Household Heterogeneity and Inequality Measurement

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Three allocation problems

- To cope with the problems raised by the fact that the basic statistical unit is household

- 1. They are heterogeneous in size and composition

  - A source of difference in needs: how to treat needs in inequality measurement ?
  - *1.Allocation of income across households*
  - A change in demographics can induce a change in social welfare
  - 2. *Allocation of individuals across households*

- 2. They are heterogeneous in sharing resources (behaviour)

  - Intra-household inequality
  - 3. *Allocation of income across the household's members*

- 3. Three issues may have to be addressed simultaneously

Outline

- 1. Differences in size
  - Assumption that household members are treated equally
- 2. Differences in sharing behaviour
  - Same size and needs
- 3. Both differences together

1 Heteregoneity in needs

- A) *Demographic composition of the population is fixed* (the marginal distribution of needs is fixed)
  – Only an allocation of income across households
  – The ordinal approach to needs
  – The cardinal approach to needs
  – The bounded dominance approach to needs

- B) *Demographic composition of the population is allowed to change*
  – + A problem of allocation of individuals accross households
  – Paradox with the Average household utility
  – Switching to Average invidual utility
  – The importance of terminal condition in dominance theory
  – Relying on the bounded dominance approach

A) Marginal distribution of needs is fixed

- singles and couples
- $p_1 + p_2 = 1$
- $f_1(y)$, $f_2(y)$: *income densities*
- *V(y,1); V(y,2) : Household « utility » functions*
- *Average household utility :*

$$W(y) = p_1 \int_0^{s_1} f_1(y) V(y,1) dy + p_2 \int_0^{s_2} f_2(y) V(y,2) dy$$

Ordinal approach to needs

- Atkinson and Bourguignon (87), Bourguignon (89)
- Classes of household utility functions where the key axiom are about the marginal social valuation across households of differents size.
- Ranking of households according to needs and therefore to social priority
- We are able to rank household
- We don’t know how much a couple is more needy wrt a single

The restrictions on the social marginal valuations

- *V(y,1); V(y,2) increasing and concave*
- *Key axioms:*
  - *The first one for the Bourguignon class,*
  - *Both axioms for the Atkinson and Bourguignon class*

$$V_y(y,2) \geq V_y(y,1), \quad \forall\, y \in \mathbb{R}_+,$$

$$V_y(y,2) - V_y(y,1) \downarrow w.r.t.\, y \forall\, y \in \mathbb{R}_+$$

The dominance criteria

- For the AB class, the Sequential Generalized Lorenz criterion (SGL)
- 1.Check that the GL test applied to the distribution of the most needy group (the couples) is satisfied
- 2.Check that the GL test applied to the most needy and the second most needy groups (the couples + the simples) is satisfied
- Why ? The first condition because the class of admissible utility functions contains utility functions for which the couples are infinitely more needy than singles
- The second condition because the class contains utility functions for which the couples are as needy as singles.

Express the SGL in terms of poverty orderings (1)

- Foster Shorrocks : stochastic dominance orderings and then GL orderings are equivalent to poverty orderings

$$\int_0^{x_2} (x_2 - y)\, f_2(y)dy$$

- is the absolute poverty gap for couples taking x_2 as the poverty line.
- First "sequence" : check that the APG for couples has decreased for all poverty lines

Express the SGL in terms of poverty orderings (ctd)

- The absolute poverty gap for all the population

$$\sum_{k=1}^{2} \int_0^x (x - y)\, p_k\, f_k(y)\, dy$$

- « Second sequence » : Check the decline of the absolute poverty gap for all households for all poverty lines

The dominance criterion for the Bourguignon Class

- Express in terms of poverty gaps but it is not sequential (even if it implies the sequential test)

- Check that the sum of absolute poverty gaps for singles and couples is decreasing for all poverty lines such that the poverty line of the couples is at least as great as the poverty line of the single

$$\sum_{k=1}^{K} p_k \int_0^{x_k} (x_k - y)\Delta f_k(y)dy \leq 0 \quad \forall (x_k)_{k=1,.2} \;\; x_2 \geq x_1 \text{ and } x_1 \in [0, \bar{s}_1]$$

Cardinal approach to needs

- A rate of conversion of the well-being of couples with respect to singles (reference type)
- The couple equivalence scale *e* means that the couple needs the *e*-fold income of a single person to reach the same standard of living
- Equivalent income of couple : y/e

The weighing problem: 2 solutions

- Weighting by the equivalence scale (number of equivalent adults from a needs perspective)
- Ebert 97, 99

$$V(y,k) = e_k U\left(\frac{y}{e_k}\right),$$

- Weighting by the size of the household (Glewwe 91, Shorrocks 04)

$$V(y,k) = n_k U\left(\frac{y}{e_k}\right)$$

Conflict between values

- Ebert’s solution
  - Satisfies the between type Pigou-Dalton principle
  - violates Pareto Indifference
  - social indifference between two social states if each individual, given her type and income in the respective states, reaches the same living standard in both states
- Shorrocks’s solution
  - Satisfies the Pareto indifference axiom
  - Violates the between type Pigou-Dalton principle

A conflict?

- The right individual indirect utility function in the microeconomic tradition in the Ebert's approach is

$$v(y,k) = \frac{e_k}{n_k} U\left(\frac{y}{e_k}\right)$$

And not

$$v(y,k) = U\left(\frac{y}{e_k}\right)$$

Lorenz criterion with Ebert

- Applied to equivalent incomes
- The population shares are computed in terms of equivalent adults

Advantages and Pitfalls of the two approaches

- *Ordinal* : robust but incomplete and contains classes of utilities functions w.r.t needs that are irrelevant
- « too coarse dominance tests »

- *Cardinal* : more discriminatory power but relies on a notion « equivalence scale at the household level » the estimation of which is problematic.

- Range of equivalence scales admissible

A middleway criterion : the bounded dominance approach

- Fleurbaey-Hagneré-Trannoy (2003) JET

build a bridge between

  - A refinement of the Bourguignon criterion
  - the Dominance for the GL applied to equivalent incomes for a range of equivalence scales

- Ooghe and Lambert (2005) MSS
  - For the AB criterion
  - The Dominance for the GL applied to equivalent incomes for a range of equivalence scales

Bounds on social priorities

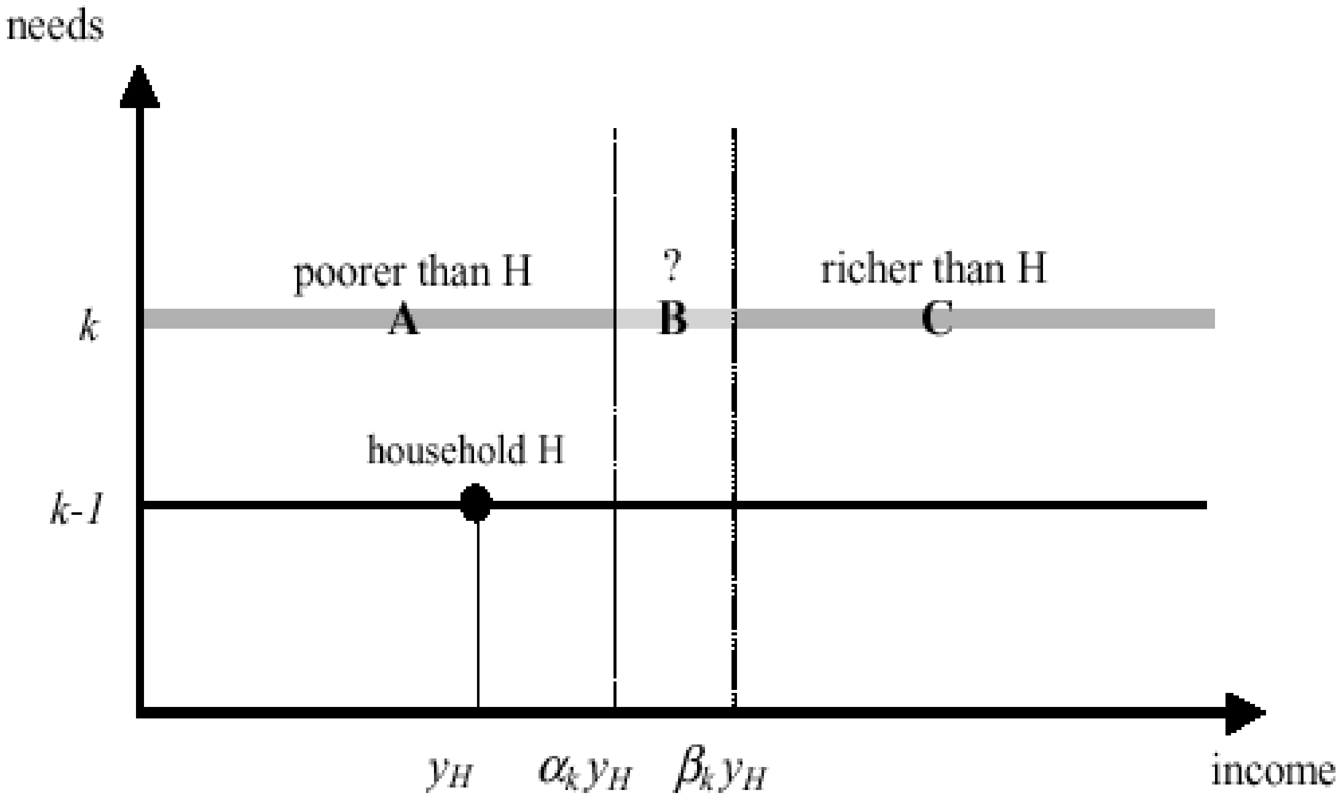
- *Lower bound* : A couple with an income equal to or lower than $\alpha y, with \alpha > 1$ is worse off than a single with income y

$$V_y(\alpha \; y, 2) \geq V_y(y, 1), \;\; \forall \; y \in \mathbb{R}_+,$$

- *Upper bound* : A couple with an income equal to or higher than $\beta y, with \beta > \alpha$ is better off than a single with income y

$$V_y(\beta y, 2) \leq V_y(y, 1), \;\; \forall \; y \in \mathbb{R}_+,$$

A grey zone to represent the divergence of opinions



A tunnel

- See whiteboard

The Extension of Bourguignon's Criterion

- Check that the sum of absolute poverty gaps for singles and couples is decreasing for all poverty lines such that the poverty line of the couple belongs to the grey zone

$$\sum_{k=1}^{K} p_k \int_0^{x_k} (x_k - y)\Delta f_k(y)dy \le 0 \quad \forall (x_k)_{k=1,.2} \;\; \alpha x_1 \le x_2 \le \beta x_1$$

$$\text{and } x_1 \in [0, \bar{s}_1, \frac{\bar{s}_2}{\beta_2}]$$

Implementation of the criterion

Unfortunately, condition (B) is not implementable since it leads to checking an infinity of conditions. One more step allows us to propose a more tractable condition.
For $K = 2$, condition (B) is written:

$$p_1 \triangle H_1(x_1) + p_2 \triangle H_2(x_2) \leq 0 \quad \forall x_1, x_2 \text{ such that}$$

$$\alpha_2 x_1 \leq x_2 \leq \beta_2 x_1 \text{ and } x_1 \in [0, \max(\bar{s}_1, \frac{\bar{s}_2}{\beta_2})]$$

It is straightforward to show that this condition is equivalent to the following:

$$p_1 \triangle H_1(x_1) + \max_{x_2 \in [\alpha_2 x_1, \beta_2 x_1]} \{p_2 \triangle H_2(x_2)\} \leq 0 \ \forall x_1 \in [0, \max(\bar{s}_1, \frac{\bar{s}_2}{\beta_2})]$$

Equivalence with the GL criterion with bounded of Equivalent Scales

- A range of equivalence scales given by

$$E(\alpha, \beta) = \{(e) \mid \alpha \le e \le \beta\}$$

- The GL dominance for this range is equivalent to the Bourguignon dominance with the grey zone

B) Change of demographic composition

- An additional allocation problem
- Allocation of people in the households
- Fleurbaey Hagneré Trannoy 07 (rev for OEP)
- Welfare in almost all the literature: the average household utility accross all households
- As long as demographic composition does not change, OK
- But when demographic composition changes, it violates the Pareto Principle. It implies a pro-family stance: the criterion favors concentrating people in large households
- A good statistical measure should be agnostic to this issue

Example : Violation of Pareto and pro-family

- Society A : A couple with income: y
- Society B : Two singles with income: y/2
- The same per-capita income z = y/2, same resources.

- u(1, z): utility of a single with z
- u (2, z): utility of a individual living in a couple with z
- Assumption : u(1,z) > u(2,z)
- Average household utility in society A: 2u(2,z)
- Average household utility in society B : u(1,z)

- While individuals are indifferent between the two societies, the criterion of average household utility gives precedence to society A as long as 2u(2, z)-u(1,z) > 0 and then favors the society made of couples

The solution

- 1) switch to welfare defined the average individual utility
- 2) When demographic differ, terminal conditions defined on the utility levels have to be added saying that « for very rich people, the difference of utilities according to household size is bounded » To pay attention to the formulation of that condition
- 3) Rely on the bounded dominance approach

- Jenkins and Lambert (1993) made use of 1) and 2) but the criterion is anti-family on part of the domain.

- If you follow these three recipe, you find extension of the bounded criteria which are flexible enough with respect to the « optimal family size issue ».

2. Heterogeneity in sharing resources

- Same needs
- Joint works with E Peluso (JET 07)

Is it our business ?

- Up to now, assumption that the resources of the household have been shared according to needs (equally)
- Intra-household inequalities are neglected
- Because an informational problem
  - Household: black-box, that forms an informational screen between the policy-maker and individual targets.
- Because a liberal point of view (it is not the matters of Gvt) (kind of subsidiarity principle)
- Because households have the right information to share resources among its members in an optimal way

Empirically, within-household inequality matters

- Underestimation of inequality (Haddad and Kanbur 1990 EJ)
- An intra-household Kuznets curve ? (an inverted-U-shape relationship between household income and its dispersion

  (BMI as a measure of well-being Sahn&Younger (2008))
- Bias in inequality assessment (Anand and Sen 1994, Lise and Seitz 2007)
- Difficulties in evaluating the effectiveness of redistributive policy

Household

Instrument :
Household income distribution

Sharing rule ?

Individuals

Goal :

Individual income distribution

Two assumptions and a question

- ***Identical individuals***

  From a normative point of view, individuals are homogeneous: same needs, then, in principle, same share of the cake

- ***Unfair intra-household behavior***

  A generalized intra-household inequality prevails, not supported by “ethical” reasons

- ***Welfare, Inequality and poverty preservation***

  What are the conditions on the sharing rules such that a dominance ranking is preserved from the household level to the individual one.

Intra-household behaviour

- Households are composed of 2 individuals

  : $\nearrow$ (poor)= dominated

  $\searrow$ (rich)=dominant

- Sharing function gives the income of "dominated " in the household i

$$p_i = f_p^i(y_i)$$

- Assumption :

  $f_p : D \to R_+$ is identical across households and

$$f_p(0) = 0$$

$$f_p(y) \leq 1/2y \quad \forall y \in R_+$$

1

Setup

$y = (y_1, y_2, \ldots. y_n)$ income distribution of n households

$D = [\underline{y}, +\infty)$

$$\mathbb{Y}_n = \{\mathbf{y} \in D^n | \text{ and } y_1 \leq y_2 \ldots \leq y_n\}$$

Generalized Lorenz(GL) dominance
Relative Lorenz (RL) dominance
Absolute Lorenz (RL) dominance

Setup

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Generalized Lorenz(GL) dominance
Relative Lorenz (RL) dominance
Absolute Lorenz (RL) dominance

Results with Lorenz criteria

- Identification of conditions on the intra-household distribution rule under which the knowledge of the Lorenz test at the household level is enough to infer the Lorenz gradient at the individual level.

Three classes of Sharing functions

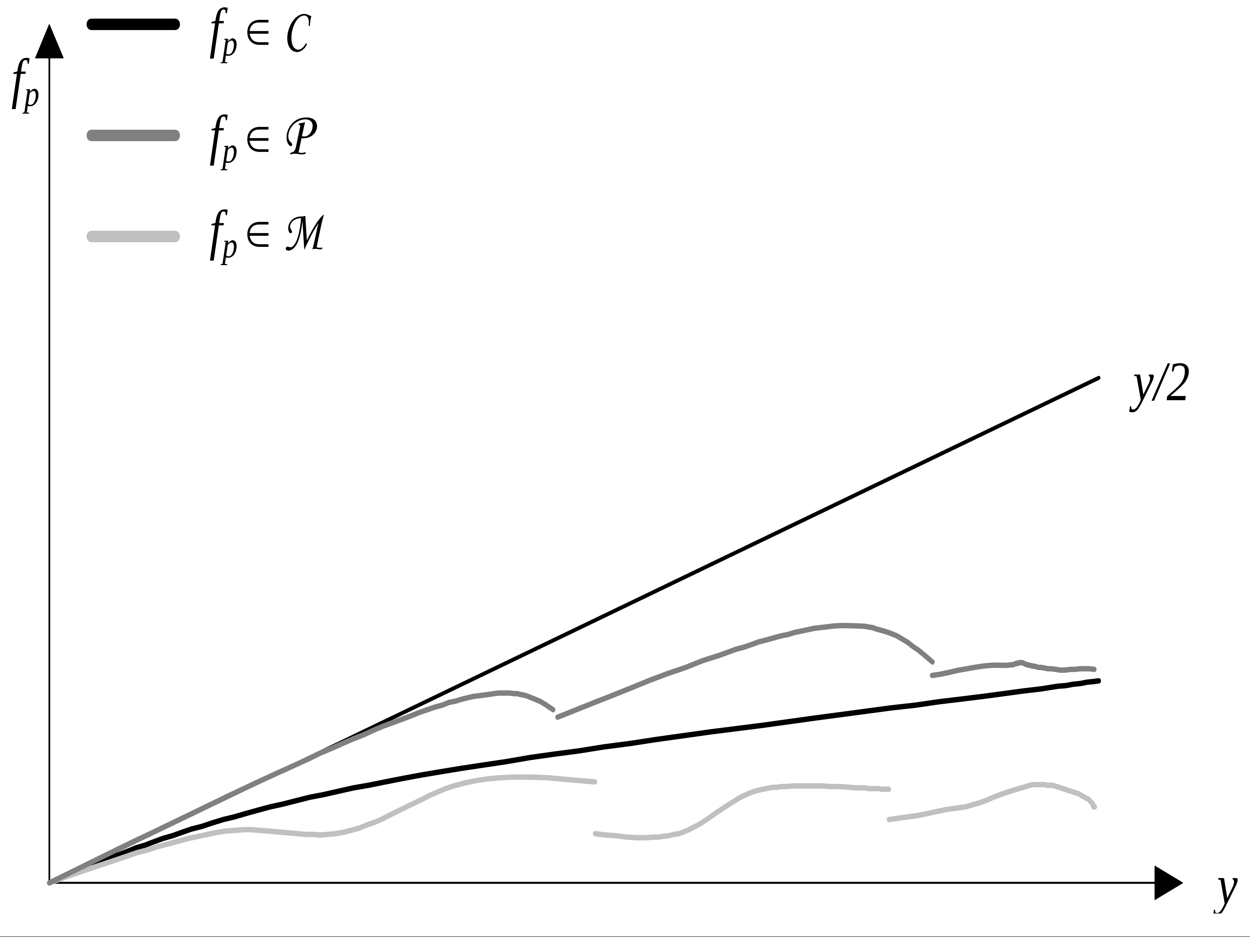
The more wealthy the household is, the more unequally it behaves

- $\mathcal{M} \subset \mathcal{F}$ "Moving away" sharing functions:

$$\frac{y}{s} - f_p(y) \uparrow \text{ with } y$$

- $\mathcal{P} \subset \mathcal{F}$ "Progressive" sharing functions: $\frac{f_p(y)}{y} \downarrow$ with y

- $\mathcal{C} \subset \mathcal{F}$ Concave functions
- $\mathcal{C} \subset \mathcal{P} \subset \mathcal{M}$


$f_p \in \mathcal{C}$
$f_p \in \mathcal{P}$
$f_p \in \mathcal{M}$
f_p
$y/2$
y

Welfare preservation

Lemma

A sharing function that preserves the GL dominance must be non decreasing and continuous.

Theorem

Let f_p non decreasing and continuous. Then:

$$f_p \in \mathcal{C} \iff [\forall\, \mathbf{y}, \mathbf{y}' \in Y_n,\ \mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \Rightarrow \mathbf{x} \succcurlyeq_{GL} \mathbf{x}'\ ].$$

Inequality: the relative point of view

- Proportional change in household income distribution.

Let $f_p \in F$.

i) f_p is progressive $\Leftrightarrow$ $[\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}), \forall \alpha \in [0,1] \text{ and } \forall \mathbf{y} \in \mathbb{Y}_n]$.

ii) f_p is regressive $\Leftrightarrow$ $[\mathbf{x}(\alpha\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}), \forall \alpha \geq 1 \text{ and } \forall \mathbf{y} \in \mathbb{Y}_n]$.

- Preservation result

Let $f_p \in \mathcal{F}$ and $\beta \in [0, \frac{1}{2}]$

$f_p = \beta y \Leftrightarrow [\forall\, \mathbf{y}, \mathbf{y}' \in \mathbb{Y}_n, \;, \mathbf{y} \succcurlyeq_{RL} \mathbf{y}' \Rightarrow \mathbf{x}(\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}')]$

- Case with $\mu_{\mathbf{y}} \leq \mu_{\mathbf{y}'}$

Let $f_p \in \mathcal{F}$ and $\mu_{\mathbf{y}} \leq \mu_{\mathbf{y}'}$

$f_p \in \mathcal{C} \Leftrightarrow [\forall\, \mathbf{y}, \mathbf{y}' \in \mathbb{Y}_n, \; \mathbf{y} \succcurlyeq_{RL} \mathbf{y}' \Rightarrow \mathbf{x}(\mathbf{y}) \succcurlyeq_{RL} \mathbf{x}(\mathbf{y}')]$.

- If the sharing function is concave,

  a progressive taxation scheme entails a lower inequality at the individual level

Relevance

- Robustness from the ethical point of view, due to the consistency with the social welfare judgement (Hardy-Littlewood-Polya Theorem)

- Specific advantage from the operational side: a test on the curvature of the sharing function is sufficient to establish whether we can neglect or not intra-household inequality in welfare and inequality analysis

- Three types

f_p = dominated
f_m = median or intermediate
f_r = dominant

- Group sharing function

$$f_g = f_p + f_m,$$

- Proposition

$f_p \in \mathcal{C}$ and $f_g \in \mathcal{C}^g \iff$
$[\mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \Rightarrow \mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}'), \ \forall\ \mathbf{y}, \mathbf{y}' \in \mathbb{Y}_n]$.

"chain condition": concavity of all the group sharing functions is necessary and sufficient to get the preservation of the GL test.

Extension to Poverty preservation

- (*Poverty gap ratio*)

$$P1\ (y,z) = \frac{1}{n}\sum_{y_i<z}\left(\frac{z-y_i}{z}\right)$$

Denoting by zc and zs the poverty line fixed at couple and individual level the immediate relation zs=zc/2 comes from the fact that the two individuals have the same needs.

To study poverty among individuals we need information about non-poor households: the "focus axiom" does not hold

Extension to family public goods

- Public sharing rule
- Private sharing rule
- « Individualized incomes » are the sum of the fraction (at least >1/2 of public expenditure) and his(her) amount of private expenditures
- Double concavity condition
- Concavity of public sharing rule is necessary!

3. Both heterogeneity together

- When households differ in sizes (singles and couples), does the knowledge of some dominance relation at the household says something about the dominance relation at the individual stage ?

Results with single and couples

- Lorenz with equalized incomes YES
- Bourguignon Criterion YES
- Atkinson and Bourguignon NO

Results with Couples and Couples with one child

- Child is not a public good, only a cost
- Constant cost whatever the wealth of the couple
- The cost is shared unequally between the two adults (a greater part for the dominated)
- Bourguignon test : OK for GL at the individual level
- Sequential GLorenz : OK for the TSD at the individual level

If you are not fed up…

- How to generate concave sharing rules ? The microeconomic foundations

- Empirical evidence about these concavity requirements (with Hélène Couprie)

- Thank you for your patience