Measuring Unfair Inequality: Reconciling Equality of Opportunity and Freedom from Poverty

Paul Hufe, Ravi Kanbur & Andreas Peichl

Canazei Winter Fun with Inequality

Alba di Canazei, 2019-01-08

Outline

Inequality in Economics

Typically, economists think about inequality as part of a trade-off:

Equity $\leftrightarrow$ Efficiency

1. But what does equity mean?
2. ... and how can we measure it?

- Let $Y^e = \{y_1^e, y_2^e, ..., y_n^e\}$ be the empirical distribution of income.
- Let the mean of the distribution be μ.
- Consider standard measures of inequality:

$$G = \frac{1}{n}\left(n + 1 - 2\frac{\sum_{i=1}^{n}(n+1-i)y_i^e}{\sum_{i=1}^{n} y_i^e}\right)$$

$$A(\epsilon) = \begin{cases} 1, & \epsilon = 0 \\ 1 - \frac{1}{\mu}\left(\prod_{i=1}^{n} y_i^e\right)^{1/n}, & \epsilon = 1 \\ 1 - \frac{1}{\mu}\left(\frac{1}{n}\sum_{i=1}^{n}(y_i^e)^{1-\epsilon}\right)^{1/(1-\epsilon)}, & \text{otherwise.} \end{cases}$$

$$\text{MLD} = \frac{1}{n}\sum_{i=1}^{n} \ln \frac{\mu}{y_i^e}$$

Norm-based inequality measurement

Each of these inequality measures can be seen as a divergence metric between the vector of observed incomes

$$Y^e = \{y_1^e, y_2^e, ..., y_n^e\}, \tag{1}$$

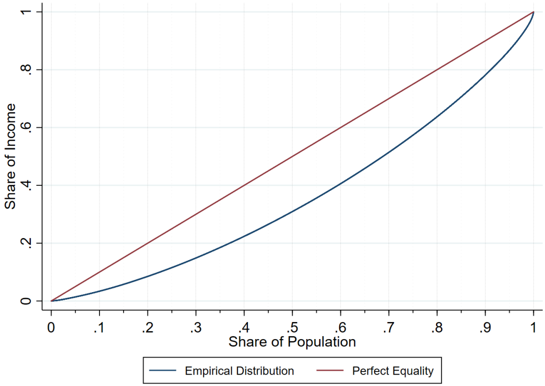
and a norm vector.
In case of standard (income) inequality measures, this norm is the vector where each element is μ, i.e.

$$M = \{\mu, \mu, ..., \mu\} \tag{2}$$

In other words, the vector where total income is distributed equally.

Measuring inequality

Figure: Lorenz-Curve Representation



- In the conventional inequality measurement literature, all the action resides in the properties of this divergence metric.
- Desirable properties for this metric include:
  - Scale Independence
  - Principle of Populations
  - Pigou-Dalton Principle of Transfers
- An additional property often used is sub-group decomposability. This property, with a few other assumptions leads to the Generalized Entropy class of inequality measures:

$$\mathrm{GE}(\alpha) = \begin{cases} \frac{1}{n}\sum_{i=1}^{n} \ln\left(\frac{\mu}{y_i^e}\right), & \alpha = 0 \\ \frac{1}{n}\sum_{i=1}^{n} \left(\frac{y_i^e}{\mu}\right) \ln\left(\frac{y_i^e}{\mu}\right), & \alpha = 1 \\ \frac{1}{n}\frac{1}{\alpha(\alpha-1)}\sum_{i=1}^{n} \left[\left(\frac{y_i^e}{\mu}\right)^{\alpha} - 1\right], & \text{otherwise.} \end{cases} \quad (3)$$

- Note that with $\alpha = 0$ we have the MLD measure.

Link to optimal taxation?

- Note that this is all to do with measuring inequality. It is a pure distributional question.
- Of course if we move to redistribution then there will be incentive effects and the mean will be affected. This leads to the large literature on optimal taxation, going back to Mirrlees (1971).
- This will NOT be the focus of this talk.

- So far, measuring inequality deals with the choice of the divergence metric between the observed distribution Y^e and the reference distribution M, the perfect equality distribution.
- But a resurgent part of the literature argues that what is at issue is not so much the metric of divergence of the actual from the reference vector, but the reference distribution itself.
- Why should we take perfect equality of outcomes as the reference, or the norm, or, in effect, the ideal? Surely the process whereby the outcomes came to be, matters as well?!?!
- The general problem is then posed as the divergence between the observed distribution Y^e and a reference or a norm distribution

$$Y^r = \{y_1^r, y_2^r, ..., y_n^r\} \quad (4)$$

- Y^r has the same mean as the observed distribution but is not necessarily M, the perfect equality distribution.

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Our Contribution

We focus on two well-established principles of distributive justice (Konow, 2003; Konow and Schwettmann, 2016), namely ...

- Equality of Opportunity (EOp)
- Freedom from Poverty (FfP)

... to derive a new empirical measure for unfair inequalities – by constructing a new norm/reference distribution.

Related literature

1. Brunori et al. (2013) propose an "opportunity sensitive poverty measure" that weighs incomes below the poverty line by the value of the individual's opportunity set. However, this is a poverty measure compliant with the focus axiom, i.e. it is invariant to income changes among the non-poor. Hence, the secondary principle – in this case EOp – carries no weight once the prioritized principle – in this case FfP – is realized.
2. Ferreira and Peragine (2016) construct "opportunity-deprivation profiles" where members of types are considered opportunity-deprived if their average outcome falls below a pre-specified deprivation threshold. Effectively, this amounts to applying standard poverty measures to types instead of individuals.

We treat EOp and FfP as co-equal principles and develop the first family of measures that is able to detect unfairness emanating from violations of EOp or FfP even if one of the two is satisfied.

Outline

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Equality of Opportunity

Which inequalities do we care about?

- The primary question concerns the construction of the norm vector.
- This is where the insurgency in the inequality measurement literature has come in recent years.
- The insurgency's premise is that what matters normatively is not equality of outcome, but equality of opportunity.
- This insurgency has deep roots in an older and esteemed philosophical literature. Metaphors associated with this view are "leveling the playing field" and "starting gate equality".
- Main philosophical accounts:
  1. Rawls (1971): Fair chance to achieve positions.
  2. Dworkin (1981a,b): Resource egalitarianism.
  3. Arneson (1989): Equal Opportunity for Welfare.
  4. Cohen (1989): Equal Access to Advantage.

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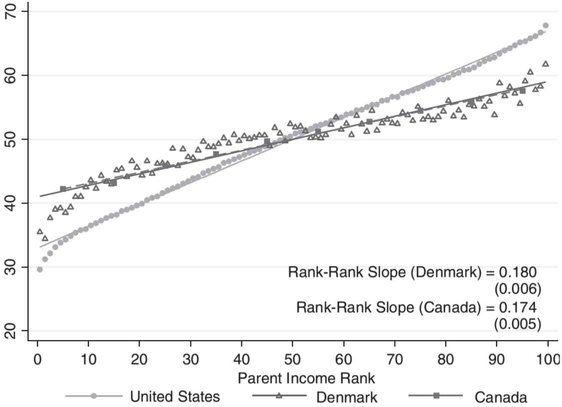
In general, Equality of Opportunity pre-supposes that all determinants of individual outcomes are the result of two sets of factors:

1. Circumstances, $C \in \Omega$: Factors beyond individual control.
   $\rightarrow$ Unfair
2. Efforts, $E \in \Theta$: Factors within the control of individuals.
   $\rightarrow$ Fair

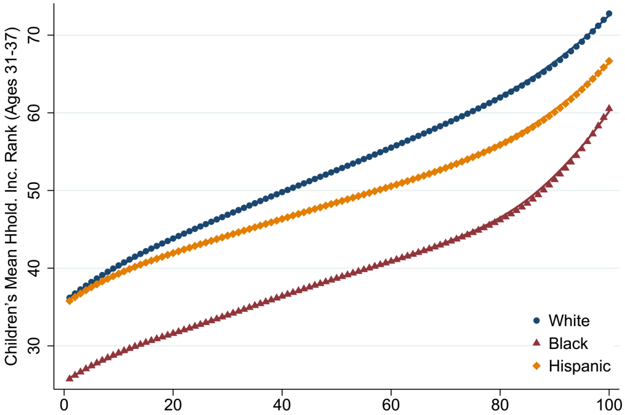
EOp measured via intergenerational mobility

Figure: Chetty et al. (2014)



IGM conditional on race

Figure: Chetty et al. (2018)



Based on these circumstances we can partition the population into types:

	High Parental Inc.	Low Parental Inc.
White	Type 1	Type 3
Non-White	Type 2	Type 4

→ Inequality between circumstance types is morally objectionable.
→ If all mean incomes across types are equal, we have EOp

The equality of opportunity principle is reflected in distributional preferences:

- Vignette studies: Faravelli (2007).
- Survey Experiments: Alesina et al. (2018).
- Lab Experiments: Cappelen et al. (2007); Krawczyk (2010); Mollerstrom et al. (2015).

Freedom from Poverty

- Are ex-post inequalities a matter of indifference for fairness evaluations?
- Some answers:
  - Fleurbaey (1995, 2008) argues for outcome egalitarianism in spheres of social interest → satisfaction of basic needs.
  - Anderson (1999) argues against pure opportunity egalitarians based on a number of examples → abandonment of negligent victims.

Based on realized outcomes we can partition the population into groups where $P = \{i : y_i \leq y_{\min}\}$ and $R = \{i : y_i > y_{\min}\}$:

	P	R
L	$y_i \leq \mu_P$	$y_i \leq \mu_R$
H	$y_j > \mu_P$	$y_i > \mu_R$

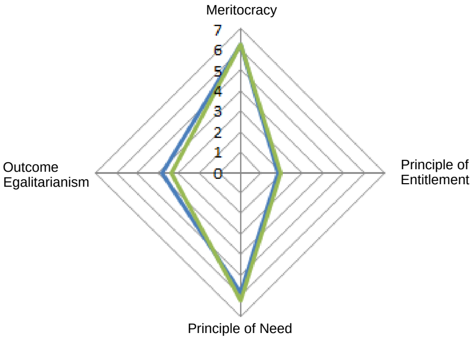
$\rightarrow$ Inequalities are objectionable (i) among individuals in P and (ii) to the extent that $\mu_P < y_{\min}$.

The freedom from poverty principle is reflected in distributional preferences:

- Vignette studies: Gaertner and Schwettmann (2007); Konow (2001).
- Lab Experiments: Cappelen et al. (2013).

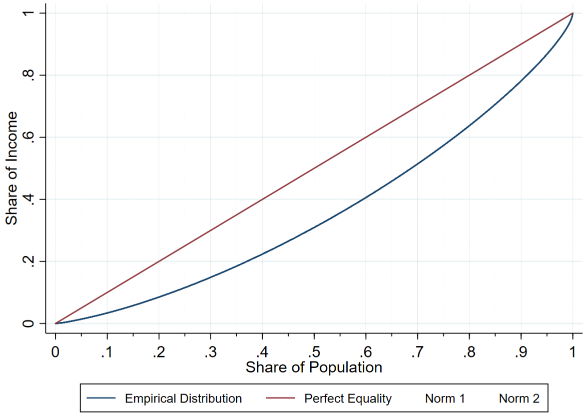
Recent evidence from Germany (Sep 2018):

Figure: Eisnecker et al. 2018



Outline

Figure: Norm-Based Inequality Measurement



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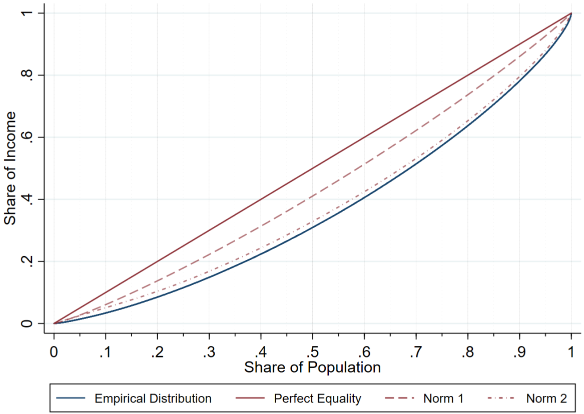

Norm Vector

Consider the following restrictions on the set of all possible income distributions D:

- Constant Resources:

$$D^1 = \left\{ D : \sum_i y_i^r = \sum_i y_i^e \right\} \tag{5}$$

- Equality of Opportunity:

$$D^2 = \{ D : \mu_t^r = \mu \ \forall \ t \in T \} \tag{6}$$

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- Freedom from Poverty:

$$D^3 = \{D : y_i^r = y_{\min} \ \forall \ i \in P\} \tag{7}$$

- Financing I:

$$D^4 = \{D : y_i^r \geq y_{\min} \ \forall \ i \in R\} \tag{8}$$

- Financing II:

$$D^5 = \left\{D : \forall t \in T, \ \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^e - y_{\min}}{y_j^e - y_{\min}} \ \forall \ i, j \in t \cap R\right\} \tag{9}$$

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The intersection $\cap_{s=1}^{5} D^s$ yields a singleton:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e < y_{\min} \\ y_i^e \left[1 - \tilde{y}_i \left(\tau^{\text{FfP}} + \tau^{\text{EOp}}(1 - \tau^{\text{FfP}})\right)\right], & \text{otherwise.} \end{cases} \qquad (10)$$

where

$$\tilde{y}_i = \left(\frac{y_i^e - y_{\min}}{y_{\min}}\right),$$

$$\tau^{\text{FfP}} = \frac{N_P(y_{\min} - \mu_P^e)}{N_R(\mu_R^e - y_{\min})},$$

$$\tau_t^{\text{EOp}} = \frac{\mu_t^e + \frac{N_{P\cap t}}{N_t}(y_{\min} - \mu_{P\cap t}^e) - \tau^{\text{FfP}}\left(\frac{N_{R\cap t}}{N_t}(\mu_{R\cap t}^e - y_{\min})\right) - \mu}{\mu_t^e + \frac{N_{P\cap t}}{N_t}(y_{\min} - \mu_{P\cap t}^e) - \tau^{\text{FfP}}\left(\frac{N_{R\cap t}}{N_t}(\mu_{R\cap t}^e - y_{\min})\right) - y_{\min}}$$

Divergence Measure

- Unfair inequality is then measured as the divergence $D(Y^e||Y^r)$ between the observed and the norm income distribution.
- Various divergence measures have been proposed in the literature: Almås et al. (2011); Cowell (1985); Magdalou and Nock (2011).
- We rely on a generalization of the generalized entropy class proposed by Magdalou and Nock (2011) with $\alpha = 0$:

$$D(Y^e||Y^r) = \frac{1}{N}\sum_i \left[\ln \frac{y_i^r}{y_i^e} + \frac{y_i^e}{y_i^r} - 1\right]. \tag{11}$$

Properties

Imagine we are indifferent to FfP. Then, the norm vector simplifies to:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e < y_{\min} \\ y_i^e \left[1 - \tilde{y}_i \left(\tau^{\text{FfP}} + \tau^{\text{EOp}}(1 - \tau^{\text{FfP}})\right)\right], & \text{otherwise.} \end{cases}$$

$$= y_i^e \left[1 - \left(\frac{\mu_t^e - \mu}{\mu_t^e}\right)\right] = y_i^e \left[\frac{\mu}{\mu_t^e}\right]$$

Using $y_i^e \left[\frac{\mu}{\mu_t^e}\right]$ in the measure of distributional change gives:

$$D(Y^e || Y^r_{\text{EOp}}) = \frac{1}{N} \sum_i \left[\ln \frac{y_i^r}{y_i^e} + \frac{y_i^e}{y_i^r} - 1 \right]$$
$$= \frac{1}{N} \sum_i \ln \frac{\mu}{\mu_t^e}.$$

This is a summary statistic of the distribution of type income means: the mean log deviation.

Imagine we are indifferent to EOp. Then, the norm vector simplifies to:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e < y_{\min} \\ y_i^e \left[1 - \tilde{y}_i \left(\tau^{\text{FfP}} + \tau^{\text{EOp}}(1 - \tau^{\text{FfP}})\right)\right], & \text{otherwise.} \end{cases}$$

$$= \begin{cases} y_{\min}, & \text{if } y_i^e < y_{\min} \\ y_i^e \left[1 - \tilde{y}_i \tau^{\text{FfP}}\right], & \text{otherwise.} \end{cases}$$

Using this norm vector in the measure of distributional change gives:

$$D(Y^e||Y^r_{\text{FfP}}) = \underbrace{\frac{1}{N}\sum_{i\in P}\ln\frac{y_{\min}}{y_i^e}}_{\text{Watts Index}} - \underbrace{\frac{1}{N}\sum_{i\in P}\left(\frac{y_{\min}-y_i^e}{y_{\min}}\right)}_{\text{Poverty Gap}} + \frac{1}{N}\sum_{i\in R}\ln\frac{y_i^r}{y_i^e} + \left(\frac{y_i^e}{y_i^r} - 1\right).$$

This incorporates two widely used poverty measures, the Watts Index and the Poverty Gap ratio.

Outline

Methods and Data

Empirics

Data:

- Cross-sectional: EU-SILC 2011.
- Longitudinal: PSID (1969-2012).

Type Partition:

- Circumstances: Sex, Occupation Parents, Education Parents, Immigration Background (Race) (36 types).

Income Concept:

- Equivalized disposable HH income (OECD equivalence scale).

Poverty Measure:

- At-Risk-Of-Poverty-Rate (60% of median income).

Results

Figure: Unfair Inequality by Country (Europe)

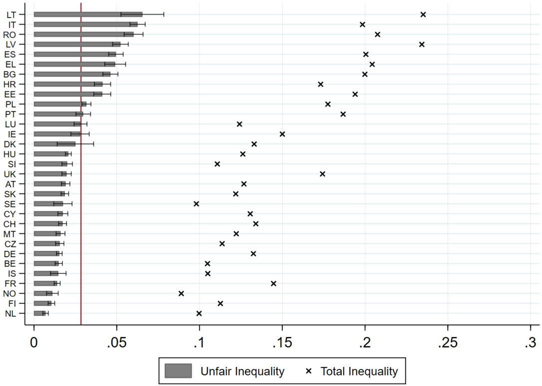


Figure: Decomposition by Country (Europe)

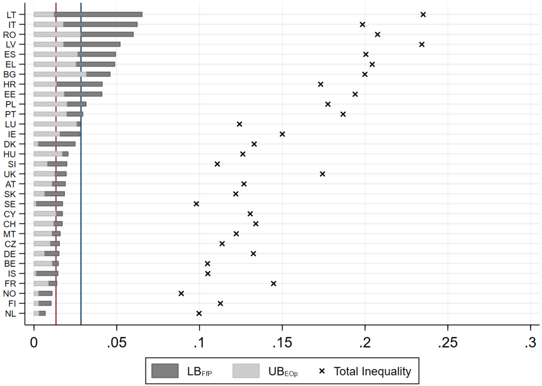
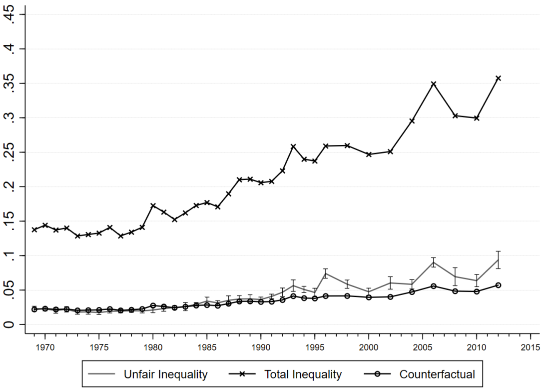


Figure: Unfair Inequality over Time (USA)



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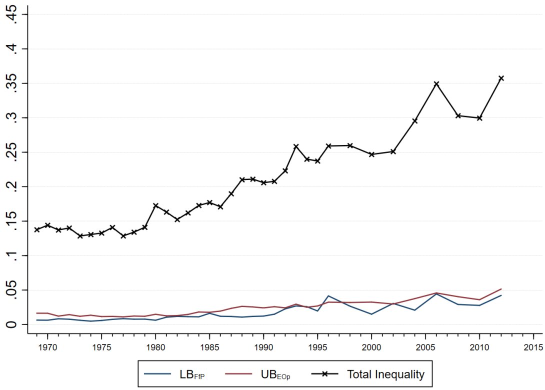

Sensitivity Checks

Sensitivity Checks

1. Varying poverty thresholds. Graph
2. Alternations in normative assumptions. Graph
3. Alternative divergence measures: Almås et al. (2011); Cowell (1985); Magdalou and Nock (2011). Table

Outline

Summary

1. The extent of unfairness/inequity in observed inequality is either overstated (standard inequality measures) or understated (EOp measures).
2. We recognize the multiplicity of fairness ideals by drawing onto the principles of EOp and FfP.
3. Combining different normative principles, i.e. EOp and FfP, yields strong upwards corrections of the unfair share of inequality.
4. The framework may be fruitfully complemented by further ideals of fairness.

Q&A

Thank you!

peichl@econ.lmu.de

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Figure: Alternative Poverty Thresholds by Country (Europe)

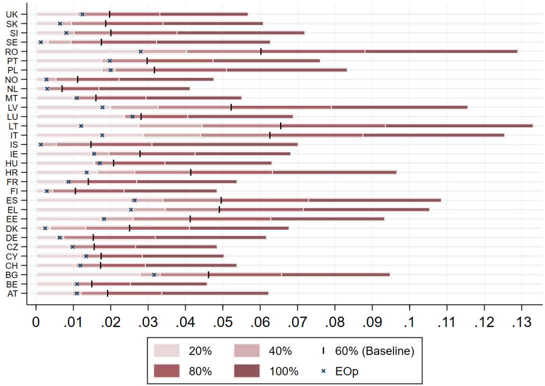
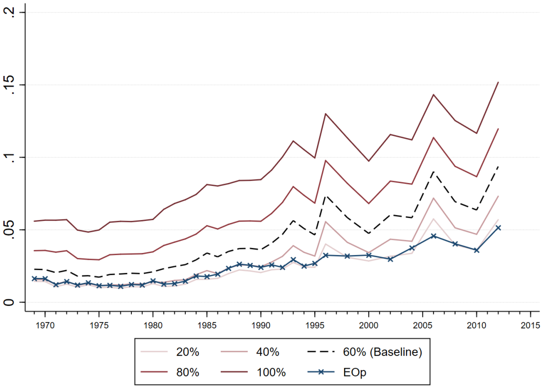
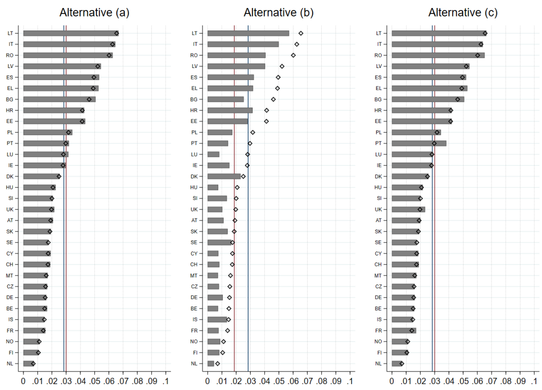


Figure: Alternative Poverty Thresholds over Time (USA)



Back

Figure: Alternative Norm Distributions by Country (Europe)



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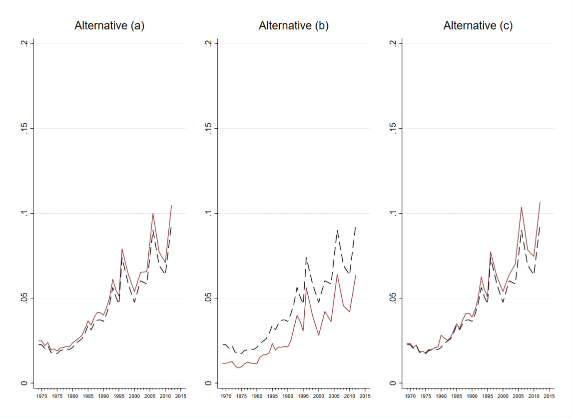

Back

Table: Rank Correlation of Measures by Country (Europe)

	Magdalou and Nock			Cowell			Almås et al.
	$\alpha = 0$ (Baseline)	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	
Magdalou and Nock							
$\alpha = 0$	1.000						
$\alpha = 1$	0.953	1.000					
$\alpha = 2$	0.911	0.982	1.000				
Cowell							
$\alpha = 0$	0.975	0.988	0.963	1.000			
$\alpha = 1$	0.953	1.000	0.982	0.988	1.000		
$\alpha = 2$	0.939	0.994	0.986	0.976	0.994	1.000	
Almås et al.							
	0.912	0.970	0.971	0.955	0.970	0.970	1.000

Table: Rank Correlation of Measures over Time (USA)

	Magdalou and Nock			Cowell			Almås et al.
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Magdalou and Nock							
$\alpha = 0$	1.000						
$\alpha = 1$	0.991	1.000					
$\alpha = 2$	0.961	0.975	1.000				
Cowell							
$\alpha = 0$	0.994	0.998	0.971	1.000			
$\alpha = 1$	0.991	1.000	0.975	0.998	1.000		
$\alpha = 2$	0.986	0.998	0.979	0.994	0.998	1.000	
Almås et al.							
	0.972	0.984	0.966	0.978	0.984	0.985	1.000

Back