Wealth inequality measurement:
Methods and evidence from HFCS

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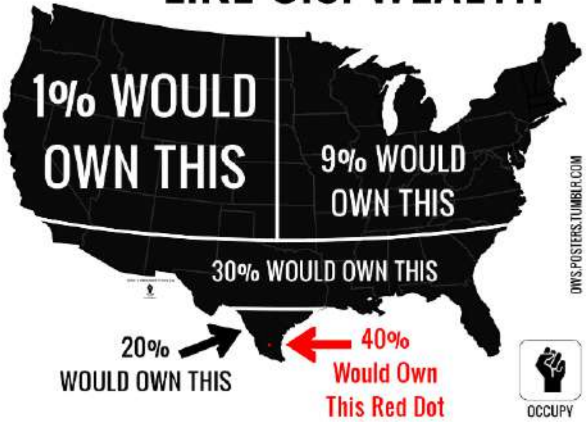
15th Winter School on Inequality and Social Welfare Theory (IT15)

Alba di Canazei, January 14 2020

IF U.S. LAND WERE DIVIDED LIKE U.S. WEALTH



Outline

Measuring household wealth inequality. How different is it from *income* inequality measurement?

1. Cowell and Van Kerm (2015), 'Wealth inequality: A survey', *Journal of Economic Surveys*, **29**(4), 671–710.
2. Cowell et al. (2017), 'Wealth, Top Incomes and Inequality', in "Wealth: Economics and Policy", K. Hamilton and C. Hepburn (Eds.), Oxford University Press.
3. Chauvel et al. (2019), 'Income and Wealth Above the Median: New Measurements and Results for Europe and the United States', in "What drives inequality?", K. Decancq and P. Van Kerm (Eds.), Research on Economic Inequality vol. 27, Emerald Publishing.

Outline

Four themes

1. Equivalence scales
2. Negative net worth
3. Age, life-cycle accumulation and wealth inequality
4. Inference

Outline

What is important but *not* covered?

1. Data collection methods: surveys vs. administrative/tax sources vs. 'indirect' methods
2. Components of household net worth (marketable wealth? incl. public pensions? incl. human capital?)
3. Valuation of (real) assets

Two sources of micro-data on wealth (and income)

- ECB Household Finance and Consumption Survey (HFCS); waves 1 (about 2011) and 2 (about 2014); wave 3 (available soon)
  https://www.ecb.europa.eu/pub/economic-research/research-networks/html/researcher_hfcn.en.html
  All Eurozone countries
- Luxembourg Wealth Study (LWS)
  http://www.lisdatacenter.org
  Many of HFCS datasets along with WAS (for UK) and SCF (for US) in harmonized form

Let’s fix ideas first

Wealth aggregates

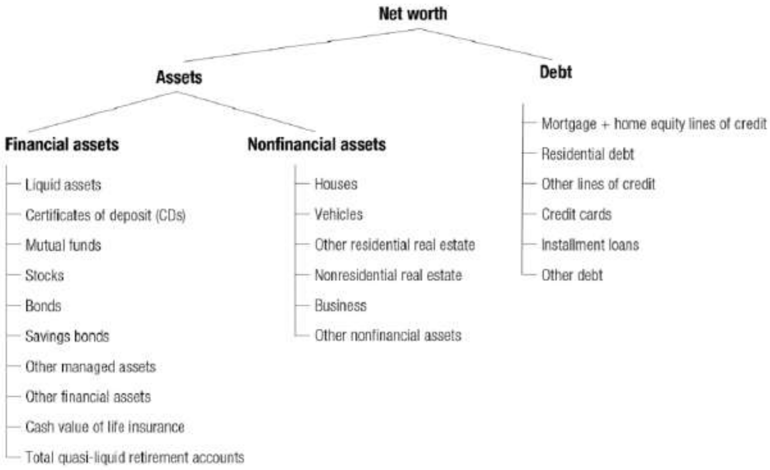


Figure 7. **SCF household portfolio.**

(Kuhn and Ríos-Rull, 2016)

Mean and median wealth across countries

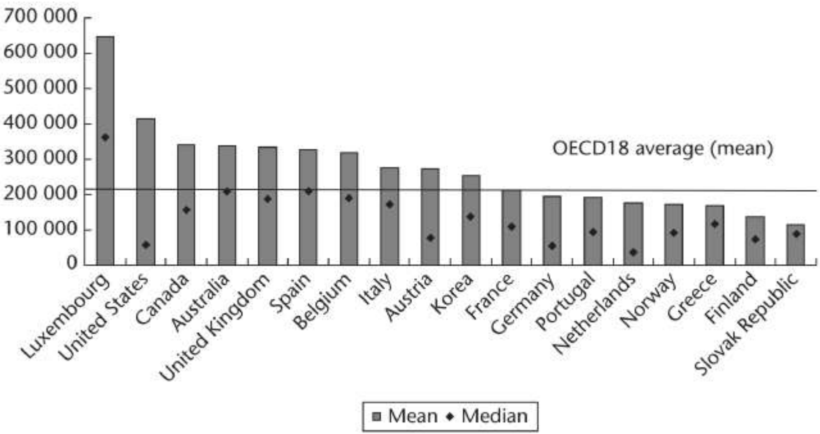
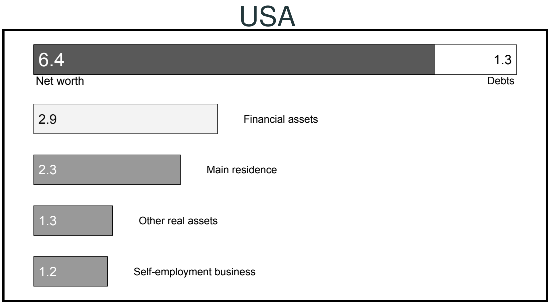
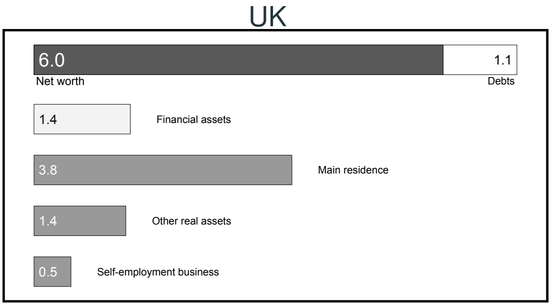


Figure 12.1. Mean and median wealth per household, selected OECD countries around 2010

Source: OECD Wealth Distribution Database

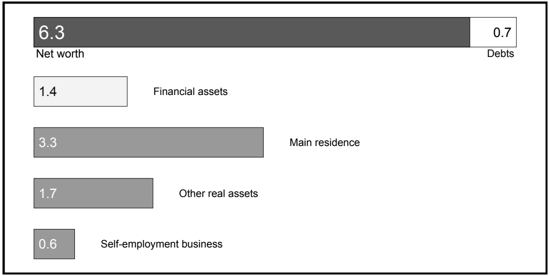
Net wealth composition



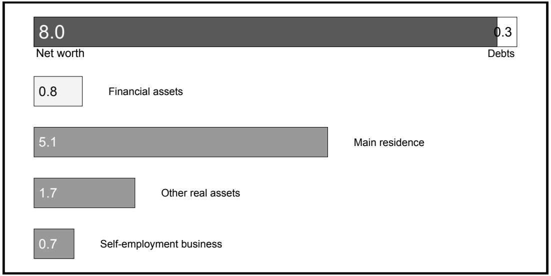


Net wealth composition

France

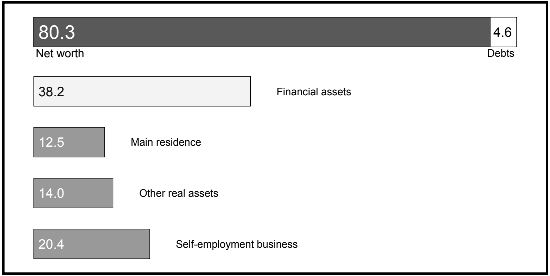


Italy

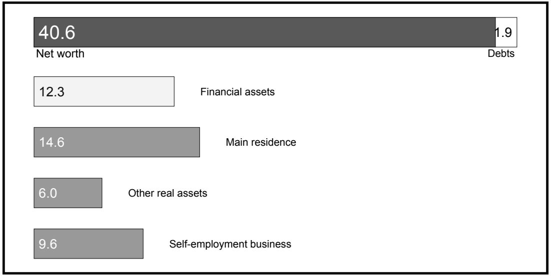


Net wealth composition (among top 5 percent)

USA

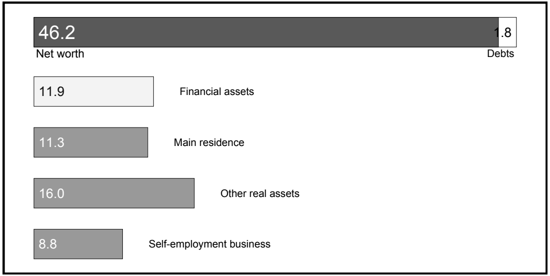


UK



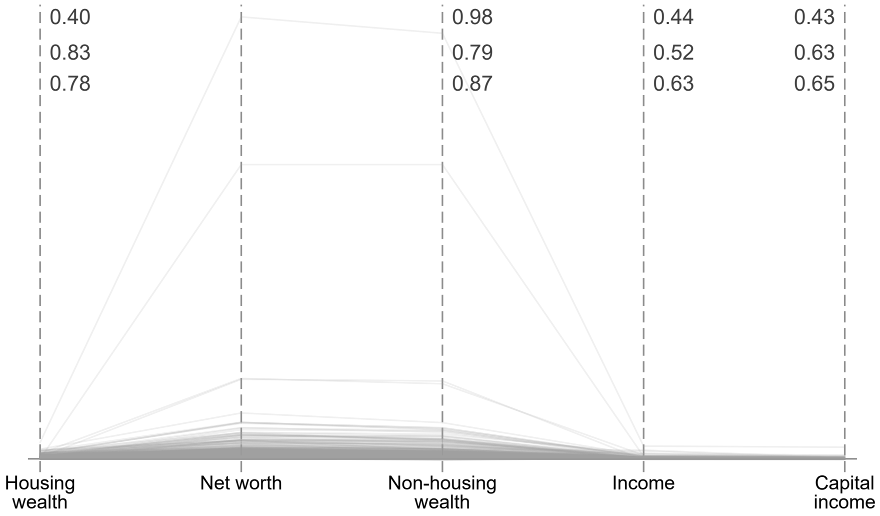
Net wealth composition (among top 5 percent)

France



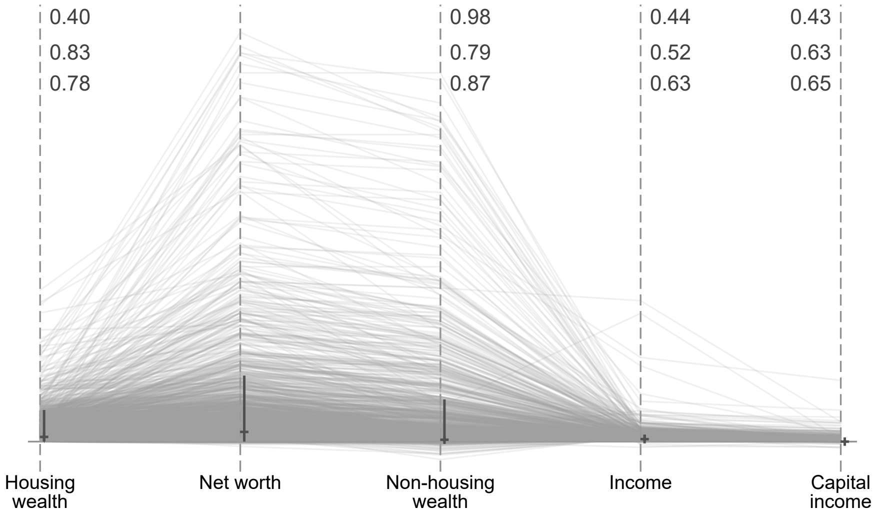
Italy

Associations in wealth and income components (in France)



Note: Capital gains are *not included* in capital income (rents, dividends, interests)!

Associations in wealth and income components (in France)

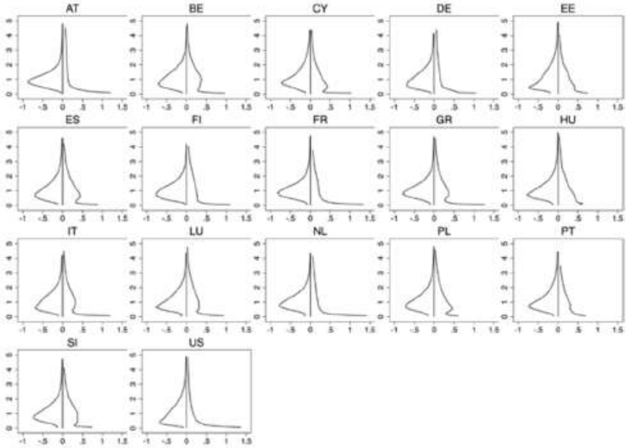


Chop the top 1% in NW off the picture

Associations in wealth and income components (in France)

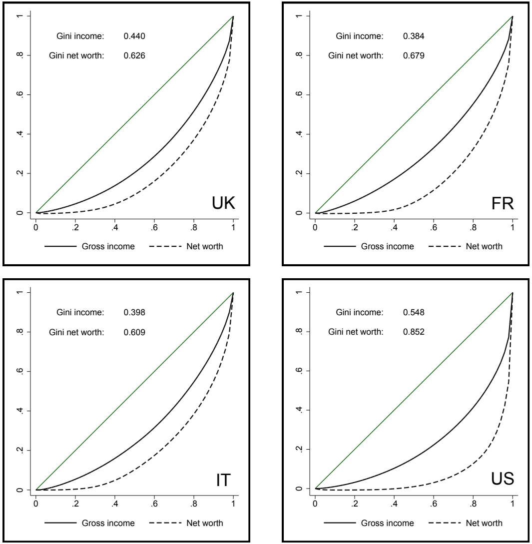
Compare the bottom 20% (blue) and those in the upper 95-99% (in red)

Income and wealth distributions have very different shapes



(Chauvel et al., 2019)

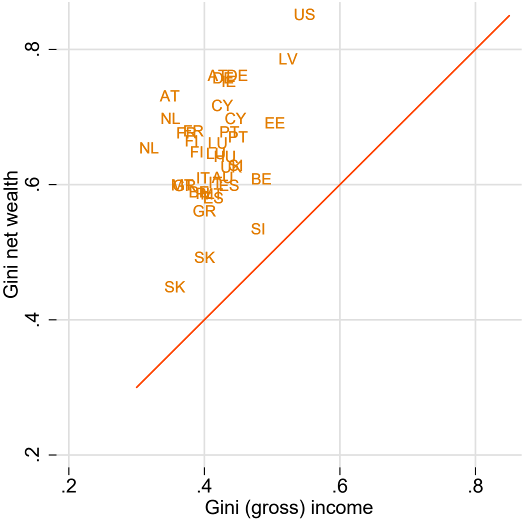
Lorenz curves



Net wealth (much?) more unequally distributed than income

The US is an outlier

Gini coefficients of net wealth and income



No clear pattern in the relationship between income and net wealth inequality

Four measurement issues

Theme One

Equivalence scales

Equivalence scales

An equivalence scale is $e(y, C)$ converts household resources y for a household of composition C into an 'equivalent amount' for reference composition C^R:

$$u(y, C) = u(e(y, C), C^R)$$

where u is some 'individual welfare function'.

In practice,

$$e(y; a, e) = \frac{y}{1 + 0.5(a - 1) + 0.3e}$$

Another classic form:

$$e(y; n) = \frac{y}{(a + \alpha e)^\theta}$$

(where, roughly, α captures different needs of children, and $0 \leqslant \theta \leqslant 1$ captures economies of scale)

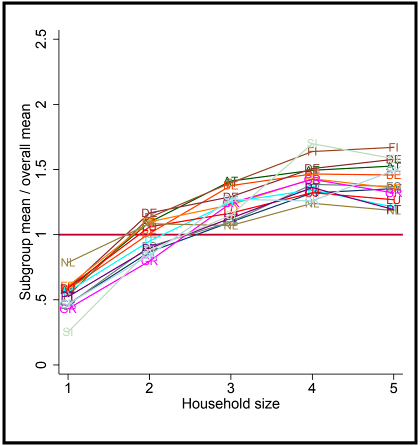
Relevance for wealth data?

But wealth is not income, so issue is controversial

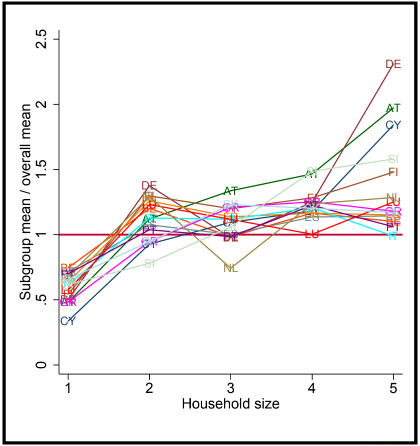
- Wealth is *not* consumed immediately: indicator of future private consumption, so future composition matters (and discard children? but what about bequests?)
- ‘Service value’ of real assets: strong economies of scale in housing ($\theta = 0$?)
- Wealth may not only be relevant for consumption but for ‘family prestige’ or ‘power’? ($\theta = 0$?)
- Capturing the national stock of capital? ($\theta = 1$)

Household size, income and net worth in HFCS

Income

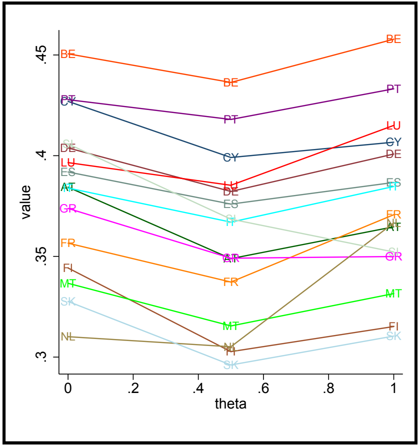


Net worth



Inequality measures for alternative scale parameters

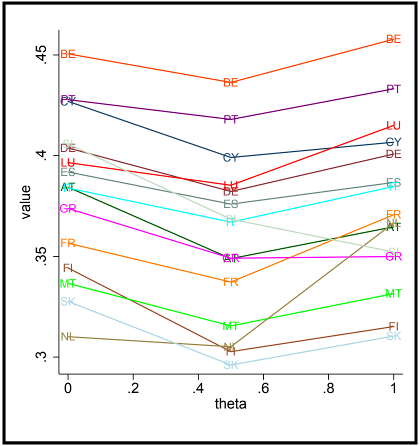
Income



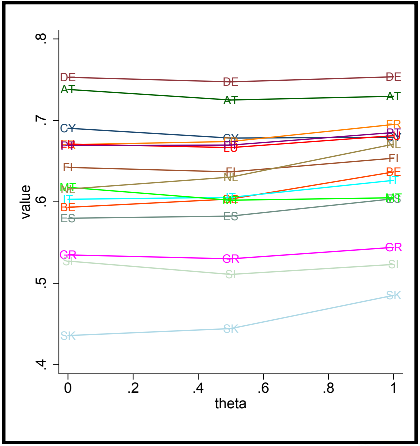
Net worth

Inequality measures for alternative scale parameters

Income



Net worth



Theme Two

Negative net worth

The significant of negative net worth

Net worth is typically the concept of choice for wealth distribution analysis, and $NW \leqslant 0$ (when debts exceed assets) is a perfectly valid outcome.

In HFCS Wave 1, for example, the fraction of households with non positive net worth reaches

Netherlands	12%
Finland	11%
Germany	9%

or 14% in the US in LWS/SCF 2010

Immediate consequence

- Many popular inequality measures based on logarithmic or fractional power transformations are undefined
  - notably: Atkinson measures, Generalized Entropy measures for $\alpha < 2$ (incl. Theil, MLD), the SD of logs
- ... and even percentile ratios or quantile share ratios based on, say, the bottom decile become undefined or somewhat ‘meaningless’
- Analysts often left with (Generalized) Gini coefficient (or other ‘linear inequality’ measures) or the CoV

$\not\Rightarrow$ the symptom of a deeper conceptual issue with ‘relative inequality measures’

(Jenkins and Jäntti, 2005)

1. Rethinking ‘maximum inequality’

We first need to rethink what ‘maximum inequality’ is!

- Is inequality maximal when one person has all wealth and everyone else has nothing (Gini equal to 1)?
    - If debt is allowed, further ‘regressive transfers’ (from a poor to a rich person) can take place by further indebting the poor household and enriching the rich household

    $\not\Rightarrow$ No theoretical ‘maximum’ (and the Gini can go beyond 1 when the Lorenz curve bends below zero)
    - Justification of ‘renormalisation approaches’ unclear (Chen et al., 1982, Berrebi and Silber, 1985)

2. Scale invariance and the principle of transfer

Relative (scale invariant) inequality measures are such that $I(\mathbf{y}) = I(\lambda\,\mathbf{y})$.

Scale independence mean that inequality does not depend on the units in which wealth is expressed.

- So, $I((-2,4)) = I((-4,8))$
  $\not\Rightarrow$ violation of 'principle of transfer' since the change involve a regressive transfer from poor to rich!
  - many relative measures are in fact undefined
  - theoretical support for available relative inequality measures beyond 'descriptive tools' becomes somewhat questionable

Absolute inequality measures?

Absolute (translation invariant) inequality measures are such that $I(\mathbf{y}) = I(\mathbf{y} + \lambda)$.

- Simplest measure is the standard deviation
- Absolute Gini indices:

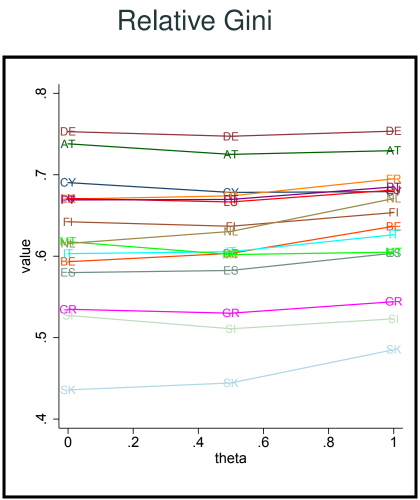
$$A(\mathbf{y}) = \mu - W(\mathbf{y})$$

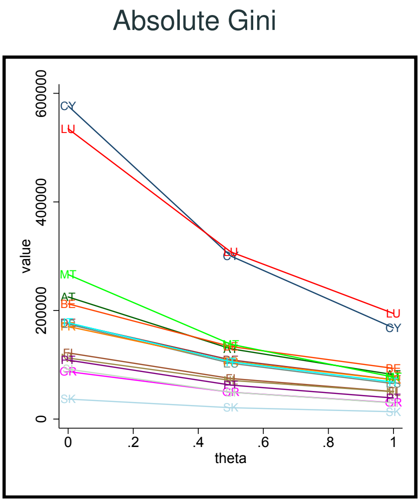
  where $W(\mathbf{y})$ is the Gini Social Welfare measure, or equally distributed equivalent wealth $W(\mathbf{y}) = n^{-1} \sum_i 2(1 - p_i) y_i$, and relative Gini is

$$G(\mathbf{y}) = 1 - \frac{W(\mathbf{y})}{\mu}$$

- but unit (currency!) matters, different normative underpinning and very different empirics!

Absolute and relative Gini coefficients

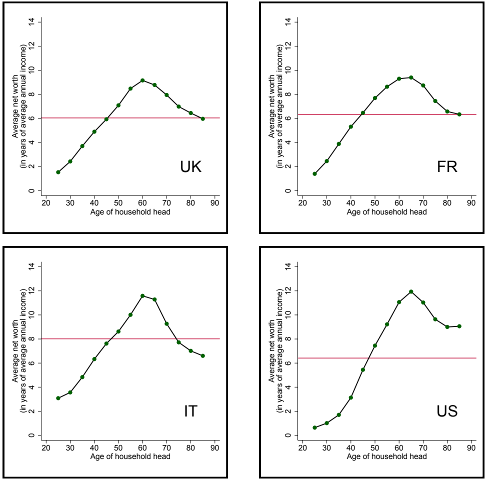




Theme Three

Age, life-cycle accumulation and wealth inequality

Age wealth profiles



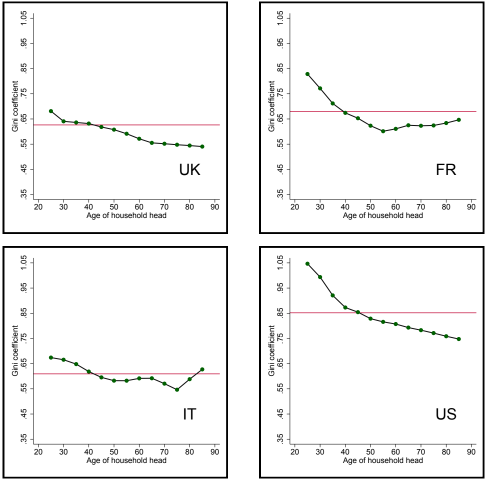
‘Hump shape’ relationship between age and wealth

Peak at 60–65

Remarkably consistent across countries

How much of wealth inequality is merely due to age mix?

Age wealth profiles



Within age-group inequality not necessarily lower than overall inequality

(if anything higher at younger ages)

Age-adjusted Gini coefficients

Decompositions into between-group vs. within-group

- neatly additive for Generalized Entropy measures... but $NW <= 0$
- Gini coefficient expressed as

$$G = \sum_{a=1}^{A} s_a \pi_a G_a + G^b + R$$

where G_a is the Gini coefficient within age group a, s_a and π_a are respectively the population share and the total wealth share of age group a, G^b is a "between-group" inequality

- Paglin's (much criticized) age-adjusted Gini (Paglin, 1975) is

$$P = G - G^b$$

(In HFCS, $P \approx \frac{2G}{3}$.)

Age-adjusted Gini coefficients

More general approaches re-express Gini as sum of all pairwise deviations from mean

$$G = \frac{1}{2\mu n^2} \sum_i \sum_j |(w_i - \mu) - (w_j - \mu))|$$

and then use an alternative wealth ‘reference’ (e.g. Almås and Mogstad, 2012)

$$AG = \frac{1}{2\mu n^2} \sum_i \sum_j |(w_i - \mu(a_i)) - (w_j - \mu(a_j))|$$

Alternative approaches tend to lead to much higher age-adjusted Gini’s than Paglin’s (much closer to unadjusted values)

Theme Four

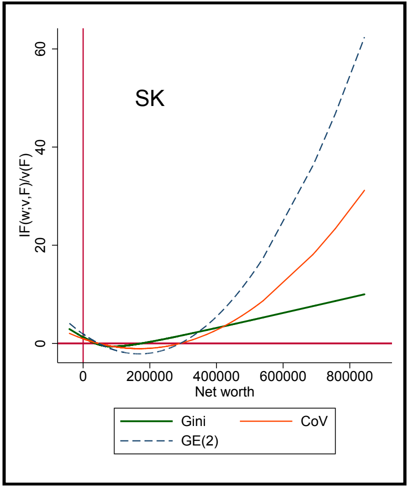
Inference

Inference with heavy-tailed distributions

Wealth distributions have a much heavier tail than income distributions. Inference problems arising from sparse, extreme data in survey samples discussed in Cowell and Flachaire (2007) and Davidson and Flachaire (2007) are compounded.

- Point estimates are sensitive to extreme data and contamination
- Imprecise estimates even in fairly large samples
- Standard methods for estimation of sampling variance and confidence intervals calculation perform poorly (both linearization and standard bootstrap methods); e.g. confidence intervals that do not cover the ‘true’ value as per the nominal level
- Non-sampling error: the ‘missing rich’ (see, e.g., Vermeulen, 2016, Kennickell, 2019)

Influence functions



The influence of extreme data is large, even for SK example

Especially large for CoV and GE(2)—hardly useable

Semi-parametric estimation for improving inference

Cowell and Flachaire (2007) and Davidson and Flachaire (2007) show that poor performance of inference is due to extreme, sparse data at the top.

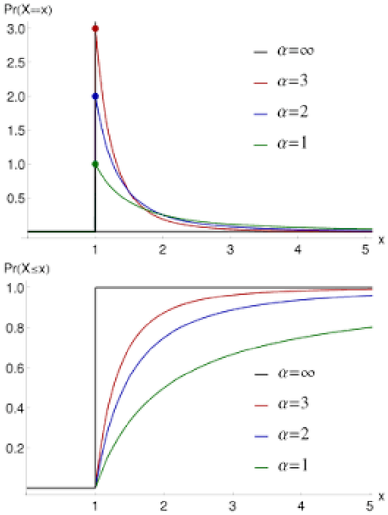
Semi-parametric approach can improve both point estimation (Cowell and Flachaire, 2007) and testing (Davidson and Flachaire, 2007):

Assume data are Pareto distributed above given threshold $\underline{w}$, so

$$\widetilde{F}(w) = \begin{cases} F(w) & w \leqslant \underline{w} \\ 1 - \beta S(w) & w > \underline{w} \end{cases}$$

(S is the survival function for a Pareto distribution)

Pareto and Power Laws



Pareto (type1)

$$S(x) = Pr[X > x] = \frac{x}{x_0}^{-\alpha}$$

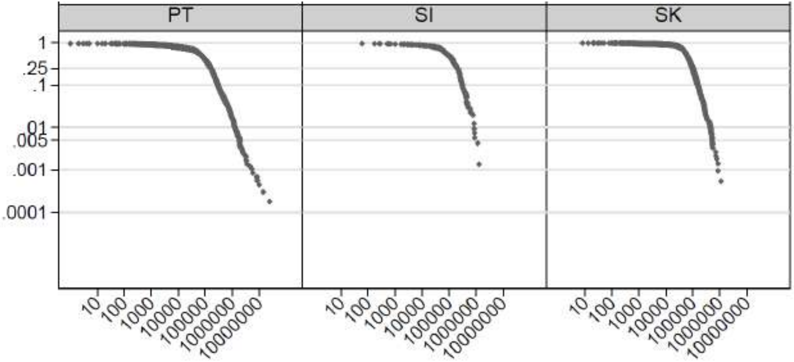
$\alpha \geqslant 1,\ x \geqslant x_0 > 0$

$$f(x) = \alpha \frac{x_0^{\alpha}}{x^{\alpha+1}}$$

May hold everywhere (for any $x > x_0$) or only asymptotically, that is for $x \to \infty$

Pareto diagram

Plot $\log(w)$ against $\log(1 - F(w))$:



A simple, practical approach

1. Estimate α by standard methods from the top k observations (Hill's index, likelihood formula)
   - Estimate for alternative k and choose value where $\hat{\alpha}$ stabilizes
   - 'robust' estimator for α even better, but typically not necessary
   - NB: k/n may be greater than β
2. Inspect Pareto diagram to select β (or $\underline{w}$), e.g., between 0.005 and 0.001,
3. Generic solution (Van Kerm, 2007, Alfons et al., 2013):
   - discard data $w_i \geqslant \underline{w}$, simulate large number of new data from Pareto distribution, reweight those draws by $\beta \times n \,/\, nsim$
   - proceed with estimation as with sample data

(see also Eckerstorfer et al., 2016, Blanchet et al., 2017, Charpentier and Flachaire, 2019)

To wrap up

Key messages and challenges

- We know it but, yes, wealth *is* very unequally distributed: the upper tail spreads out far away
- Age-wealth profiles are clearly marked ... but within-age-group wealth inequality is not much smaller than overall inequality

Measuring wealth inequality is *not* just like income inequality

- (Wealth definition, collection and valuation difficult and crucial!)
- Implication of nature of wealth (negative and not directly ‘consumed’) on standard concepts and methods need to be appreciated
- Extend ‘toolbox’ to include absolute and age-adjusted measures
- Inference issues are compounded by the heavy tail of the distribution

Some basic accounting identities

- Wealth accumulation (savings and capital gains)

$$a_{it+1} = a_{it}(1 + q_{(i)t}) + \Delta_{it} + s_{it}$$

- Income allocation by source and purpose

$$y_{it} = y_{it}^{L} + y_{it}^{K} + y_{it}^{TB} = s_{it} + c_{it}$$

- Capital and labour income (wage times employment)

$$y_{it}^{L} = w_{(i)t} l_{it} \qquad\qquad y_{it}^{K} = r_{(i)t} a_{it}$$

- Net tax-benefit transfer

$$y_{it}^{TB} = b_{it} - \tau_{(i)t}^{L} y_{it}^{L} - \tau_{(i)t}^{L} y_{it}^{K})$$

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