Minimum income schemes and redistribution.

January 13-16 , 2020. Alba di Canazei, Italy

Winter School on
Inequality and Social Welfare Theory

A Social Welfare Approach for Measuring Welfare Protection

The Measurement of Social Welfare Gains in Social Assistance Programs

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A Social Welfare Approach for Measuring Welfare Protection

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MOTIVATION (1)

How alterative designs of Minimum income programmes impact on social welfare?

No consensus → Analysis based on two dimensions

a) Adequacy: compares level of benefit provided with the general living standard

b) Coverage: proportion of target population who receive it.

↓

Social welfare assessments may be different as we evaluate with one indicator or another.

MOTIVATION (2)

Adequacy

Evidence shows that welfare systems are quite insufficient to keep benefit levels in line with the general living standard (Van Mechelen and Marchal, 2013).

The economic crisis also affected the abilities of governments to provide adequate levels of benefits (Marchal *et al.*, 2014): growth of social needs and growing constraint on the allocation of resources.

Adequacy problems could vary greatly among different jurisdictions or countries.

How adequacy rates are measured is therefore very important:

- Most comparisons across regions or countries have been made in terms of the gaps between benefits and poverty lines (Ravallion, 2007)
- Over the last years there has been a shift in focus towards measures that relate benefits to in-work or net disposable income (Immervoll, 2010; Vandenbroucke *et al.*, 2013).

MOTIVATION (3)

Coverage

Eligibility and take-up rates stand as two of the main factors that drive the coverage rate.

Eligibility rules limit coverage by design, either by introducing categorical conditions that exclude potential beneficiaries or by setting the income threshold for entitlement too low.

Figari *et al.* (2013): in several countries, a large proportion of individuals are ineligible for welfare benefits even when they fall below a low poverty line.

High non take-up levels also involve that some eligible households do not receive benefits: targeting errors (Duclos, 1996), stigma and transaction costs (Moffitt, 1983; Whelan, 2010), information asymmetries (Currie, 2006), expectations regarding long-term unemployment and/or levels of social assistance payments (Bargain *et al.*, 2012), or endogenous government policy (Ayala and Triguero, 2017)

MOTIVATION (4)

Regional or country rankings and levels of inequality in terms of welfare protection can be very different depending upon the chosen outcome.

Governments can choose to maintain high levels of generosity while promoting low access to benefits by different means.

MOTIVATION (5)

Our Contributions

1.- We propose to assess the social welfare protection provided by welfare schemes based on SWF that finally are related to coverage and adequacy.

2.- We propose a parameterized family of indices that can be interpreted in terms of social welfare. These indices satisfy desirable properties.

3.- The methodology can be extrapolated to any decentralized welfare program.

Illustration: Spanish regional welfare programs, that are the last safety net for low-income households and they are completely decentralized.

ANALYTICAL FRAMEWORK (1)

Let $\aleph = \{1,2,\ldots,N\}$ be a population composed by N individuals.

$x = (x_i)_{i\in\aleph} = (x_1, x_2, \ldots, x_N)$ the distribution of income of the target population before receiving the MIP.

B the level of income guaranteed by MIP.

$\wp = \{1,2,\ldots,R\}$ the group of recipients of MIP.

$\Im = \{1,2,\ldots,L\}$ the target group or potential claimants, $\wp \subset \Im$.

$y_\Im = (y_i)_{i\in\Im} = (y_1, y_2, \ldots, y_L)$ the distribution of income of the target population after receiving the MIP

$y_i = B$ for recipients of MIP ($i \in \wp$).

$y_i = x_i$ for potential claimants that do not receive MIP ($i \in \overline{\wp}$).

ANALYTICAL FRAMEWORK (2)

Different criteria to define potential claimants but we consider a level of income.

Definition 1. An individual belongs to the target group $\mathfrak{I}$:

$$i \in \mathfrak{I} \quad \text{if } x_i \leq k,$$

where k is a certain level of income, with $k \leq B$.

The level of protection provided by MIP can be measured through the adequacy or coverage rate.

ANALYTICAL FRAMEWORK (3)

Definition 2. The coverage provided by the MIP, C is

$$C = R/L$$

where R is the number of recipients and L the number of potential claimants.

Definition 3. The adequacy of a MIP, A is

$$A = B/z$$

the ratio between the level of guaranteed income and the poverty line (z). We will consider that z is chosen such that z>B.

The literature has focused on quantifying adequacy and coverage but its consequences in terms of well-being have received little attention. We are not interested in the adequacy or coverage per se, but in the effects of both on economic welfare.

A SWF TO EVALUATE PROTECTION PROVIDED (1)

We define the social welfare associated with an income distribution as the social welfare corresponding to the potential claimants.
We do not consider the entire population.

We assume some standard properties of the SWF (Lambert, 1993; Cowell, 1995):

- Individualistic:

  SWF depends on individuals' utilities and on nothing else, there are no externalities.

- Additively separable:

  SWF is the summation of individuals' utilities.

  Each individual having the same utility function that depends only on her income

A SWF TO EVALUATE PROTECTION PROVIDED (1)

We assume some standard properties of the SWF (Lambert, 1993; Cowell, 1995):

- Strictly increasing function of income:

$$y > y' \Rightarrow W(y) < W(y')$$

  social welfare increases when, ceteris paribus, any potential claimant's income rises

- Symmetric:

  individuals play identical roles.
  Let $(y_i)_{i \in \mathcal{L}}, (y'_i)_{i \in \mathcal{L}}$ be two distributions such that y' is obtained from y by a permutation of incomes, then

$$W(y) = W(y')$$

- Focus:

  independent of the incomes of individuals who are not in the target group (non-potential claimants).

A SWF TO EVALUATE PROTECTIONI PROVIDED (2)

SWF can be written as

$$W(y) = \frac{1}{L}\sum_{i=1}^{L} U(y_i)$$

Conditions on U(.):

- increasing social utility function that depends only on the individuals' own incomes (Cowell, 1995).
- U(.) strictly concave and therefore inequality averse.
- Social marginal utility has constant elasticity (α-1).

A SWF TO EVALUATE PROTECTIONI PROVIDED (3)

Utility function is

$$U_\alpha\left(\frac{y_i}{z}\right) = \begin{cases} \frac{1}{\alpha}\left(\frac{y_i}{z}\right)^\alpha & 0 \neq \alpha < 1 \\ ln\left(\frac{y_i}{z}\right) & \alpha = 0 \end{cases}$$

Social Welfare function

$$W(y|z,\alpha) = \frac{1}{\alpha}\sum_{i=1}^{L}\frac{\left(\frac{y_i}{z}\right)^\alpha}{L}, \qquad 0 \neq \alpha < 1$$

$$W(y|z,0) = \sum_{i=1}^{L}\frac{ln\left(\frac{y_i}{z}\right)}{L}, \qquad when\ \alpha = 0$$

α is a measure of inequality aversion (how sharply curved the SWF is). The lower α the more the social welfare function takes the lower incomes into account.

A SWF TO EVALUATE PROTECTIONI PROVIDED (4)

In a general setting in which a MIP provides an income level B to the recipients

$$W(y|z,\alpha) = \frac{1}{\alpha}\sum_{i\in\wp}\frac{\left(\frac{B}{z}\right)^{\alpha}}{L} + \frac{1}{\alpha}\sum_{\substack{i\notin\wp \\ i\in\Im}}\frac{\left(\frac{y_i}{z}\right)^{\alpha}}{L} = \left(\frac{B}{z}\right)^{\alpha}\frac{R}{L}\frac{1}{\alpha} + \frac{1}{L}\sum_{\substack{i\notin\wp \\ i\in\Im}}\frac{\left(\frac{y_i}{z}\right)^{\alpha}}{\alpha}, \quad 0 \neq \alpha < 1$$

The lower α, the higher is the sensitivity of the social welfare to lower incomes, that is, those potential claimants that do not receive the guaranteed minimum income.

$$W(y|z,0) = \frac{R}{L}ln\left(\frac{B}{z}\right) + \frac{1}{L}\sum_{\substack{i\notin\wp \\ i\in\Im}} ln\left(\frac{y_i}{z}\right) \quad , \qquad when\ \alpha = 0.$$

A SWF TO EVALUATE PROTECTIONI PROVIDED (5)

Sometimes, these programs are targeted to households that have no income

$$W(y|z,\alpha) = \frac{1}{\alpha}\sum_{i\in\wp}\frac{\left(\frac{B}{z}\right)^{\alpha}}{L} = \frac{R}{L}\left(\frac{B}{z}\right)^{\alpha}\frac{1}{\alpha}, \qquad 0 \neq \alpha < 1$$

The protection provided by a MIP depends on the level of adequacy and coverage, and also on the level of income of those potential claimants that are not recipients. The balance will depend on the alternative value judgments summarized by α.

AN INDEX OF WELFARE PROTECTION (1)

We can calculate the equally distributed income level ξ such that $W(y|Z,\alpha) = W(\xi 1|Z,\alpha)$.

The index:

$$P = \frac{\xi}{z}$$

It is the proportionate welfare loss caused by having potential claimants with incomes below this threshold after receiving benefits.

$$P = \left\{\frac{1}{L}\sum_{i=1}^{L}\left(\frac{y_i}{z}\right)^{\alpha}\right\}^{1/\alpha}, \qquad 0 \neq \alpha < 1$$

$$P = \left\{\prod_{i=1}^{L}\left(\frac{y_i}{z}\right)\right\}^{1/L}, when\ \alpha = 0.$$

AN INDEX OF WELFARE PROTECTION (2)

In a general setting in which the MIP provides an income level B the former expression can be decomposed into:

$$P = \left\{ \left(\frac{B}{Z}\right)^{\alpha} \frac{R}{L} + \frac{1}{L} \sum_{\substack{i \notin \wp \\ i \in \Im}} \left(\frac{y_i}{Z}\right)^{\alpha} \right\}^{\frac{1}{\alpha}}$$

Special case $\rightarrow$ programs targeted to households with no income:

$$P = \left\{ \frac{1}{L} \sum_{i \in \wp} (B/Z)^{\alpha} \right\}^{1/\alpha} = \frac{B}{Z} \left(\frac{R}{L}\right)^{1/\alpha}, 0 \neq \alpha \leq 1$$

P is the Adequacy rate weighted by a function of Coverage rate (alternative value judgments on the relevance of adequacy and coverage).

PROPERTIES (1)

The measure that quantifies the protection provided by a MIP, P, satisfies a list of properties:

Property 1. Anonymity.
Let $(y_i)_{i \in \aleph}, (y'_i)_{i \in \aleph}$, be two distributions such that y' is obtained from y by a permutation of incomes, then

$$P(y') = P(y),$$

the only thing that matters is income.

Property 2. Focus.
Let $(y_i)_{i \in \aleph}, (y'_i)_{i \in \aleph}$ be two distributions such that $y_i = y'_i$ $\forall\, i \in \mathcal{L}$, then

$$P(y') = P(y).$$

P is independent of the incomes of individuals who are not in the target group (non-potential claimants).

PROPERTIES (2)

Property 3. Scale invariance.

If γ is a positive scalar then P(γ y)=P(y).

It is satisfied if z is defined as a function of y of degree 1.

Property 4. Replication invariance.

$(y_i)_{i\in\aleph^k}$ is generated by the k-fold replication of an original income distribution $(y_i)_{i\in\aleph}$, then

$$P\left((y_i)_{i\in\aleph^k}\right) = P((y_i)_{i\in\aleph})$$

This property allows comparisons of indexes for different populations in which the number of individuals is different.

Property 5. Normalization.

If all the targeted individuals receive B=z, then $P(y) = 1$, otherwise $0 \leq P(y) \leq 1$.

PROPERTIES (3)

Property 6. Subgroup consistency

Given a society composed of G subgroups, exhaustive and mutually exclusive, it holds that

$$P(n,y) = \Phi(P(n^1,y^1), P(n^2,y^2), \dots, P(n^G,y^G))$$

where Φ is a continuous and strictly increasing function in each of the first G arguments.

If a given population subgroup's index increases, and everything else remains constant, then the index for the whole population should increase.

PROPERTIES (4)

The measure that quantifies the protection provided by a MIP, P, satisfies a list of properties:

Property 1. Monotonicity with respect to coverage.

$$\frac{\partial P(y)}{\partial C} > 0$$

The greater the proportion of recipients, all other things being equal, the more protection the MIP provides, and therefore the greater the value of the index.
This property is satisfied for $\alpha > 0$.
That is why we restrict to $0 \leq \alpha < 1$.

PROPERTIES (5)

Property 2. Monotonicity with respect to adequacy**.**

$$\frac{\partial P(y)}{\partial A} > 0$$

The higher the income provided by a MIP (B), keeping z and C fixed, the higher the index. That is, the higher the income provided by a MIP, all else being equal, the higher the protection provided.

PROPERTIES (6)

Property 3. General preference for poorer recipients.

Let $(x_i)_{i \in \aleph}$, $(y'_i)_{i \in \aleph}$, $(y''_i)_{i \in \aleph}$, be three distributions such that $y''_l = B$ and $y''_i = \mathrm{x}_i, \forall\, i \neq l$ and $i, l \in \mathcal{L}$ and y' is obtained from x in such a way that $y'_j = B$ and
$y'_i = \mathrm{x}_i, \forall\, i \neq j$ and $i, j \in \mathcal{L}$; and $x_j < x_l$, then

$$P(y') > P(y'')$$

That is, transfers directed to poorer individuals are preferred and therefore associated with higher levels of the index.

$$y' = (x_1, x_2, \ldots, y_j = B, \quad \ldots, x_l \quad, x_L)$$
$$y'' = (x_1, x_2, \ldots, x_j, \quad \ldots, y_l = B, x_L)$$

$P(y') > P(y'')$

PROPERTIES (7)

Property 4. Preference for poorer recipients given a budget restriction.

Let $(x_i)_{i\in\aleph}$, $(y'_i)_{i\in\aleph}$, $(y''_i)_{i\in\aleph}$ be three distributions such that y' is obtained from x in such a way that $y''_l = B$ and $y''_i = \mathrm{x}_i, \forall\, i \neq l$ and $i, l \in \Im$; and $y'_j = x_j + (B - x_l)$ and $y'_i = \mathrm{x}_i, \forall\, i \neq j$ and $i, j \in \Im$ and $x_j < x_l$, then

$$P(y') > P(y'')$$

$$y' = (x_1, x_2, \ldots, y_j = x_j + B - x_l, \quad \ldots, x_l \quad , x_L)$$
$$y'' = (x_1, x_2, \ldots, x_j, \quad \ldots, y_l = B = x_l + B - x_l, x_L)$$

We prefer the recipients that are poorer.

PROPERTIES (6)

Property 5. Preference for multiple small improvements.

Given a fixed amount of transfers mT to be distributed, let $(y'_i)_{i\in\aleph}$, $(y''_i)_{i\in\aleph}$ be two distributions derived from two alternative MIP such that in y' , mT is equally distributed among m recipients — all of whom have the same income before receiving an MIP — while in y'', the whole amount of transfers mT is assigned to s of the m recipients in such a way that they have income B after receiving it, being $s < m$. Then

$$P(y') > P(y'')$$

That is, the level of protection of a MIP that distributes mT among m individuals should be higher than the one resulting from a small proportion of those individuals receiving income B at cost mT as MIP.

2. APPLICATION TO SPANISH REGIONS (1)

DATA

- Spanish minimum income programs
- Completely decentralized → extreme model of fiscal federalism

2. APPLICATION TO SPANISH REGIONS (1)

a) Adequacy rates:

- Benefits in each region (Department of Social Services) are compared with the national poverty line (Spanish sample of EU-SILC)

b) Coverage rates:

- Number of recipients (Department of Social Services) as a proportion of no income households (Labour Force Survey)

Table 1. Legal amount of minimum Income Benefits in Spanish Regions, 2007 and 2013 (euros per month)

	2007			2013		
Region	Single person	Couple, 2 children	Single-parent, 2 children	Single person	Couple, 2 children	Single-parent, 2 children
AN	353.8	490.7	445.1	400.1	555.0	503.3
AR	336.0	629.0	524.2	441.0	749.7	661.5
AS	396.7	610.9	547.4	443.0	682.1	611.3
IB	364.5	583.2	546.7	425.7	681.1	638.6
CN	342.8	410.5	376.6	472.2	584.0	534.3
CB	286.8	418.2	383.6	426.0	585.8	532.5
CM	349.4	464.8	426.3	372.8	454.8	413.8
CL	374.4	499.2	464.3	426.0	639.0	596.4
CT	385.0	514.2	473.8	423.7	589.6	534.3
EX	374.4	494.2	454.3	399.4	585.8	532.5
GA	374.4	524.2	484.2	399.4	516.5	463.3
MD	340.0	578.0	510.0	375.6	532.5	532.5
MC	300.0	498.0	422.0	300.0	498.0	442.0
NC	456.5	656.2	599.1	548.5	898.0	832.8
PV	585.6	831.9	818.6	662.5	941.1	941.1
RI	335.4	518.0	464.7	372.8	372.8	372.8
VC	364.5	414.5	400.5	385.2	434.9	416.2
Mean	371.8	537.4	490.7	427.9	605.9	562.3

Note: Andalusia: AN; Aragón: AR; Asturias: AS; Balearic Islands: IB; Canary Islands: CN; Cantabria: CB; Castile-La Mancha: CM; Castile and León: CL; Catalonia: CT; Extremadura: EX; Galicia: GA; Madrid: MD; Murcia: MC; Navarre: NC; Basque Country: PV; Rioja: RI; Valencia: VC.

2. APPLICATION TO SPANISH REGIONS (2)

Rank	Coverage rate, 2007		Adequacy rate, 2007		Coverage rate, 2013		Adequacy rate, 2013	
1	Basque Country	1.00	Basque Country	0.66	Basque Country	1.00	Basque Country	0.77
2	Asturias	1.00	Navarre	0.52	Navarre	1.00	Navarre	0.73
3	Cantabria	0.73	Aragón	0.50	Asturias	0.68	Aragón	0.61
4	Navarre	0.51	Asturias	0.48	Cantabria	0.61	Asturias	0.55
5	Galicia	0.38	Balearic Islands	0.46	Rioja	0.58	Balearic Islands	0.55
6	Murcia	0.36	Madrid	0.46	Aragón	0.51	Castile and León	0.52
7	Andalusia	0.29	Galicia	0.42	Castile and León	0.34	Catalonia	0.48
8	Madrid	0.26	Rioja	0.41	Galicia	0.33	Cantabria	0.48
9	Catalonia	0.24	Catalonia	0.41	Andalusia	0.32	Extremadura	0.48
10	Aragón	0.14	Castile and León	0.40	Catalonia	0.26	Canary Islands	0.47
11	Castile and León	0.14	Murcia	0.40	Madrid	0.23	Andalusia	0.45
12	Balearic Islands	0.11	Extremadura	0.39	Valencia	0.14	Madrid	0.43
13	Canary Islands	0.11	Andalusia	0.39	Canary Islands	0.12	Galicia	0.42
14	Valencia	0.09	Castile-La Mancha	0.37	Murcia	0.12	Murcia	0.40
15	Rioja	0.09	Cantabria	0.33	Balearic Islands	0.09	Castile-La Mancha	0.37
16	Extremadura	0.06	Valencia	0.33	Extremadura	0.08	Valencia	0.35
17	Castile-La Mancha	0.04	Canary Islands	0.33	Castile-La Mancha	0.05	Rioja	0.30

2. APPLICATION TO SPANISH REGIONS (2)

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Rank	Coverage rate, 2007		Adequacy rate, 2007		Coverage rate, 2013		Adequacy rate, 2013	
1	Basque Country	1.00	Basque Country	0.66	Basque Country	1.00	Basque Country	0.77
2	Asturias	1.00	Navarre	0.52	Navarre	1.00	Navarre	0.73
3	Cantabria	0.73	Aragón	0.50	Asturias	0.68	Aragón	0.61
4	Navarre	0.51	Asturias	0.48	Cantabria	0.61	Asturias	0.55
5	Galicia	0.38	Balearic Islands	0.46	Rioja	0.58	Balearic Islands	0.55
6	Murcia	0.36	Madrid	0.46	Aragón	0.51	Castile and León	0.52
7	Andalusia	0.29	Galicia	0.42	Castile and León	0.34	Catalonia	0.48
8	Madrid	0.26	Rioja	0.41	Galicia	0.33	Cantabria	0.48
9	Catalonia	0.24	Catalonia	0.41	Andalusia	0.32	Extremadura	0.48
10	Aragón	0.14	Castile and León	0.40	Catalonia	0.26	Canary Islands	0.47
11	Castile and León	0.14	Murcia	0.40	Madrid	0.23	Andalusia	0.45
12	Balearic Islands	0.11	Extremadura	0.39	Valencia	0.14	Madrid	0.43
13	Canary Islands	0.11	Andalusia	0.39	Canary Islands	0.12	Galicia	0.42
14	Valencia	0.09	Castile-La Mancha	0.37	Murcia	0.12	Murcia	0.40
15	Rioja	0.09	Cantabria	0.33	Balearic Islands	0.09	Castile-La Mancha	0.37
16	Extremadura	0.06	Valencia	0.33	Extremadura	0.08	Valencia	0.35
17	Castile-La Mancha	0.04	Canary Islands	0.33	Castile-La Mancha	0.05	Rioja	0.30

2. APPLICATION TO SPANISH REGIONS (3)

Synthetic index (couple, 2 children)

	2007			2013		
Region	(α = 0.5)	(α = 0.75)	(α = 1)	(α = 0.5)	(α = 0.75)	(α = 1)
Andalusia	0.03	0.07	0.11	0.05	0.10	0.15
Aragón	0.01	0.04	0.07	0.16	0.25	0.31
Asturias	0.49	0.49	0.49	0.26	0.33	0.38
Balearic Islands	0.01	0.03	0.05	0.00	0.02	0.05
Canary Islands	0.00	0.02	0.03	0.01	0.03	0.06
Cantabria	0.18	0.22	0.24	0.18	0.25	0.29
Castile and León	0.01	0.03	0.05	0.06	0.12	0.18
Castile-La Mancha	0.00	0.00	0.01	0.00	0.01	0.02
Catalonia	0.02	0.06	0.10	0.03	0.08	0.13
Extremadura	0.00	0.01	0.02	0.00	0.02	0.04
Galicia	0.06	0.11	0.16	0.05	0.10	0.14
Madrid	0.03	0.07	0.12	0.02	0.06	0.10
Murcia	0.05	0.10	0.14	0.01	0.02	0.05
Navarre	0.14	0.21	0.27	0.73	0.73	0.73
Basque Country	0.66	0.66	0.66	0.77	0.77	0.77
Rioja	0.00	0.02	0.03	0.10	0.15	0.18
Valencia	0.00	0.01	0.03	0.01	0.02	0.05

CONCLUSION

- We propose a measure that uses a social welfare approach to evaluate the protection provided by minimum income programs.
- We have built a new index that combines adequacy and coverage.
- We analyze both dimensions using a unified framework.
- Coverage rates can be balanced to summarize alternative value judgements about the relevance of each dimension in the social welfare measurement of minimum income protection.
- This proposal may help to a better understanding of the performance of regional welfare programs considering different dimensions under a unified framework.

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The Measurement of Social Welfare Gains in Social Assistance Programs

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MOTIVATION (1)

In most Western welfare states: benefits targeted to low-income act as a last economic safety net.

- Restrictive reforms to foster transitions from welfare to work:
  - limiting the increase in benefit levels
  - establishing stricter time limits and
  - imposing more onerous obligations upon recipients

- Major topic of public concern: whether these programs improve the welfare levels of the corresponding society through lower levels of poverty.

MOTIVATION (2)

POTENTIAL EFFECTS OF SOCIAL ASSISTANCE PROGRAMS?

- Many studies on effect over poverty incidence and intensity.
- Less research on effects over possible gains or losses in terms of social welfare.

Recent reforms motivated research on the overall outcomes of these policy changes:

- Measures that quantify the protection provided by social assistance benefits usually focus on either adequacy or coverage.
- A measure that combines both dimensions to assess the level of protection provided: Ayala and Bárcena-Martín (2018).

MOTIVATION (3)

KEY:

The assessment of the impact of social assistance benefits on social welfare requires the measurement of the change in welfare derived from the implementation of the scheme, rather than measuring the final level of social welfare achieved.

OUR PAPER

1.- Use of a welfare function combining adequacy and coverage proposed by Ayala and Bárcena-Martín (2018).
2.- We propose measures based on the well-known FGT poverty indices (Foster et al. 1984): we propose to sum up the welfare gains in different jurisdictions in a way that is consistent with the value judgements conducted in the literature on economic inequality adapting the approach used in the segregation literature by Del Río and Alonso-Villar (2018).

SOCIAL WELFARE GAINS (1)

How to construct a measure of the gains derived from social benefits schemes that satisfies normative properties usually accepted in the literature on economic inequality?

The unit used: geographical area (a country —that can be decomposed into mutually exclusive areas, e.g., regions) that have implemented alternative designs of minimum income programs (MIP).

3 steps:

1. Define welfare function for a jurisdiction
2. Measure the change in welfare for a jurisdiction
3. Measure overall welfare gain in the society

SOCIAL WELFARE GAINS (2)

Step 1: Measurement of the protection provided by MIP by Ayala and Bárcena-Martín (2018)

Consider a censored income distribution that caps individual incomes at the poverty threshold.

$$t_i = min\{y_i, z\}$$

Normalized by the poverty threshold, z, it can be expressed in terms of normalized poverty gaps:

$$\frac{t_i}{z}$$

Define the welfare function of the potential claimants as

$$W(y|z,\alpha) = \frac{1}{\alpha}\sum_{i=1}^{L}\frac{\left(\frac{t_i}{z}\right)^{\alpha}}{L}, \qquad 0 \neq \alpha \leq 1$$

where L is the number of potential claimants

SOCIAL WELFARE GAINS (3)

Step 1: Measurement of the protection provided by MIP by Ayala and Bárcena-Martín (2018)

In a general setting in which a MIP provides an income level B to the recipients, $\wp$, and no changes occur for potential claimants, $\Im$, that are not recipients:

$$W(y|z,\alpha) = \frac{1}{\alpha}\sum_{i\in\wp}\frac{\left(\frac{B}{z}\right)^{\alpha}}{L} + \frac{1}{\alpha}\sum_{\substack{i\notin\wp \\ i\in\Im}}\frac{\left(\frac{y_i}{z}\right)^{\alpha}}{L} = \left(\frac{B}{z}\right)^{\alpha}\frac{R}{L}\frac{1}{\alpha} + \frac{1}{L}\sum_{\substack{i\notin\wp \\ i\in\Im}}\frac{\left(\frac{y_i}{z}\right)^{\alpha}}{\alpha}, 0 \neq \alpha < 1,$$

SOCIAL WELFARE GAINS (4)

Step 2: Welfare change in jurisdiction k

Denote K the number of jurisdictions.

x initial income and y final income

z^k the poverty line of jurisdiction k .

Welfare change of jurisdiction k

$$\Phi^k = W^k\left(y^k \middle| z^k, \alpha\right) - W^k\left(x^k \middle| z^k, \alpha\right)$$

$$= \frac{1}{\alpha L_k} \sum_{i \in \wp_k} \left(\frac{B^k}{z^k}\right)^\alpha + \frac{1}{\alpha L_k} \sum_{\substack{i \notin \wp_k \\ i \in \Im_k}} \left(\frac{y_i^k}{z^k}\right)^\alpha - \frac{1}{\alpha L_k} \sum_{i \in \Im_k} \left(\frac{x_i^k}{z^k}\right)^\alpha$$

$$= \frac{1}{\alpha} \left(\frac{B^k}{z^k}\right)^\alpha \frac{R_k}{L_k} - \frac{1}{\alpha} \frac{1}{L_k} \sum_{\substack{i \in \wp_k \\ i \in \Im_k}} \left(\frac{x_i^k}{z^k}\right)^\alpha$$

It depends on the coverage and adequacy rate in that jurisdiction, the aversion parameter and the poverty line.

SOCIAL WELFARE GAINS (5)

Step 3: Measurement of the overall social welfare gains

Define $g = (g_1, \ldots, g_i, \ldots, g_K)$ as the vector resulting from giving each jurisdiction the maximum between the social welfare change in that jurisdiction and zero:

$$g_i = \max\{\Phi^i, 0\}.$$

Adapt the family of poverty indices proposed by Foster et al. (1984) to construct an index of the overall social welfare gains from the gains in each jurisdiction. We consider G the number of jurisdictions with social welfare gains, being $G \leq K$

$$W\mathrm{G}_\varepsilon(g) = \frac{1}{L}\sum_{i=1}^{\mathrm{G}} {g_i}^\varepsilon L_i$$

$0 < \varepsilon < 1$ measure of social welfare propensity.

SOCIAL WELFARE GAINS (6)

Step 3: Measurement of the overall social welfare gains

Alternatively

$$WG_{\varepsilon}(g) = \frac{1}{L}\sum_{i=1}^{G} g_i L_i {g_i}^{\varepsilon-1} \ ,$$

where $\frac{1}{L} L_i {g_i}^{\varepsilon-1}$ is the weight given to welfare gains and, ceteris paribus, increases as the welfare gain is smaller for $0 < \varepsilon < 1$.

SOCIAL WELFARE GAINS (7)

Step 3: Measurement of the overall social welfare gains

For $\varepsilon = 0$, $$WG_0 = \frac{1}{L}\sum_{i=1}^{G} L_i = \frac{\tilde{L}}{L}$$

where $\tilde{L}$ is the number of potential claimants belonging to jurisdictions in which there is an increase in social welfare. WG_0 measures the incidence of social welfare gains.

For $\varepsilon = 1$, $$WG_1 = \frac{1}{L}\sum_{i=1}^{G} g_i L_i$$

gives the mean welfare gain per potential claimant. That is, the intensity of welfare gains.

For $0 < \varepsilon < 1$, WG_ε is the average social welfare gain per potential claimant when the gains are weighted in a way that makes them satisfy normative properties usually accepted in the literature on economic inequality.

SOCIAL WELFARE GAINS (8)

PROPERTIES:

Our index satisfies the properties *of:*

1. *Symmetry*: For all, $g, g' \in \mathbb{R}^K_{++}$ $WG_\varepsilon(g) = WG_\varepsilon(g')$ whenever g' is obtained after applying a permutation of jurisdictions on g.

This property guarantees that the index is based only on social welfare gains no matter the jurisdiction that experiences them.

2. *Replications invariance.* For all, $g, g' \in \mathbb{R}^K_{++}$ $WG_\varepsilon(g) = WG_\varepsilon(g')$ whenever g' is obtained after applying a φ-replication of the economy.

This property allows comparisons of indices for different populations in which the number of jurisdictions is different.

SOCIAL WELFARE GAINS (9)

PROPERTIES:

Our index satisfies the properties *of:*

3. *Strict monotonicity*. Given the vector $g = \{g_1, \dots, g_i, \dots, g_j, \dots, g_q, \dots, g_n\} \in \mathbb{R}^K_{++}$ consider any $\delta \in \mathbb{R}_{++}$ such that $g^{\delta(i)} = \{g_1, \dots, g_i + \delta, \dots, g_j, \dots, g_q, \dots, g_n\} \in \mathbb{R}^K_{++}$. For all $g \in \mathbb{R}^K_{++}, WG_\varepsilon\left(g^{\delta(i)}\right) > WC_\varepsilon(g)$.

It implies that the higher the value of the social welfare gains the higher the index of overall social welfare gains, reflecting that the increase in well-being in a jurisdiction represents an increment in the overall social welfare.

SOCIAL WELFARE GAINS (10)

PROPERTIES:

Our index satisfies the properties *of:*

4. *Preference for equality*, or the transfer axiom. Given the vector $g = \{g_1, \dots, g_i, \dots, g_j, \dots, g_q, \dots, g_n\} \in \mathbb{R}^K_{++}$ consider any $\delta \in \mathbb{R}_{++}$ such that $g' = \left\{g_1, \dots, g_i + (\frac{\delta L_j}{L_i}), \dots, g_j - \delta, \dots, g_q, \dots, g_n\right\} \in \mathbb{R}^K_{++}$. For all $g, g' \in \mathbb{R}^K_{++}$, $WG_\varepsilon(g') > WG_\varepsilon(g)$ for $0 < \varepsilon < 1$.

That is, if a disadvantaged jurisdiction, i.e., one with low social welfare gains, increases its social welfare gain while a less disadvantaged jurisdiction reduces its social welfare gain in the same amount, the value of the index increases, because there is preference for equality in the social welfare gain distribution due to the concavity of the function.

SOCIAL WELFARE GAINS (11)

PROPERTIES:

Our index satisfies the properties *of:*

5. *Focus*. For all, $g, g' \in \mathbb{R}^K_{++}, WG_\varepsilon(g') = WG_\varepsilon(g)$ whenever g' is obtained from g by a change in social welfare losses of jurisdictions that still remain having social welfare losses.

SOCIAL WELFARE GAINS (12)

PROPERTIES:

Our index satisfies the properties *of*

6. WG_ε are *additively decomposable*. The contribution of a group of jurisdictions to overall welfare gains can be estimated.

$$WG_\varepsilon(g) = \frac{1}{L}\sum_{i=1}^{J} \hat{g}_i{}^{\varepsilon}\hat{L}_i \quad \text{where} \quad \hat{g}_i{}^{\varepsilon} = \frac{1}{\hat{L}_j}\sum_{i=1}^{M_j} g_i{}^{\varepsilon} L_i$$

WG_ε can also be decomposed in terms of the mutually exclusive components of the MIP:

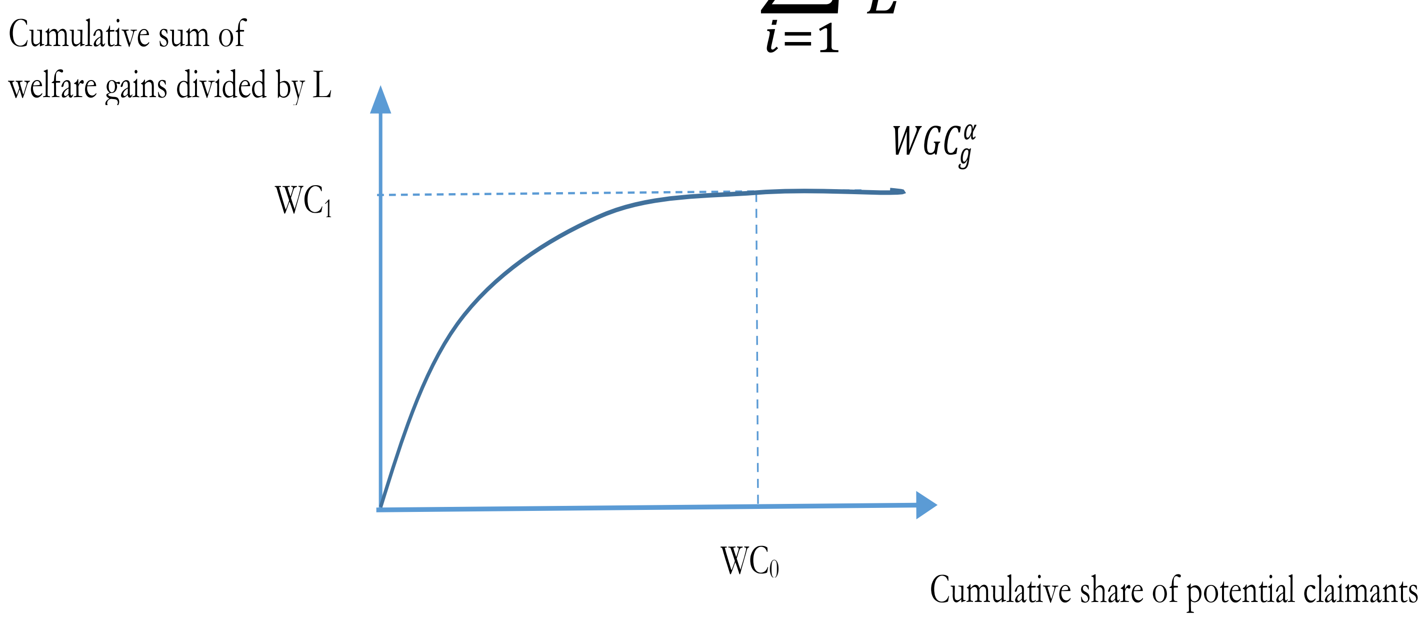
$$WG_\varepsilon(g) = \frac{1}{L}\sum_{i=1}^{G}\sum_{j=1}^{Q} g_{ij}{}^{\varepsilon} L_{ij}$$

SOCIAL WELFARE GAINS CURVE (1)

Assume in vector g gains are ranked from highest to lowest.

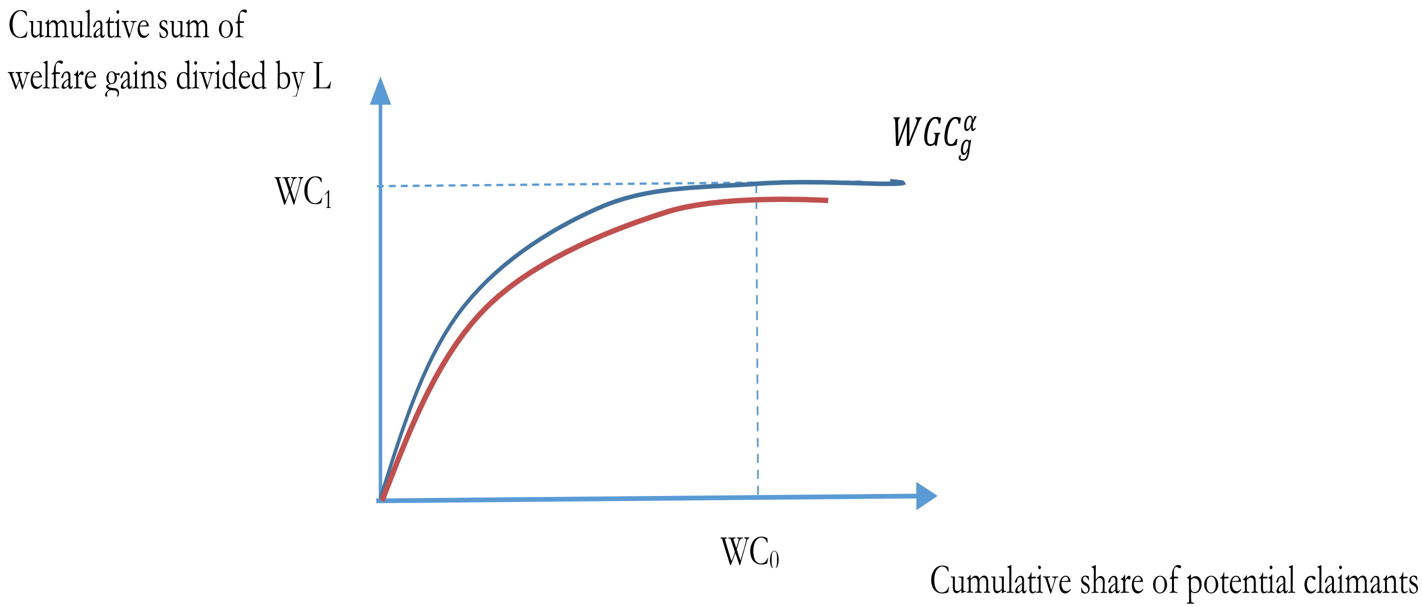
Definition. We define the "*social welfare gain curve associated with MIP*", the WGC curve — denoted by $WGC_g^{\alpha}(p^s)$ — at point p^s, as the sum of the social welfare gains of the first s jurisdictions, each one weighted by its population share ($\frac{L_i}{L}$).

$$WGC_g^{\alpha}(p^s) = \sum_{i=1}^{s} \frac{L_i}{L} g_i$$



SOCIAL WELFARE GAINS CURVE (2)

Definition. We say that vector (g, L^*) dominates in social welfare gains $(g', L^{*\prime})$ if $g \neq g'$ and $WGC_g^\alpha(p) \geq WGC_{g'}^\alpha(p)$ for all $p \in [0,1]$ with at least one point of strict inequality.



SOCIAL WELFARE GAINS CURVE (2)

Let us denote by $H \in R^m$ $(m \geq 2)$ the set of vectors g and $\Psi^*: H \rightarrow R$ the class of functions that are *symmetric, replication-invariant, strictly monotonic and that increases when the distribution of welfare gains becomes relatively more unequal* (equivalent to the transfer axiom).

Result. Let us denote by (g, L^*) and $(g', L^{*\prime})$ two different economies. Vector (g, L^*) "dominates in social welfare gains associated with MIP" vector $(g', L^{*\prime})$ if and only if $\Psi(g) > \Psi(g')$ for all $\Psi \in \Psi^*$

AN EMPIRICAL ILLUSTRATION (1)

Measuring welfare gains resulting from MIP in years 2004 and 2015: contain recession.

19 EU countries: Austria, Belgium, the Czech Republic, Estonia, France, Germany, Ireland, Latvia, Lithuania, Luxembourg, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, the Netherlands, and the United Kingdom.

Limited by data availability on MI benefits in the Mutual Information System on Social Protection of the European Union and household income in EU-SILC (Eurostat).

AN EMPIRICAL ILLUSTRATION (2)

Information needed:

B: MI benefits for three types of families: single person, a couple with two children and a single parent with two children provided by MISSOC

Restrict the sample to individuals living in any of the three types of families considered

R: those who receive social exclusion benefits. (EU-SILC)

L: those who live in households with 0 income plus those who are recipients. (EU-SILC)

z: 60 % of median equivalent income of the country. (EU-SILC)

AN EMPIRICAL ILLUSTRATION (3)

Table 1. $WG_{\varepsilon}(g)$ for 19 European countries

	Year	L	WG_0 α=0.5	$WG_{0.5}$ α=0.5	WG_1 α=0.5	WG_0 α=1	$WG_{0.5}$ α=1	WG_1 α=1
TOTAL	2004	11099701	1.000	1.069	1.188	1.000	0.676	0.484
Single person	2004	4351438	1.000	0.879	[illegible]	[illegible]	[illegible]	[illegible]
Coupl[illegible]	[illegible]	[illegible]	1.000	1.166	1.381	[illegible]	[illegible]	[illegible]
Single[illegible]	[illegible]	[illegible]	1.000	1.238	1.557	1.000	0.811	0.674
TOTAL	[illegible]	[illegible]	[illegible]	[illegible]	0.953	1.000	0.565	0.357
Single[illegible]	[illegible]	[illegible]	1.000	0.734	0.601	1.000	0.431	0.217
Couple 2 children	2015	3906462	1.000	1.108	1.240	1.000	0.669	0.455
Single parent 2 children	2015	2360296	1.000	1.177	1.392	1.000	0.742	0.557

$WG_0 = 1$, all countries have a minimum income program

Intensity was smaller in 2015 than in 2004, no matter the type of family analyzed nor the sensitivity parameter of the welfare function used, even when lower gainers are given more weight

it seems that the economic crisis reduced the effects of MIP on social welfare

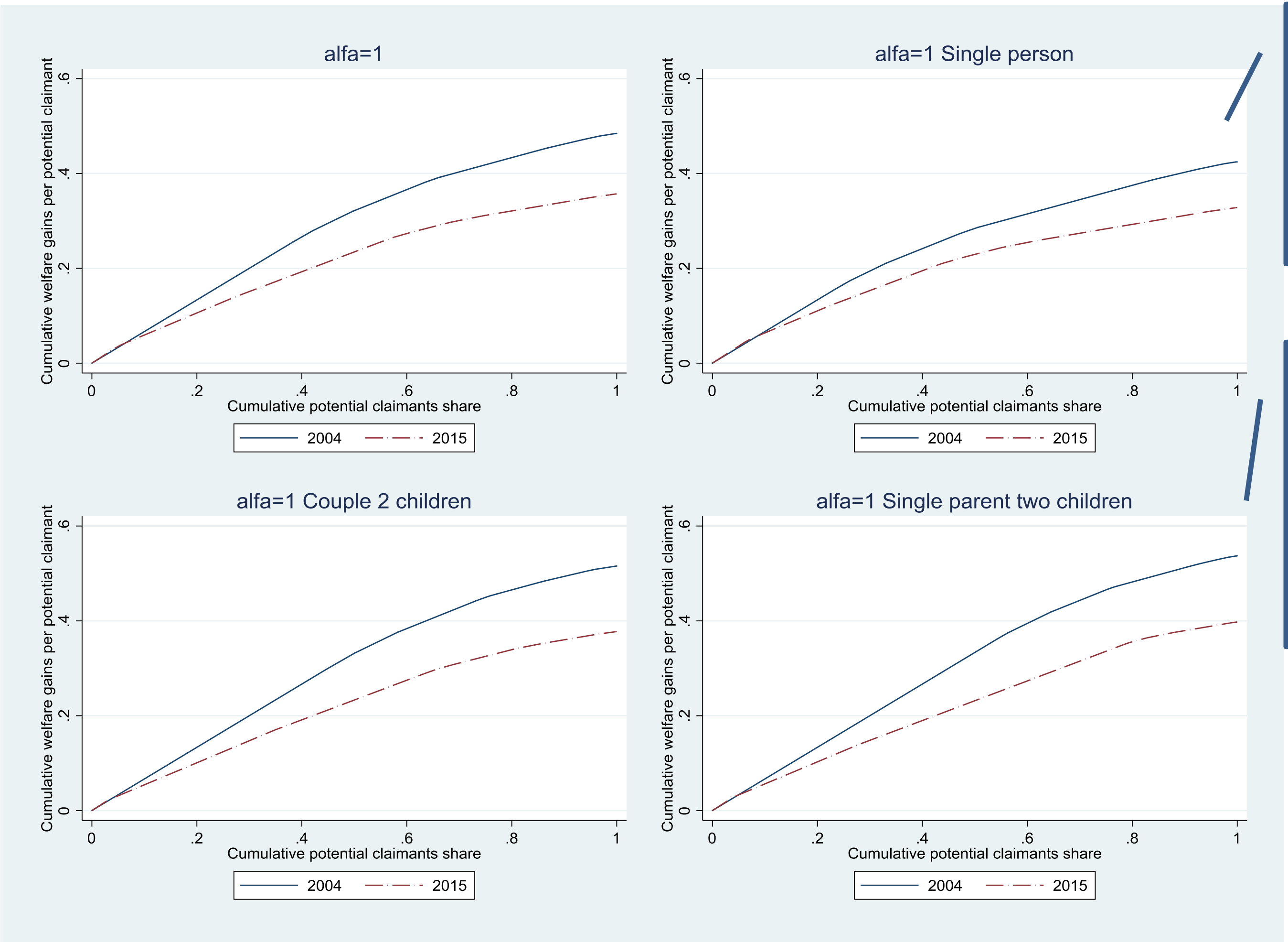
AN EMPIRICAL ILLUSTRATION (4)

By family type:

- Differences in social welfare gains among groups reduce when less emphasis is placed on lower gainers (greater ε)
- The relative contribution of single parent with two children families is higher than its expected value given the relatively lower demographic weight.

AN EMPIRICAL ILLUSTRATION (5)

Figure 2. Welfare gains curve associated with MIP, α=1.



Lowest SW gains for single-person households.

Greatest SW gain for single parents with two children.

Dominance of the 2004 curves over the 2015 curves (overall and each group)→deterioration of the protection of MIP

AN EMPIRICAL ILLUSTRATION (6)

By country:

- Luxembourg and the Netherlands have the highest contribution, and increased between 2004 and 2015.
- Slovakia, the Czech Republic and the United Kingdom had great contributions in 2004 but reduced them in 2015 in favor of Austria and Slovenia.

- Estonia and Ireland, persistently showing the smallest contributions.
- Spain had a small contribution in 2004 but a larger one in 2015.
- The opposite took place in Germany.

Conclusions

- Interesting for policy makers and analysts: proposed methodology allows both the results of each jurisdiction to be aggregated and the contribution of a specific jurisdiction to the total social welfare gains to be identified.
- We have analyzed the welfare gains caused by MIP in EU countries. Our results yield some interesting results:
  - The economic crisis reduced social welfare gains
  - Single parents with children are the household type where the social welfare gain derived from these benefits is the highest and also where they reduced more
  - The reduction in countries' welfare gains were not homogeneous in the EU countries in 2004-2015

Minimum income schemes and redistribution.

January 13-16 , 2020. Alba di Canazei, Italy

AN EMPIRICAL ILLUSTRATION (5)

Table 2. Contribution of each type of family to the total welfare gain with different indices

	year	P_j	C_j $WG_{0.5}$ α=0.5	Cj/Pj $WG_{0.5}$ α=0.5	C_j WG_1 α=0.5	Cj/Pj WG_1 α=0.5	C_j $WG_{0.5}$ α=1	Cj/Pj $WG_{0.5}$ α=1	C_j WG_1 α=1	Cj/Pj WG_1 α=1
Total	2004	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Single person	2004	39.2%	32.2%	82.3%	26.2%	66.8%	31.3%	79.9%	24.6%	62.8%
Couple 2 children	2004	39.7%	43.3%	109.1%	46.1%	116.3%	43.4%	109.2%	46.0%	115.9%
Single parent 2 children	2004	21.1%	24.4%	115.8%	27.7%	131.1%	25.3%	119.9%	29.4%	139.1%
Total	2015	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Single person	2015	49.4%	38.7%	78.4%	31.2%	63.1%	37.7%	76.2%	30.0%	60.8%
Couple 2 children	2015	31.5%	37.3%	118.4%	41.0%	130.0%	37.3%	118.3%	40.2%	127.5%
Single parent 2 children	2015	19.1%	23.9%	125.7%	27.8%	146.0%	25.0%	131.3%	29.8%	156.1%