On Collective Identification Procedures with Independent Qualified Certification

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Abstract

This paper studies Collective Identification Procedures in a finite distributive lattice when the standard arrowian Independence axiom is dropped and replaced with an Independent Qualified Certification requirement.

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1 Introduction

Collective identification procedures (CIPs) are game forms that model the wide class of more or less formal protocols that are used in order to identify the legitimate members of certain associations, clubs, or communities (including scientific communities, political parties, sports clubs, religious fraternities). The role and impact of associative structures and social networks has always been a prominent topic in the social sciences, and has recently attracted a good deal of interest among economists as a possible explanatory factor of some remarkable discrepancy in the observed performance of similar economic and political institutions in different social environments. Indeed, associations of different types may arguably influence, say, social cohesion, political stability or economic performance in several distinct ways[1]. Moreover, one might plausibly conjecture that *inclusiveness* of an association is a key factor for a proper assessment of its impact on cohesion and other relevant socioeconomic dimensions. Therefore, inclusion and exclusion rules are conceivably an important characteristics of an association, and a possible predictor of its effects on the social environment. In particular, all of the above suggests the significance of a classification of associations, clubs and communities in terms of *admission rules*.

In the last decade, some work has been devoted to the formal social-choice-theoretic study of CIPs. The bulk of the extant literature has focussed on two essentially disjoint classes of procedures. One class comprises those procedures which satisfy a counterpart of the arrowian social-choice-theoretic *Independence* property including of course the self-certification-based 'libertarian' rule (see e.g. Samet and Schmeidler (2003), Çengelci and Sanver (2005), Sung and Dimitrov (2005), Houy (2007), and the seminal Kasher and Rubinstein (1997)). The other class includes procedures which rely on some *cooptation* principle (see again Kasher and Rubinstein (1997), Dimitrov, Sung and Xu (2003), Houy (2006)). Unfortunately, the Independence property is very restrictive and rules out many commonly used admission rules. Indeed, in the present collective identification setting, Independence

[1] Admittedly, identification of the 'mechanisms' that might channel that influence through individual behavior is still to a large extent a matter of plausible conjectures informed by casual observations and common sense. However, some promising theoretical and experimental work on the impact of social identification and 'categorization' on individual behavior is now under way (see, among others, Fryers and Jackson (2002), Charness, Rigotti, and Rustichini (2006), and their extensive bibliographic references).

establishes that membership of each population unit does only depend on the assessment of her qualifications on the part of all population units to the effect of disregarding the qualifications of units to assess each other. Thus, all non-trivial admission rules which require nomination/sponsorship by some member(s) are excluded by the Independence requirement. The most obvious versions of nomination rules seem to rely on cooptation. Now, cooptation principles and practices amount to allowing some asymmetries among more or less active and passive members in the nomination process, including possibly the distinction between 'founding' members and 'others'. In many associations (including most contemporary political parties and organizations) such asymmetries are much disputed and often regarded as highly undesirable. Therefore, one should also consider collective identification procedures that do not satisfy Independence *and* eschew cooptation practices. *But then, are the requirements of internal/qualified sponsorship and no-cooptation mutually consistent?*

This paper is mainly devoted to those CIPs that rely on the principle of Independent Qualified Certification (IQC): *membership requires certification/approval by another qualified unit namely by another member.* The IQC principle is essentially incompatible with Independence (but see Section 3 below for a careful discussion of this point). However, IQC does not rule out cooptation altogether. Therefore, a further Collective Self-Determination property (a generalization of a condition due to Samet and Schmeidler (2003)) is introduced to the effect of eschewing cooptation. A Participatory Certification condition ensuring that membership is voluntary is also considered. A special emphasis is put on the *most inclusive* CIPs which satisfy several combinations of the foregoing properties.

A major characteristic feature of the present paper concerns *the basic underlying 'algebra' of admissible coalitions/associations.* Indeed, while the extant literature is typically concerned with the boolean lattice of subsets of the universal (finite) set of agents, our analysis is pursued -along the lines of Monjardet (1990)[2]- in the *more general framework of an arbitrary finite distributive lattice.* The choice of such a general environment allows the

[2]Monjardet (1990) is in fact concerned with arrowian i.e. 'independent' aggregation rules in semi-lattices. Our IQC-consistent CIPs may be regarded as specialized 'non-independent' aggregation rules in lattices. In a more specialized setting Miller (2006) does also heavily rely on semi-latticial-theoretic properties ('join-separability' and 'meet-separability'). In contrast, Ballester and García-Lapresta (2005) is focussed on sequential identification procedures with several degrees of membership.

accommodation of the cases of abstention and of many-valued memberships (see section 2 below).

Several CIPs satisfying various combinations of Independent Qualified Certification, Collective Self-Determination and Participatory Certification are identified and analyzed. Simple characterizations of certain CIPs, based upon those three properties, are also provided. It turns out that a basic common feature of CIPs that satisfy IQC is the existence of circles of members that mutually certify each other's qualifications. Moreover, such circles may be partially overlapping or even disjoint to the effect of producing several subcommunities under the same denomination (and possibly occurrence of the well-known phenomenon of contested identities).

The present paper is organized as follows: Section 2 is devoted to a presentation of the model, and the results, including a short discussion of manipulability and core-stability properties when each agent only cares about her own affiliation. Section 3 includes a discussion of some related literature. Section 4 provides some short comments on the results of Section 2, and remarks about possible extensions of the analysis.

2 Model and main results

2.1 Basic notation, definitions and preliminaries

Let $\mathbf{L} = (L, \leqslant)$ be a finite *distributive lattice* namely a finite partially ordered set[3] such that for any $x, y \in L$ both the greatest lower bound $x \wedge y$ and the least upper bound $x \vee y$ of $\{x, y\}$ do exist[4], and $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ for any $x, y, z \in L$. A *join irreducible* element of $\mathbf{L}$ is any $j \in L$ such that $j \neq \wedge L$ and for any $x, y \in L$ if $j = x \vee y$ then $j \in \{x, y\}$. The set of all join irreducible elements of $\mathbf{L}$ is denoted J^*: it is also assumed that $\#J^* \geq 2$ in order to avoid tedious qualifications or trivialities. The following analysis refers to an arbitrary but fixed $J_L \subseteq J^*$such that $\#J_L \geq 2$. The suggested interpretation is the following: J^*denotes the set of all population units, and J_L the set of *relevant* population units i.e. those agents[5] who have a say in

[3]Thus, by definition, $\leqslant$ is a transitive, reflexive and antisymmetric binary relation on L.

[4]For any $A \subseteq L$, $\wedge_{x \in A} x$ and $\vee_{x \in A} x$ are defined in the obvious way.

[5]The term 'agent' shall be henceforth used to denote any relevant population unit namely any element of J^L.

the collective identification process under consideration.

A *Collective Identification Procedure (CIP)* on J_L is a function $F : L^{J_L} \rightarrow L$. In particular, for any $i, j \in J_L$, it will be said that j *accepts/nominates* i at profile $x = (x_1, ..., x_{\#J_L})$ whenever $i \leqslant x_j$.

In order to justify the interpretation of CIPs we are going to suggest, a basic property of (finite) distributive lattices is to be recalled, namely

Fact: (see e.g. Grätzer (1998), Monjardet (1990)). Let $(L', \leqslant)$ be a finite distributive lattice, and J^* the set of join-irreducible elements of $(L', \leqslant)$. Then, i) for any $x \in L'$ there exists a unique $J(x) = \{j_1, ..., j_k\} \subseteq J^*$such that $x = \vee J(x)$ and $x < \vee J'$ for any $J' \subset J(x)$; ii) for any $j \in J^*$, and any $l_1, ..., l_h \in L'$, if $j < l_1 \vee ... \vee l_h$ then there exists $i \in \{1..., h\}$ such that $j \leqslant l_i$.

Thus, for any CIP $F : L^{J_L} \rightarrow L$, and any $x \in L^{J_L}$, there exists a unique (minimal or irredundant) $J(F(x)) = \{j_1, ..., j_k\} \subseteq J^*$ such that $F(x) = j_1 \vee ... \vee j_k$. Hence *the identity* $F(x) = j_1 \vee ... \vee j_k$ *may be taken to denote* ***with no ambiguity*** *that CIP* F *at opinion profile* x *identifies units* $j_1, ..., j_k$ *and their join-irreducible lower bounds*[6] *as the only legitimate members of the association under consideration.*

For any pair F, F' of CIPs on J_L, it will be written $F' \leqq F$ whenever $F'(x) \leqslant F(x)$ for all $x \in L^{J_L}$ i.e. when F is *more inclusive* than F'.

In order to illustrate the scope of the ensuing analysis, let us introduce a few prominent examples which are special instances of our model[7].

Example 1: Binary collective identification without abstention

That is the case the extant literature on collective identification procedures is typically focussed on: each agent either is a member or is not a member, and provides a positive or negative assessment of each agent. Thus, the lattice is $(\mathcal{P}(N), \subseteq)$, where N is the finite population of agents. In that lattice, the join-irreducible elements are the atoms i.e. the singletons. Thus

[6] Thus, the members are precisely the join-irreducibles in $(j_i] = \{x : x \in L, x \leqslant j_i\}$ for some $i \in \{1, ..., k\}$.

[7] An important case that, on the contrary, is *not* covered by our analysis is collective identification of groups or partitions (see e.g. Kasher, Rubinstein (1996), and Houy (2007)). Indeed, in that case one has to deal with the lattice of partitions of N, which is *not* distributive. The ensuing difficulties, partly reflected in the somewhat paradoxical results obtained by Houy (2007), will be explored elsewhere.

the standard case with set of agents N reduces to a special instance of our model with $L = 2^N$, $\leqslant = \subseteq$ and $J_L = J^* \simeq N$.

Example 2: Three-valued collective identification with abstention

In this case, agents may be declared members, non-members or of uncertain status, and are allowed to abstain. Hence, one has $L = 3^N$, while $\leqslant$ reduces to the component-wise partial order, and $J_L \subset J^*$, $J_L \simeq N$.

Example 3: Collective Identification with (Finitely) Many Degrees of Membership and Impact Rankings

This is apparently the more general case of interest in the present setting, and encompasses the previous ones as special cases. Let $(K, \leq)$ denote the finite chain of degrees of memberships. Then, $L = K^N$, and $\leqslant$ reduces again to the component-wise partial order on K^N, and $J_L \subseteq J^*$.

Notice that, under a suitable reinterpretation of N and K, the lattice $(K^N, \leqslant)$ might also accommodate some models of academic influence assessment[8], by taking N to denote a reference set of academic journals or institutions, K a suitably chosen set of relevant percentiles of quotations or publications, and $N \simeq J_{K^N}$.

The present work will be mainly focussed on the following properties of CIPs:

Independent Qualified Certification (IQC): For any $j \in J_L$ and $x \in L^{J_L}$ such that $j \leqslant F(x)$ there exists $i \in J_L, i \neq j$ such that $i \leqslant F(x)$ and $j \leqslant x_i$.

Participatory Certification (PC): For any $x \in L^{J_L}$ and any $j \in J_L$, if $j \leqslant F(x)$ then $j \leqslant x_j$.

[8]Under such a reinterpretation, a CIP would concern the many-valued identification/ranking of influential academic 'agents': but see Palacios-Huerta and Volij (2004) for a thorough axiomatic treatment of impact rankings based upon the richer structure of nonnegative real matrices. Notice that CIPs that satisfy Independent Qualified Certification (as defined in the text) share their somewhat 'recursive' structure with the most widely use impact ranking methods, including Google's celebrated PageRank.

Collective Self-Determination (CSD): For any $x, x' \in L^{J_L}$ if $[j \leqslant x_i$ iff $i \leqslant x'_j$ for any $i, j \in J_L]$ then $F(x) = F(x')$.

Clearly, IQC establishes that membership requires certification of eligibility by another *member*, while PC simply requires voluntariness of membership[9]. Finally, CSD is a no-cooptation property which amounts to imposing identity of the set of members under reversal of roles between certificators and nominees.

We shall then proceed to introduce and characterize some CIPs, relying on the foregoing axioms and looking for the *most inclusive* procedures which satisfy some suitable combinations of them[10].

2.2 Independent Qualified Certification

To begin with, we consider two CIPs which satisfy IQC while not disallowing cooptation.

Definition 1 *The **Extended Qualified Nomination (EQN)** procedure*[11]*: for any* $x \in L^{J_L}$

$$F^E(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \textit{there exist } k \in \mathbb{Z},\ k \geq 2\ ,\ \{i_1, .., i_k\} \subseteq J_L \\ \textit{and } i \in \{i_1, .., i_k\} \setminus \{j\}\ \textit{ such that} \\ h \leqslant x_{h+1(\mathrm{mod}\, k)} \textit{ for any } h \in \{i_1, .., i_k\}\ \textit{ and } j \leqslant x_i \end{array} \right\}$$

In plain words, EQN identifies as members the nominees of some agent in some circle of nominees where each agent is nominated by her successor/neighbour.

[9] The IQC property is related to two axioms introduced by Houy (2006) under the labels 'Cooptation' and 'Robustness of the Js'. 'Cooptation' establishes that an agent is accepted *if* and only if she is nominated by a member. 'Robustness of the Js' requires that if an agent i is regarded as a legitimate member under a certain profile of opinions then her membership should not be affected by a change of her own opinion about legitimate members, (provided she identifies some legitimate members) . Thus, IQC is implied by (a suitably generalized version of) the 'only if' part of 'Cooptation' plus 'Robustness of the Js'. It should also be mentioned here that 'Cooptation' is definitely a misnomer for the foregoing property.

[10] Observe that the dual issues concerning the *least inclusive CIPs* satisfying (suitable combinations of) IQC, CSD and PC all admit trivial answers since the constant Universal Rejection CIP (see Section 3 below for an explicit definition) satisfies each of them.

[11] The suffix $(\mathrm{mod}\, k)$ (following an addition) which is introduced below denotes k-modular sum namely sum in the finite group $\mathbb{Z}/k$ (where k is any positive integer).

Definition 2 *The **Participatory Extended Qualified Nomination (PEQN)** procedure: for any* $x \in L^{J_L}$

$$F^{PE}(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \text{there exist } k \in \mathbb{Z},\ k \geq 2\ ,\ \{i_1, .., i_k\} \subseteq J_L \\ \text{and } i \in \{i_1, .., i_k\} \setminus \{j\} \text{ such that} \\ h \leqslant x_h \wedge x_{h+1(\mathrm{mod}\, k)} \text{ for any } h \in \{i_1, .., i_k\} \text{ and} \\ j \leqslant x_j \wedge x_i \end{array} \right\}$$

Thus, PEQN identifies as members the *consenting* nominees of some agent who belong to some circle of nominees where each agent is nominated by her successor/neighbour and *declares herself a member.*

First, it is shown below that the EQN procedure can be readily characterized as the *most inclusive CIP that satisfies IQC.*

Proposition 3 *F^E satisfies IQC. Moreover, for any CIP $F : L^{J_L} \to L$, if F satisfies IQC then $F \leqq F^E$.*

Proof. Let us consider $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F^E(x)$. Then there exist $k \in \mathbb{Z}$, $k \geq 2$, $\{i_1, .., i_k\} \subseteq J_L$ and $i \in \{i_1, .., i_k\} \setminus \{j\}$ such that $\# \{i_1, .., i_k\} = k$, $i_h \leqslant x_{i_{h+1(\mathrm{mod}\, k)}}$ for any $h \in \{1, .., k\}$, and $j \leqslant x_i$. Moreover, by definition, $i_h \leqslant F^E(x)$ for any $h \in \{1, .., k\}$ and $j \neq i$ hence F^E does indeed satisfy IQC.

Now, let F be a CIP that satisfies IQC.

For any $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F(x)$, there exists $i_1 \in J_L$, $i_1 \neq j$ such that $i_1 \leqslant F(x)$ and $j \leqslant x_{i_1}$, by IQC. But then, by IQC again, there exists $i_2 \in J_L$, $i_2 \neq i_1$ such that $i_2 \leqslant F(x)$ and $i_1 \leqslant x_{i_2}$. By repeating the argument, and in view of finiteness of J_L, we may conclude that there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\# \{i_1, .., i_k\} = k$, $\{i_1, .., i_k\} \subseteq J_L$ such that $h \leqslant x_{h+1(\mathrm{mod}\, k)}$ and $h \leqslant F(x)$ for any $h \in \{i_1, .., i_k\}$. Thus, by definition, $j \leqslant F^E(x)$ as required. ■

Similarly, PEQN is *the most inclusive CIP that satisfies IQC and PC.*

Proposition 4 *F^{PE} satisfies IQC and PC. Moreover, for any CIP $F : L^{J_L} \to L$, if F satisfies IQC and PC then $F \leqq F^{PE}$.*

Proof. Let us consider $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F^{PE}(x)$. Then there exist $k \in \mathbb{Z}$, $k \geq 2$, $\{i_1, .., i_k\} \subseteq J_L$ and $i \in \{i_1, .., i_k\} \setminus \{j\}$ such that $\# \{i_1, .., i_k\} = k$, $i_h \leqslant x_{i_h} \wedge x_{i_{h+1(\mathrm{mod}\, k)}}$ for any $h \in \{1, .., k\}$, and $j \leqslant x_j \wedge x_i$. Moreover, by definition, $i_h \leqslant F^{PE}(x)$ for any $h \in \{1, .., k\}$ and $j \neq i$, hence F^{PE} does indeed satisfy IQC. It is straightforward to check that, by definition, F^{PE}satisfies PC as well.

Let F be a CIP that satisfies IQC and PC. For any $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F(x)$, there exists $i_1 \in J_L$, $i_1 \neq j$ such that $i_1 \leqslant F(x)$ and $j \leqslant x_j \wedge x_{i_1}$, by IQC and PC. But then, by IQC again, there exists $i_2 \in J_L$, $i_2 \neq i_1$ such that $i_2 \leqslant F(x)$ and $i_1 \leqslant x_{i_2}$. By repeating the argument, and in view of finiteness of J_L, we may conclude that there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\# \{i_1, .., i_k\} = k$, $\{i_1, .., i_k\} \subseteq J_L$ such that $h \leqslant x_h \wedge x_{h+1 (\operatorname{mod} k)}$ and $h \leqslant F(x)$ for any $h \in \{i_1, .., i_k\}$. Thus, by definition, $j \leqslant F^{PE}(x)$. ■

Clearly enough, both EQN and PEQN explicitly allow cooptation of members (namely, nominees who are not members of the basic 'circle' are coopted by some member within such 'circle').

2.3 Independent Qualified Certification without cooptation

Let us then turn to some CIPs that satisfy IQC *and* disallow cooptation practices.

Definition 5 *The* ***Restricted Qualified Nomination (RQN)*** *procedure: for any* $x \in L^{J_L}$

$$F^R(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \text{there exist } k \in \mathbb{Z},\ k \geq 2 \text{ and } \{i_1, .., i_k\} \subseteq J_L \\ \text{such that } j \in \{i_1, .., i_k\} \\ \text{and } h \leqslant x_{h+1 (\operatorname{mod} k)} \text{ for any } h \in \{i_1, .., i_k\} \end{array} \right\}$$

Hence, by definition, the RQN procedure identifies as members those agents who belong to some circle of nominees where each agent is nominated by her successor/neighbour.

Definition 6 *The* ***Participatory Restricted Qualified Nomination (PRQN)*** *procedure: for any* $x \in L^{J_L}$

$$F^{PR}(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \text{there exist } k \in \mathbb{Z},\ k \geq 2 \text{ and } \{i_1, .., i_k\} \subseteq J_L \\ \text{such that } j \in \{i_1, .., i_k\} \\ \text{and } h \leqslant x_h \wedge x_{h+1 (\operatorname{mod} k)} \text{ for any } h \in \{i_1, .., i_k\} \end{array} \right\}$$

Again, the PRQN procedure is a voluntary version of RQN, namely it identifies as members those agents who belong to some circle of nominees where each agent is nominated by her successor/neighbour and *declares herself to qualify as a member.*

Collective identification procedures RQN and PRQN are also amenable to easy characterizations in terms of our axioms[12]. In particular, procedure RQN is *the most inclusive CIP that satisfies IQC and CSD*, as testified by the following result:

Proposition 7 *F^R satisfies IQC and CSD. Moreover, for any CIP $F : L^{J_L} \to L$, if F satisfies IQC and CSD then $F \leqq F^R$.*

Proof. Let us consider $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F^R(x)$. Then there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\{i_1, .., i_k\} \subseteq J_L$ such that $\#\{i_1, .., i_k\} = k$, $j = i_{h^*} \in \{i_1, .., i_k\}$ for some $h^* \in \{1, .., k\}$, and $i_h \leqslant x_{i_{h+1(\mathrm{mod}\, k)}}$ for any $h \in \{1, .., k\}$. Moreover, by definition, $i_h \leqslant F^R(x)$ for any $h \in \{1, .., k\}$. Since $k \geq 2$ it must be the case that in particular $j \neq i_{h^*+1(\mathrm{mod}\, k)}$ hence F^R does indeed satisfy IQC.

Moreover, let $x, x' \in L^{J_L}$ be such that $[j \leqslant x_i$ iff $i \leqslant x'_j$ for any $i, j \in J_L]$. Next, take any $j \in J_L$ such that $j \leqslant F^R(x)$. Then, by definition, there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\{i_1, .., i_k\} \subseteq J_L$ such that $\#\{i_1, .., i_k\} = k$, $j \in \{i_1, .., i_k\}$ and $h \leqslant x_{h+1(\mathrm{mod}\, k)}$ for any $h \in \{i_1, .., i_k\}$. But then, $h + 1(\mathrm{mod}\, k) \leqslant x'_h$ for any $h{+}1(\mathrm{mod}\, k) \in \{i_1, .., i_k\} = \{i'_1, .., i'_k\}$ where $i'_h = i_{k-h+1(\mathrm{mod}\, k)}$, $h = 1, .., k$. Therefore, equivalently, $h \leqslant x'_{h+1(\mathrm{mod}\, k)}$ for any $h \in \{i'_1, .., i'_k\}$. It follows that, by definition, $j \leqslant F^R(x')$ whence $F^R(x) \leqslant F^R(x')$. Since $(x')' = x$, by a similar argument $F^R(x') \leqslant F^R(x)$. Thus, $F^R(x) = F^R(x')$ i.e. F^R satisfies CSD.

Now, let F be a CIP that satisfies IQC and CSD.

For any $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F(x)$, there exists $i_1 \in J_L$, $i_1 \neq j$ such that $i_1 \leqslant F(x)$ and $j \leqslant x_{i_1}$, by IQC. But then, by IQC again, there exists $i_2 \in J_L$, $i_2 \neq i_1$ such that $i_2 \leqslant F(x)$ and $i_1 \leqslant x_{i_2}$. By repeating the argument, and in view of finiteness of J_L, we may conclude that there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\#\{i_1, .., i_k\} = k$, $\{i_1, .., i_k\} \subseteq J_L$ such that $h \leqslant x_{h+1(\mathrm{mod}\, k)}$ and $h \leqslant F(x)$ for any $h \in \{i_1, .., i_k\}$, and $j \leqslant F(x)$. If $j \in \{i_1, .., i_k\}$ then $j \leqslant F^R(x)$ and we are done. Let us then assume that $j \notin \{i_1, .., i_k\}$ for any such set $\{i_1, .., i_k\} \subseteq J_L$. Hence in particular $i_h \nleqslant x_j$ for any $i_h \in \{i_1, .., i_k\}$ (otherwise, one might consider $\{i_1, .., i_h, j\} \subseteq J_L$, which would violate our

[12] Incidentally, the constant CIP $F^{\vee J_L}$ that always declares all relevant agents to be legitimate members may also be characterized as the most inclusive CIP that satisfies Collective Self-Determination.

previous assumption). Next, take $x' \in L^{J_L}$ such that $[i \leqslant x'_l$ iff $l \leqslant x_i$ for any $i, l \in J_L]$. Therefore, $j \nleqslant x'_{i_h}$ for any such $\{i_1, .., i_k\} \subseteq J_L$ and any $i_h \in \{i_1, .., i_k\}$. By IQC it follows that $j \nleqslant F(x')$, while CSD entails $j \leqslant F(x')$, a contradiction. Thus, $j \in \{i_1, .., i_k\}$ whence $j \leqslant F^R(x)$. ■

Similarly, procedure PQRN is *the most inclusive CIP that satisfies IQC, PC and CSD.*

Proposition 8 F^{PR} *satisfies IQC, CSD and PC. Moreover, for any CIP* $F : L^{J_L} \rightarrow L$*, if* F *satisfies IQC, CSD and PC then* $F \leqq F^{PR}$.

Proof. Let us consider $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F^{PR}(x)$. Then there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\{i_1, .., i_k\} \subseteq J_L$ such that $\#\{i_1, .., i_k\} = k$, $j = i_h \in \{i_1, .., i_k\}$, $j \leqslant x_{i_h} \wedge x_{i_{h+1(\mathrm{mod}\, k)}}$ and $i_{h+1(\mathrm{mod}\, k)} \leqslant x_{i_{h+2(\mathrm{mod}\, k)}}$. Clearly, by definition, $i_h \leqslant F^{PR}(x)$ for any $h \in \{1, .., k\}$. Since $k \geq 2$ it must be the case that $j \neq i_{h+1(\mathrm{mod}\, k)} \neq i_{h+2(\mathrm{mod}\, k)}$ hence by definition again, F^{PR} satisfies IQC.

Moreover, let $x, x' \in L^{J_L}$ be such that $[j \leqslant x_i$ iff $i \leqslant x'_j$ for any $i, j \in J_L]$. Next, take any $j \in J_L$ such that $j \leqslant F^{PR}(x)$. Then, by definition, there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\{i_1, .., i_k\} \subseteq J_L$ such that $\#\{i_1, .., i_k\} = k$, $j \in \{i_1, .., i_k\}$ and $h \leqslant x_h \wedge x_{h+1(\mathrm{mod}\, k)}$ for any $h \in \{i_1, .., i_k\}$. But then, $h+1(\mathrm{mod}\, k) \leqslant x'_{h+1} \wedge x'_h$ for any $h+1(\mathrm{mod}\, k) \in \{i_1, .., i_k\} = \{i'_1, .., i'_k\}$ where $i'_h = i_{k-h+1(\mathrm{mod}\, k)}$, $h = 1, .., k$. Therefore, equivalently, $h \leqslant x'_h \wedge x'_{h+1(\mathrm{mod}\, k)}$ for any $h \in \{i'_1, .., i'_k\}$. It follows that, by definition, $j \leqslant F^{PR}(x')$ whence $F^{PR}(x) \leqslant F^{PR}(x')$. Since $(x')' = x$, by a similar argument $F^{PR}(x') \leqslant F^{PR}(x)$. Thus, $F^{PR}(x) = F^{PR}(x')$ i.e. F^{PR} also satisfies CSD.

Finally, F^{PR} clearly satisfies PC, by definition.

Now, let F be a CIP that satisfies IQC, CSD and PC.

For any $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F(x)$, there exists $i_1 \in J_L$, $i_1 \neq j$ such that $i_1 \leqslant F(x)$ and $j \leqslant x_{i_1}$, by IQC. Moreover, $j \leqslant x_j$ by PC. But then, by IQC and PC again, there exists $i_2 \in J_L$, $i_2 \neq i_1$ such that $i_2 \leqslant F(x)$, $i_2 \leqslant x_{i_2}$ and $i_1 \leqslant x_{i_2}$. By repeating the argument, and in view of finiteness of J_L, we may conclude that there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\#\{i_1, .., i_k\} = k$, $\{i_1, .., i_k\} \subseteq J_L$ such that $h \leqslant x_h \wedge x_{h+1(\mathrm{mod}\, k)}$ for any $h \in \{i_1, .., i_k\}$, and $j \leqslant F(x)$. But then, take $x' \in L^{J_L}$ such that $[j \leqslant x_i$ iff $i \leqslant x'_j$ for any $i, j \in J_L]$. Therefore, by CSD, $j \leqslant F(x')$ which violates IQC. Thus, $j \in \{i_1, .., i_k\}$ whence $j \leqslant F^{PR}(x)$.

If $j \in \{i_1, .., i_k\}$ then $j \leqslant F^{PR}(x)$ and we are done. Let us then assume that $j \notin \{i_1, .., i_k\}$ for any such set $\{i_1, .., i_k\} \subseteq J_L$. Hence in particular

$i_h \nleqslant x_j$ for any $i_h \in \{i_1, .., i_k\}$ (otherwise, one might consider $\{i_1, .., i_h, j\} \subseteq J_L$, which would violate our previous assumption). Next, take $x' \in L^{J_L}$ such that $[i \leqslant x'_l$ iff $l \leqslant x_i$ for any $i, l \in J_L]$. Therefore, $j \nleqslant x'_{i_h}$ for any such $\{i_1, .., i_k\} \subseteq J_L$ and any $i_h \in \{i_1, .., i_k\}$. By IQC it follows that $j \nleqslant F(x')$, while $j \leqslant F(x)$ and CSD entail $j \leqslant F(x')$, a contradiction. Thus, $j \in \{i_1, .., i_k\}$ whence $j \leqslant F^{PR}(x)$. ■

Concerning the independence of the IQC, PC, and CSD axioms, notice that, as it is easily checked, F^{PE}satisfies IQC and PC but violates CSD, while F^R satisfies IQC and CSD but violates PC. Next, consider the following well-known identification rule

Definition 9 *The **Libertarian (L^*)** procedure: for any $x \in L^{J_L}$,*
$F^{L^*}(x) = \bigvee \{j \in J_L : \ j \leqslant x_j \}$[13]

Of course, L^* accepts as members precisely all agents who declare themselves to qualify. It is straightforward to check that F^{L^*} satisfies PC[14] and CSD but fails to satisfy IQC. Thus, IQC, PC and CSD are *mutually independent axioms.*

2.4 Manipulability and cooperative stability in simple environments

As mentioned above, CIPs are strategic game forms with opinions as strategies. It therefore makes sense to enquire about their manipulability or, more generally, their solvability with respect to some suitable noncooperative or cooperative game-theoretic solution concepts, once the set of admissible preferences over outcomes of each agent is specified. Here I shall focus on a very simple set of admissible preference profiles, leaving a more general, full-fledged analysis as a topic for further research.

Indeed, let us consider the most elementary case of self-oriented preferences, where each agent $i \in N$ only cares about her own status with respect to association. Then, in the basic case of binary identification agent i will

[13]Hence, the libertarian rule identifies as members precisely those agents who declare themselves to qualify as members.

[14]Indeed, $F \leqq F^{L^*}$ for any CIP $F : L^{J_L} \to L$ that satisfies PC.

partition the outcome set L into *two* equivalence classes, namely the two sets of best and worst outcomes, characterized by consistency with her own preferred and dispreferred membership status, respectively. This case motivates the general notion of a *simple* environment as made precise by the following

Definition 10 *Let* $\mathbf{L} = (L, \leqslant)$ *be a finite distributive lattice,* J^* *the set of its join-irreducible elements,* $J_L \subseteq J^*$*, and* $x = (x_j)_{j \in J_L} \in L^{J_L}$*. A* J_L*-profile* $(\succcurlyeq_i)_{i \in J_L}$ *of binary (preference) relations on* L^{J_L} *is* simple *w.r.t.* x *iff for any* $i \in J_L$*, and any* $y, z \in L^{J_L}$*:* $y \succcurlyeq_i z$ *if and only if either* $[j \leqslant y$ *and* $j \leqslant x_j]$ *or* $[j \nleqslant y$ *and* $j \nleqslant x_j]$*. The* simple *environment (on* $(\mathbf{L}, J_L)$*) consists of the set* $\mathcal{S}^{J_L}$ *of all preference* J_L*-profiles on* L^{J_L} *that are simple w.r.t. some* $x \in L^{J_L}$*.*

Notice that the significance of the simple environment rests not only on its remarkable tractability but also, and foremost, on the fact that it apparently leaves as little scope as possible for strategic manouvering and manipulation. The relevant notion of non-manipulability is a straightforward adaptation to the present setting of the standard concept of strategy-proofness for voting mechanisms, namely

Definition 11 $\mathbf{L} = (L, \leqslant)$ *be a finite distributive lattice,* J^* *the set of its join-irreducible elements, and* $J_L \subseteq J^*$*. A CIP* $F : L^{J_L} \to L$ *is* strategy-proof *on the simple domain* $\mathcal{S}^{J_L}$ *iff for any* $x \in L^{J_L}$*,* J_L*-profile* $(\succcurlyeq_i)_{i \in J^L} \in \mathcal{S}^{J_L}$ *that is simple w.r.t.* x*,* $i \in J_L$*, and* $y_i \in L$

$F(x) \succcurlyeq_i F(y_i, x_{-i})$.

However, it turns out that even on a very restricted domain such as the simple environment, the inclusive IQC-consistent CIPs studied in this paper are indeed manipulable, namely

Proposition 12 *Let* $F : L^{J_L} \to L$ *be a CIP ,* $F \in \{F^E, F^{PE}, F^R, F^{PR}\}$ *as defined above. Then* F *is* not *strategy-proof on the simple domain* $\mathcal{S}^{J_L}$*.*

Proof. Let $J_L = \{j_1, .., j_k\}$. Then, consider an (opinion) profile $x = (x_j)_{j \in J_L}$ such that $j_h \leqslant x_{j_{h+1}}$, $h = 1, .., k-1$, $j_h \leqslant x_{j_h}$, $h = 1, .., k$, and $j_h \nleqslant x_i$ otherwise. Clearly, by definition, $j \nleqslant F(x)$ for any $j \in J_L$, and $F \in \{F^E, F^{PE}, F^R, F^{PR}\}$. Now, take $x'_{j_1} \in L$ such that $j_1 \leqslant x'_{j_1}$ and $j_k \leqslant x'_{j_1}$. It is easily checked that $j_1 \leqslant F(x'_{j_1}, x_{-j_1})$, for any $F \in \{F^E, F^{PE}, F^R, F^{PR}\}$. Hence, by definition of simple profile w.r.t. x,

$F(x'_{j_1}, x_{-j_1}) \succcurlyeq_{j_1} F(x)$ and *not* $F(x) \succcurlyeq_{j_1} F(x'_{j_1}, x_{-j_1})$

i.e. for any $F \in \{F^E, F^{PE}, F^R, F^{PR}\}$, F is *not* strategy-proof on $\mathcal{S}^{J_L}$. ■

Next, let us turn to cooperative solution concepts and related solvability issues.

Definition 13 *Let* $\mathbf{L} = (L, \leqslant)$ *be a finite distributive lattice,* J^* *the set of its join-irreducible elements,* $J_L \subseteq J^*$, $F : L^{J_L} \to L$ *a CIP,* $x \in L^{J_L}$, *and* $(\succcurlyeq_i)_{i\in J^L} \in \mathcal{S}^{J_L}$ *a* J_L*-profile that is simple w.r.t.* x. *Then, a* J_L*-profile* $y \in L^{J_L}$ *is a* coalitional equilibrium with threats *of* F *at* $(\succcurlyeq_i)_{i\in J^L}$ *iff for any* $T \subseteq J_L$ *and* $z_T \in L^T$ *there exists* $w_{N\setminus T} \in L^{N\setminus T}$ *and* $i \in T$ *such that*

$F(y) \succcurlyeq_i F(w_{N\setminus T}, z_T)$.

Moreover, a coalitional equilibrium with threats y *of* F *at* $(\succcurlyeq_i)_{i\in J^L}$ *is a* strong equilibrium *(of* F *at* $(\succcurlyeq_i)_{i\in J^L}$*) iff in particular for any* $T \subseteq J_L$ *and* $z_T \in L^T$ *there exists* $i \in T$ *such that*

$F(y) \succcurlyeq_i F(y_{N\setminus T}, z_T)$.

Remark 14 *Of course, any strong equilibrium of* F *at* $(\succcurlyeq_i)_{i\in J^L}$ *is a coalitional equilibrium with threats of* F *at* $(\succcurlyeq_i)_{i\in J^L}$ *, but not vice versa. Also notice that coalitional equilibrium with threats is the strategic counterpart of the* core, *namely any* $l \in L$ *is a* core outcome *of* F *at* $(\succcurlyeq_i)_{i\in J^L}$ *iff there exists a coalitional equilibrium with threats* y *of* F *at* $(\succcurlyeq_i)_{i\in J^L}$ *such that* $l = F(y)$.

It turns out that the IQC-consistent CIPs introduced above are indeed solvable with respect to strong equilibrium on the simple domain (hence with respect to coalitional equilibrium with threats as well), namely

Proposition 15 *Let* $F : L^{J_L} \to L$ *a CIP,* $F \in \{F^E, F^{PE}, F^R, F^{PR}\}$, $x \in L^{J_L}$, *and* $(\succcurlyeq_i)_{i\in J^L} \in \mathcal{S}^{J_L}$ *a* J_L*-profile that is simple w.r.t.* x. *Then, the set of strong equilibria of* F *at* $(\succcurlyeq_i)_{i\in J^L}$ *is nonempty.*

Proof. Take profile $y = (y_j)_{j\in J_L}$ defined as follows:

for any $i \in J_L$, $j \leqslant y_i$ for all $j \in J_L$ if if $i \leqslant x_i$, and

$j \nleqslant y_i$ for all $j \in J_L$ if if $i \nleqslant x_i$.

Now, let us partition J_L into the following blocks

$N_1 = \{i \in J_L : i \leqslant x_i \text{ and } i \leqslant F(y)\}$

$N_2 = \{i \in J_L : i \nleqslant x_i \text{ and } i \leqslant F(y)\}$

$N_3 = \{i \in J_L : i \leqslant x_i \text{ and } i \nleqslant F(y)\}$

$N_4 = \{i \in J_L : i \nleqslant x_i \text{ and } i \nleqslant F(y)\}$.

Next observe that for any $i \in J_L$, and any $F \in \{F^E, F^{PE}, F^R, F^{PR}\}$,

$i \leqslant F(y)$ if and only if $i \leqslant x_i$
(by definition of F): hence, in fact $N_2 \cup N_3 = \emptyset$.
Let us now assume that there exist $T \subseteq J_L$ and $z_T \in L^T$ such that
$F(y_{N \setminus T}, z_T) \succ_i F(y)$ for each $i \in T$.
Since $F(y)$ is $\succ_i$-optimal for any $i \in N_1 \cup N_4$ -by definition of $(\succcurlyeq_i)_{i \in J^L}$ - it must be the case that $T \cap (N_1 \cup N_4) = \emptyset$, i.e. $T \subseteq N_2 \cup N_3 = \emptyset$.
Therefore, $(y_{N \setminus T}, z_T) = y$, a contradiction.
It follows that y is indeed a strong equilibrium of F at $(\succcurlyeq_i)_{i \in J^L}$. ■

Therefore, the IQC-consistent CIPs characterized in the previous subsections enjoy a reasonable amount of cooperative stability. In particular, they are strong equilibrium solvable, hence a fortiori core-stable at least on the simple domain[15].

3 Related literature

The IQC principle requires certification by another *qualified* agent as a necessary condition to qualify for membership. Thus, the assessment of the qualifications of any agent may well also depend on the assessment of the qualifications of *other* agents. This is in stark contrast with the idea that membership of any agent should only depend on the assessment of *her own* credentials, as established by the most straightforward adaptation of *arrowian Independence* to collective identification problems, namely

Independence (IND): For any $x, x' \in L^{J_L}$ and any $j \in J_L$ if [for all $i \in J_L$: $j \leqslant x_i$ iff $j \leqslant x'_j$] then [$j \leqslant F(x)$ iff $j \leqslant F(x')$]

To be sure, IQC and IND are not strictly speaking mutually inconsistent. For instance, the constant CIP $F^{\wedge L}$ establishing Universal Rejection[16] satisfies both IQC and IND. However, it can be shown that Unanimity (UN)[17]

[15] A strategic game form G is solvable (or stable) with respect to a certain solution concept on a certain domain D of preference profiles, if at each preference profile $\succcurlyeq$, the game $(G, \succcurlyeq)$ has a nonempty set of solutions.

[16] Hence, the CIP $F^{\wedge L}$ is defined as follows: for any $x \in L^{J_L}$, $F^{\wedge L}(x) = \wedge L$.

[17] A CIP $F : L^{J_L} \rightarrow L$ satisfies *Unanimity* (or *Idempotence*) if $F(x) = x^*$ whenever $x = (x^*, x^*, ..., x^*)$.

is inconsistent with IQC, and conversely the conjunction of IQC and Souvereignty (S)[18] is inconsistent with IND, namely

Claim 16 *Let $F : L^{J_L} \rightarrow L$ be a CIP . Then i) if F satisfies UN then it must violate IQC; ii) if F satisfies S and IQC then it must violate IND.*

Proof. i) Straightforward: consider profile $x \in L^{J_L}$ such that for any $i, j \in J_L$, $j \leqslant x_i$ iff $j = j^*$. Hence, by UN, $j \leqslant F(x)$ iff $j = j^*$i.e. $F(x) = j^*$which violates IQC.

ii) For any $x \in L^{J_L}$, and any $i \in J_L$, posit
$N_i(x) = \{j \in J_L : i \leqslant x_j\}$.

Then, choose a $j \in J_L$, and take a profile $x \in L^{J_L}$ such that $j \leqslant F(x)$ (by S, there exists a profile with such a property). Next, consider a (new) profile $x' \in L^{J_L}$ defined by the following condition: for any $j' \neq j$, $N_{j'}(x') = \emptyset$, while $N_j(x) = N_j(x')$.

Clearly, $j \nleqslant F(x')$ by IQC. Therefore, F violates IND as claimed. ■

We have already observed in the previous note that the IQC-consistent CIPs studied in the present paper invariably satisfy Souvereignty. Moreover, the inconsistency between Unanimity and Independent Qualified Certification does also involve Independence in that the most interesting CIPs that satisfy Independence satisfy Unanimity as well. Indeed, the Libertarian CIP F^{L^*}introduced above satisfies both.

Another well-known identification rule is the following

Definition 17 *The **Unanimous Consent (UC)** procedure: for any $x \in L^{J_L}$,*
$F^{UC}(x) = \bigvee \{j \in J_L : \ j \leqslant x_i \ \ \textit{for all } i \in J_L \ \}$[19]

Clearly, UC identifies as members precisely those agents who are unanimously nominated.

[18] A CIP $F : L^{J_L} \rightarrow L$ satisfies *Souvereignty* iff it is a surjective function. Obviously Unanimity implies Souvereignty, but not vice versa. Notice that each one of the IQC-consistent CIPs introduced and characterized in the present paper do satisfy Souvereignty.

[19] Hence, the Unanimous Consent procedure identifies as members precisely those agents that are unanimously declared to qualify for membership. That is far from being an irrelevant, far-fetched procedure: indeed, it may be regarded as a convenient model of those voluntary affiliations where qualifications consist of essentially verifiable information (e.g. professional and related associations).

Again, UC also satisfies IND and UN (and therefore violates IQC): moreover, it satisfies PC but also violates CSD (to check the last claim, just consider the case of a profile that unanimously declares exactly one agent as the only qualified one).

The (Quota) Consent Rules requiring a minimum number of nominations (rejections) for membership acceptance (rejection)- as introduced and characterized by Samet and Schmeidler (2003)- also satisfy UN and IND[20].

Remark 18 *It is worth noticing that the Libertarian procedure, the Unanimous Content procedure, and the Quota Consent Rules mentioned above do also satisfy Monotonicity (see note 20 for a precise definition). Now, it is easily checked that each CIP which satisfies Independence and Monotonicity is also* strategy-proof *on the simple domain* $\mathcal{S}^{J_L}$ *as defined in the previous section. To see this, observe that, by Independence, each agent can only affect her membership by her own self-evaluation, while, by Monotonicity, each agent with 'simple' preferences can never ameliorate her status by submitting a false self-evaluation. It follows that, in particular, the Libertarian and Unanimous Consent procedures, and the Quota Consent Rules, are strategy-proof on the simple domain. Moreover, the foregoing CIPs are also* strong-equilibrium-solvable *on the simple domain. To see this just consider, for any opinion profile* x*, the opinion profile* y *such that for any* $i, j \in J_L$*,* $i \leqslant y_j$ *iff* $i \leqslant x_i$*. At this profile, each agent is the ultimate opinion leader about her own qualifications as declared by herself at* x *: therefore profile* y *does not allow profitable coalitional deviations at the simple preference profile ineuced by* x*.*

As mentioned in the Introduction, some CIPs which do *not* satisfy IND have also been introduced and studied in the relevant literature (see e.g. Kasher and Rubinstein (1997)[21], Dimitrov, Sung and Xu (2003), Houy (2006)), namely

[20] Indeed, in Samet and Schmeidler (2003) the Quota Consent Rules are characterized as the CIPs which satisfy IND, *Monotonicity* (i.e. $F(x) \leqslant F(x')$ whenever $(x_i)_{i\in J_L} = x \leqslant x' = (x'_i)_{i \in J_L}$ i.e. $x_i \leqslant x'_i$ for any $i \in J_L$), and *Symmetry* (i.e. $F(\sigma x) = \sigma F(x)$ for any permutation σ of J_L, where $\sigma x = (\sigma x_i)_{i \in J_L}$, with $j \leqslant \sigma x_i$ iff $\sigma(j) \leqslant x_{\sigma(i)}$, and $j \leqslant \sigma F(x)$ iff $\sigma^{-1}(j) \leqslant F(x)$).

[21] To be sure, Kasher and Rubinstein use 'Independence' as a label for a much weaker conditional version of the arrowian-like Independence defined above. Indeed, such a Kasher-Rubinstein weakened independence is satisfied by the UCQN and LMQN procedures defined below as well as by the IQC-consistent procedures previously considered in the text.

Definition 19 *The **Liberal Multi-level Qualified Nomination (LMQN)** procedure: for any* $x \in L^{J_L}$

$$F^{LM}(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \textit{there exist } k \in \mathbb{Z},\ k \geq 1, \textit{ and } \{i_1, .., i_k\} \subseteq J_L \\ \textit{such that} \\ i_1 \leqslant x_{i_1} \\ h \leqslant x_{h+1(\operatorname{mod} k)} \textit{ for any } h \in \{i_1, .., i_k\} \textit{ and } j \leqslant i_k \end{array} \right\}$$

Thus, LMQN may be regarded as a multi-stage procedure which identifies as members both the agents who declare themselves to qualify as members *and* their (direct and indirect) nominees.

Definition 20 *The **Consensual Multi-level Qualified Nomination (CMQN)** procedure is defined as follows: for any* $x \in L^{J_L}$

$$F^{CM}(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \textit{there exist } k \in \mathbb{Z}, k \geq 1, \textit{ and } \{i_1, .., i_k\} \subseteq J_L \\ \textit{such that} \\ i_1 \leqslant x_i \textit{ for each } i \in J_L \\ h \leqslant x_{h+1(\operatorname{mod} k)} \textit{ for any } h \in \{i_1, .., i_k\} \textit{ and } j \leqslant i_k \end{array} \right\}$$

Clearly, CMQN may also be regarded as a multi-stage procedure which identifies as members precisely all the agents who are *unanimously* declared to qualify *and* their (direct and indirect) nominees.

Definition 21 *The **Indirectly Consensual Nomination (ICN)** procedure is defined as follows: for any* $x \in L^{J_L}$

$$F^{IC}(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \textit{for all } h \in J_L \textit{ there exist } k \in \mathbb{Z},\ k \geq 1, \\ \textit{and } \left\{ i_1^{(h)}, .., i_k^{(h)} \right\} \subseteq J_L \\ \textit{such that} \\ h = i_1^{(h)},\ j = i_k^{(h)}, \textit{ and} \\ i_l^{(h)} \leqslant x_{i_{l+1}^{(h)}} \textit{ for any } l \in \{1, ..., k-1\} \end{array} \right\}$$

Thus, ICN (due to Houy (2006)) declares members precisely all the agents that are directly or indirectly nominated by each agent.

It is worth considering, for the sake of comparisons, the behavior of the foregoing procedures in terms of the axioms considered in the present work.

Indeed, it is easily checked that F^{LM} and F^{IC} fail to satisfy IQC, CSD, or PC.[22]

In contrast, it can be easily shown that F^{CM} does satisfy IQC but violates both PC and CSD. One might easily devise a 'participatory' version of F^{CM} (to be defined in the obvious way) which would satisfy IQC and PC while violating CSD.

4 Concluding remarks

The main point of the present work is to show by example that there is a rich variety of collective identification procedures worth considering that do not satisfy the classic arrowian independence condition, while reconciliating the apparently conflicting requirements of independent qualified certification or sponsorship and no-cooptation.

The procedures discussed in the current paper are probably best regarded as stylized 'ideal' paradigms, mostly useful as benchmarks for classificatory purposes. However, this is not to say they are unrelated to 'real' collective identification procedures. On the contrary, it seems to me that for virtually all the procedures considered in the former sections a rather close similarity to some classes of historically relevant examples may be claimed. For instance, while affiliations to most political parties in contemporary European democracies essentially rely on libertarian procedures such as L^*, admissions to some of their former counterparts operating in clandestinity under nazi-fascist regimes used to rely on versions of the PEQN procedure as introduced above in Section 2.1. Moreover, one might perhaps claim that 16th century's vicious conflict between Catholics and mainstream Protestant denominations on one side and Anabaptists on the other concerning the validity of early (e.g. infant) baptism is at least to some extent captured by the contrast between EQN and PEQN. On a more frivolous tone, PEQN (as opposed to, say, L^*) is arguably a rather good stylized version of the typical admission procedures used by the best tennis clubs, while L^* is perhaps more characteristic of respectable clubs of soccer teams' fans.

[22] Of course, they also violate IND. In particular, the characterization of ICN proposed by Houy (2006) is wrong in that it implicitly relies on the false claim that ICN satisfies IQC. To see this, just take the lattice $(\mathcal{P}(N), \subseteq)$ with $N = \{1,2,3\}$, and consider opinion profile $x = \{\{1\},\{1\},\{1\}\}$. Clearly, $F^{IC}(x) = \{1\}$. However, there is no $j \neq 1$ such that $\{1\} \subseteq x_j$ *and* $\{j\} \subseteq F^{IC}(x)$, whence F^{IC} violates IQC.

Be it as it may, this is certainly not the place to dwell on a serious discussion of those putative historical or current common-life examples. Rather, a few specific comments on the IQC-consistent procedures introduced and studied in the present paper are in order here.

First, it should be clear at this point that the IQC principle entails the existence of some 'virtuous circles' of mutually sustaining certifications, which may be regarded as a social analogue of certain interactive catalytic chemical reactions as nicely epitomized by Eigen's well-known 'hypercycles'[23] (see e.g. Hofbauer and Sigmund (1988)). In particular, such 'virtuous circles' may be particularly appropriate as an idealized model of the 'constitutional' phase of an association. Of course, my use of 'virtuous' as a qualifier alludes to the positive role of those circles of mutual certification in bringing to existence and sustaining the association under consideration, in accordance with its admission rules. Therefore, the foregoing circles are certainly 'virtuous' from the point of view of the association itself i.e. of its supporters. However, it may be the case that such circles simply result from (and/or foster) some possibly unpalatable segregation process.

It should also be remarked that CIPs that satisfy IQC may well be consistent with (indeed, partly explanatory of) the reality of 'contested identities', namely the existence of several, possibly disjoint, subcommunities claiming the same identity.

Of course, CIPs designed to prevent such conflicts involving members (and to oppose undesirable forms of segregation) might be identified by introducing appropriate supplementary requirements.

Also, it is worth emphasizing again that all the IQC-consistent CIPs characterized in Section 2 do satisfy both Monotonicity and Symmetry as defined above (see Note 19), along the lines of Samet and Schmeidler (2003): characterizing the entire class of CIPs that satisfy IQC, Monotonicity and

[23] Eigen's 'hypercycles' consist of k (types of) selfreplicating macromolecules (e.g. polynucleotides) arranged in a closed loop in such a way that each macromolecule catalyses the replication of its successor. This catalytic interaction provides a plausible way out of the dilemma arising from the observation that

i) the lenght of selfreplicating macromolecules is limited by the (replication) error bound;

ii) the error bound is in fact abated thanks to the action of enzymes i.e. polypeptides (chains of amino acids) working as biocatalists; but

iii) polypeptides are coded by polynucleotides which in turn consist of *very long* sequences of nucleotides (e.g. adenine, cytosine, guanine, thymine, uracil).

Admittedly, the analogy with self-certifying 'virtuous' circles I am suggesting here involves two very remote domains, but in my view is indeed too close to resist mentioning.

Symmetry is an interesting open issue which has not been addressed here.

Finally, it should be noticed again that CIPs are just *strategic game forms of a highly specialized sort.* Therefore, one may consider their solvability properties with respect to several noncooperative and cooperative game-theoretic solution concepts on suitably *general* domains of preference profiles. Moreover, other structural properties of CIPs are amenable to further analysis through the study of their concept lattices along the lines of Vannucci (1999).

All those topics, however, are best left as the subjects of some further research.

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