Equality of opportunity and optimal effort decision under uncertainty

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Example

<u>$t = 0$</u>

Example

***t* = 0**

External Shock

Example

$t = 0$ **$t = 1$**


External Shock

UCL


Example

$t = 0$




External Shock

$t = 1$

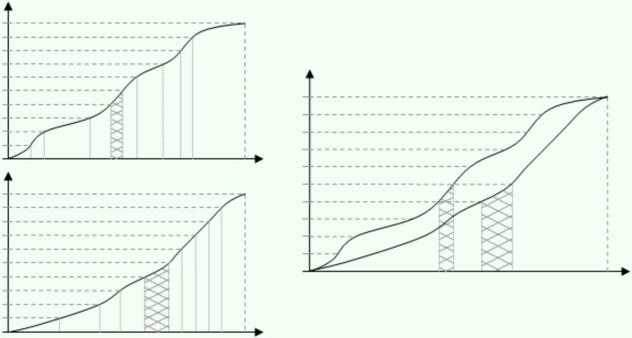
$t = 2$

Roemer's approach

- Equality of opportunity (EOp). Roemer's approach:

  Outcome: "Opportunity" and "Responsibility"

- Intuition and weakness of this framework?:



Objectives

1. I analyse the effect of uncertainty (luck) on income distribution.

2. I characterise the individual's optimal effort decision as a solution of an explicit intertemporal utilitarian maximization problem.

3. The planner's optimal EOp policy is also studied.

Results

1. Luck affects income distribution in a biased and persistent way.

2. Therefore, opposite to the to generally assumed neutral effect of luck on income, we assume that such an effect **does call** for social compensation.

3. Traditional results in the literature also fit within our setting.

4. The planner:
    4.1 She can affect the individuals' effort decision so as to smooth down the effect of luck on income.
    4.2 As usual, the optimal EOp policy implies compensations just within effort groups.

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Preliminaries I

- The society is made up of a finite number of individuals: $\mathcal{M} = \{1, \ldots, m, \ldots, M\}$.
- Finite income space: $X \in R^n_+$, with $x_1 < \ldots < x_i < \ldots < x_n$.
- $\mathcal{M}(w_j)$ is the set of agents with circumstances $j \in \{1, \ldots, J\}$.
- The effort level chosen by agent m is a real number $\widetilde{e}$ in the closed interval $\left[\widetilde{e}^L_w, \widetilde{e}^H_w\right]$.

Preliminaries II

- Conditional probability:

$$\begin{aligned}\Pr\left[x = x_i \mid \widetilde{e}, w\right] &= p_i\left(\widetilde{e}, w\right); \forall i \in \{1, \ldots, n\} \\ &= e p_i^H\left(w\right) + \left(1 - e\right) p_i^L\left(w\right)\end{aligned}$$

- Expected income:

$$\begin{aligned}&\overline{x}_m\left(e_m\left(t\right), w_m\left(t\right)\right) \\ = &\sum_{i=1}^{n}\left[e_m(t) p_i^H\left(w_m\left(t\right)\right) + (1 - e_m(t)) p_i^L\left(w_m\left(t\right)\right)\right] x_i\end{aligned}$$

Preliminaries III

- Individual's utility:

$$U_m\left(e_m\left(t\right), w_m\left(t\right)\right) = u_m\left(x_i\right) - c_m\left(e_m\left(t\right)\right)$$

- Income utility and effort cost functions:

$$\begin{aligned} u_m\left(x_i\right) &= x_i \\ c_m\left(e_m\left(t\right)\right) &= \left[\left(1 - e_m\left(t\right)\right)^{-\sigma_m} - 1\right] \end{aligned}$$

- Every period, a fraction α of the present income is saved as next period initial wealth.

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Optimal effort decision I

- The individual maximises her intertemporal expected utility:

$$\left.\begin{array}{c} \max\limits_{\{e_m(s)\}} \sum\limits_{s=t}^{T} \beta^{s-t} \left((1-\alpha) \left[\overline{x}_m(e_m(s), w_m(s)) \right] - \left[(1-e_m(s))^{-\sigma_m} - 1 \right] \right) \\ s.t.: \ e_m(s) \in [0,1] \\ \overline{w}_m(s+1) = \alpha \left[\overline{x}_m(e_m(s), w_m(s)) \right] \\ w_m(0) = w_0^m; \forall m \in \mathcal{M} \end{array} \right\}$$

Optimal effort decision II

Proposition

The individual's optimal effort decision at any period $t \in [0, T]$ is given by the following expression:

$$e_m^*(t) = 1 - \left[\frac{(1-\alpha)}{\sigma_m} \sum_{i=1}^{n} k_i(w_m(t))\, x_i (1+\Delta_t) \right]^{\frac{-1}{1+\sigma_m}}$$

where: $w_m(t) \sim \Phi(w_0^m, \sigma_m)$.

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Optimal effort decision III

Claim

-Circumstances, personal choices and luck are the determinants of the individual's income.
-The luck factor has a biased and persistent effect on income, and hence it calls for social compensation in order to assure equality of opportunity.

Corollary

If circumstances are required to be fixed, the individual would make a constant level of effort. Moreover, the planner could infer exactly the individuals' level of responsibility by means of a simple updating of beliefs mechanism.

Static solution

Optimal effort decision III

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Static solution

Optimal effort decision IV

- I introduce a social planner that is concerned about inequality of opportunity.
- Such a planner aims at equalizing incomes of those agents who exert a comparable degree of effort.
- The planner cannot infer exactly the individuals' level of responsibility.
- Function $g_m(t) = f(\overline{x}(\cdot, t))$ summarizes the planner's beliefs about all her past experiences with agent m up to period t.

Optimal effort decision V

Proposition

The individual's optimal effort decision at any period $t \in [0, T]$ is given by the following expression:

$$e_m^*(t) = 1 - \left[\frac{(1-\alpha)}{\sigma_m} \sum_{i=1}^{n} k_i(w_m(t))\, x_i \left[1 + \Delta_t + \Psi_t\right] + \Gamma_t \right]^{\frac{-1}{1+\sigma_m}}$$

where: $w_m(t) \sim \Phi(w_0^m, \sigma_m)$.

Max. prog. Effort decision

EOp problem I

Definition

There is EOp in the society if and only if $\forall m, m' \in \mathcal{M} : g_m(t) = g_{m'}(t)$ *the following condition holds:*

$$\overline{x}_m\left(\widehat{e}_m^*(t), w_m(t)\right) = \overline{x}_m\left(\widehat{e}_{m'}^*(t), w_{m'}(t)\right)$$

EOp condition

EOp problem II

Proposition

-The social planner can affect the individuals' optimal choice of effort, and hence the income distribution, in order to assure equal opportunity within the society.

-As usual, the equal opportunity feature is concerned about income inequalities within effort groups. Income differences between those groups only represent diverse rewards of people's autonomous choices and will not be considered unfair.

Planner's Max. Prob.

Concluding remarks I

- EOp is generally considered the *fairest* principle at the time of evaluating outcome and opportunity.
- Optimal effort decision depends on: the individual's circumstances, her preferences, and the sort of luck experienced in the past.
- The introduction of luck through the uncertainty of income exhibits here very interesting features.

Concluding remarks II

- Social planner concerned about inequality in opportunity terms.

- She can design a redistribution policy so as to affect the individuals' distribution of income.

- Education (health) as outcome. Investment in early stages of education has a deeper impact on the students' future success.

Thank you very much

Grazie Mille

Additional slide 2

Static solution

- If there is no relation between periods, the individual's optimal effort turns into:

$$e_m^*(t) = 1 - \left[\frac{1}{\sigma_m}\sum_{i=1}^{n} k_i\left(w_0^m\right)x_i\right]^{\frac{-1}{(\sigma_m+1)}}$$

Corollary

Under incomplete information, if individuals are making a constant level of effort, the planner can infer exactly (after a certain finite number of periods) the individual's level of responsibility by means of a simple updating of beliefs process.

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Additional slide 3

Planner's maximization problem

- The planner maximises the following program:

$$\left.\begin{array}{c} \max_{\left\{\gamma_m^{w_j(t)}\right\}} \sum_{m=1}^{M} \left((1-\alpha) \left[\overline{x}_m(\cdot, t) + g_m(t)\, \gamma_m^{w_j(t)} \right] - \left[\left(1 - \widehat{e}_m^*(t)\right)^{-\widehat{\sigma}_{t,m}} - 1 \right] \right) \\ s.t.: \sum_{m=1}^{M} g_m(t)\, \gamma_m^{w_j(t)} = \kappa \\ \widehat{e}_m^*(t) = 1 - \left[\frac{\Upsilon_m}{\widehat{\sigma}_{t,m}} \right]^{\frac{-1}{1+\widehat{\sigma}_{t,m}}} ; \forall m \in \mathcal{M} \\ \overline{x}_m\left(\widehat{e}_m^*(t)\right) = \overline{x}_{m'}\left(\widehat{e}_{m'}^*(t)\right); \forall m, m' \in \mathcal{M} : g_m(t) = g_{m'}(t) \end{array}\right\}$$

back

Additional slide 4

Optimal effort decision I

Proposition

The individual's optimal effort decision at any period $t \in [0, T]$ is given by the following expression:

$$e_m^*(t) = 1 - \left[\frac{(1-\alpha)}{\sigma_m} \sum_{i=1}^{n} k_i (w_m(t)) \, x_i (1 + \Delta_t) \right]^{\frac{-1}{(\sigma_m + 1)}}$$

where: $k_i(\cdot) = p_i^H(\cdot) - p_i^L(\cdot)$; $\Delta_t = \sum_{j=t+1}^{T} (\beta\rho)^{j-t}$, $\rho = \alpha \left[\frac{\partial \overline{x}_m(\cdot, t+1)}{\partial w_m(t+1)} \right]$, *and* $w_m(t) = \Phi(w_0^m, \sigma_m, \alpha)$.

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Additional slide 5

Optimal effort decision II

Proposition

The individual's optimal effort decision at any period $t \in [0, T]$ is given by the following expression:

$$e_m^*(t) = 1 - \left[\frac{(1-\alpha)}{\sigma_m} \sum_{i=1}^{n} k_i(w_m(t))\, x_i \left[1 + \Delta_t + \Psi_t\right] + \Gamma_t \right]^{\frac{-1}{(\sigma+1)}}$$

where:

$$\begin{aligned}
\Delta_t &= \sum_{j=t+1}^{T} (\rho\beta)^{j-t} \\
\Psi_t &= \frac{1}{\alpha} f_{\overline{x}} \left(\frac{\partial \overline{x}(\cdot, t+1)}{\partial w_m(t+1)} \right) \sum_{j=t+1}^{T} \rho^{j-t} \sum_{j=t+2}^{T} \beta^{j-t} \gamma^{w_m(j)} \\
\Gamma_t &= f_{\overline{x}} \left(\sum_{i=1}^{n} k_i(w_m(t))\, x_i \right) \sum_{j=t+1}^{T} \beta^{j-t} \gamma^{w_m(j)}
\end{aligned}$$

Additional slide 6

EOp condition

Proposition

There is EOp in the society if and only if $\forall m, m' \in \mathcal{M} : g_m(t) = g_{m'}(t)$ *the following condition holds:*

$$\sum_{i=1}^{n}\left[\left(1-\left(\frac{1}{\widehat{\sigma}_t}\Upsilon_m\right)^{\frac{-1}{(\widehat{\sigma}_t+1)}}\right)k_i\left(w_m(t)\right)+p_i\left(w_m(t)\right)\right]x_i$$

$$=\sum_{i=1}^{n}\left[\left(1-\left(\frac{1}{\widehat{\sigma}_t}\Upsilon_{m'}\right)^{\frac{-1}{(\widehat{\sigma}_t+1)}}\right)k_i\left(w_{m'}(t)\right)+p_i\left(w_{m'}(t)\right)\right]x_i$$

where $\Upsilon_m = (1-\alpha)\left(\sum_{i=1}^{n} k_i\left(w_m(t)\right)x_i\left[1+\Delta_t+\Psi_t\right]+\Gamma_t\right)$, *and* $\widehat{\sigma}_{t,m}$ *is the level of disutility of effort that in period* t *the planner guesses from agent* m.

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Additional slide 7

Individual's EOp maximization

- The individual maximises her intertemporal expected utility:

$$\left.\begin{array}{c}\max_{\{e_m(s)\}}\sum_{s=t}^{T}\beta^{s-t}\left(\left(1-\alpha\right)\left[\overline{x}_m\left(\cdot\right)+g_m\left(s\right)\gamma^{w_m(s)}\right]-\left[\left(1-e_m\left(t\right)\right)^{-\sigma_m}-1\right]\right)\\ s.t.:\ e_m(s)\in[0,1]\\ \overline{w}_m\left(s+1\right)=\alpha\left[\overline{x}_m\left(e_m(s),w_m\left(s\right)\right)\right]\\ w_m\left(0\right)=w_0^m;\forall m\in\mathcal{M}\\ g_m\left(s+1\right)-g_m\left(s\right)=f\left(\overline{x}\left(\cdot,s\right)\right)\\ g_m\left(0\right)=g_m^0\in\mathbb{R}_+\end{array}\right\}$$

back