions compared is equal.
- We highlight the importance symmetry around the median principle in obtaining robust rankings in the context of absolute socioeconomic health inequalities.
- Is this the only way one can obtain robust orderings of absolute socioeconomic health inequality?
  - If at the second order $\rightarrow$ Yes.
  - If willing to move to higher orders $\rightarrow$ No.

Identification of Robust Orderings: Higher Order

- Higher order pro-poor transfer sensitivity (e.g., Wagstaff, 2002).
  - We show that if there is not intersection of the GC curves at the second order then there exist no higher order for which dominance can be established under the principle of income related health transfer.
  - We show that if the two curves intersect at the second order, then there will be an order for which dominance can be established.
- Higher order pro-extreme rank transfer sensitivity (e.g., Erreygers, Clarke and van Ourti, 2012).
  - We derive a classical non intersection dominance condition.