# On MD poverty comparisons

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# Overview

- Introduction: 3 problems
- Notation
- Axioms
- Result
- Discussion

# Problem 1: ‘transferability’

- Are all attributes ‘transferable’?

- We split up all attributes into
  - ‘transferable’ (typically cardinal) attributes
  - ‘non-transferable’ (typically ordinal/nominal) attributes

- Whether an attribute is transferable is
  - not a physical characteristic of the attribute, but depends on whether the attribute should be included in the definition of certain ‘transfer’-type axioms
  - …, thus a ‘normative’ choice

# Problem 2: defining the poor

- Given a poverty bundle $z$, should we use
  - an intersection approach,
  - a union approach, or
  - an intermediate approach?

- Roughly speaking, we start by measuring poverty via

  $\sum_i \pi_z(x^i)$, with $x^i = \left(x_1^i, x_2^i, \ldots\right)$ and $z = \left(z_1, z_2, \ldots\right)$

  and define the poor as individuals with bundles $x$ s.t.

  $\pi_z(x) > \pi_z(z)$.

# Problem 3: priority to the poor

- a MD generalization of the FGT poverty family:

$$\sum_i I\left(x^i \ll z\right)\prod_j \left(z_j - x_j^i\right)^{\alpha_j},$$

- Consider
  - 2 dimensions (with all $\alpha_j=1$) and a poverty bundle $z=(1,1)$
  - 2 individuals with bundles $(0.4,0.6)$ & $(0.65,0.4)$
- Both individuals are poor, but who is poorest?
- Consider
  - an indivisible amount 0.05 of the 1st attribute, say income
  - who should get it? priority!
  - but …

# Notation

- Set of individuals $I$
- Set of attributes $J = T \cup N$ (recall problem 1)
- An attribute bundle $x = (x_T, x_N)$, element of $B = \mathcal{R}_+^{|J|}$
- A poverty bundle $z$ in $B$
- A distribution $X = (x^1, x^2, \ldots)$, element of $D = B^{|I|}$
- A poverty ranking ('better-than' relation) $\succeq_z$ on $D$

# Representation

- Representation (R): There exists a $C^1$-map $\pi_z : B \to \mathcal{R}$, with $\pi_z(z) = 0$, s.t. for all $X$, $Y$ in $D$, we have

$$X \succeq_z Y \Leftrightarrow \sum_{i \in I} \pi_z\left(x^i\right) \leq \sum_{i \in I} \pi_z\left(y^i\right)$$

- the poverty ranking $\succeq_z$ is assumed to be
  - Complete, transitive & continuous(ly differentiable)
  - Separable (decomposable)
  - Anonymous
  - Normalization
- strong, but not unusual (A&B, 1982; F, 1984)

# Focus

- Recall problem 2 ( = defining the poor)
- The set of poor individuals in $X$ is defined as

$$\mathcal{P}(X, \succeq_z) = \{ i \in I \mid (x^i, x^i, \ldots, x^i) \prec_z (z, z, \ldots, z) \}$$

- Given (R), the poor are those with $\pi_z(x) > 0$
- Focus (F): for all $X$ in $D$, $X \sim_z Y$, with $Y$ obtained from $X$ by a 'change' in the bundle of a non-poor in $X$, while keeping him/her non-poor in $Y$.
- Given (R) & (F), the non-poor have $\pi_z(x) = 0$

# Monotonicity

- Monotonicity (M):

  for all $X, Y$ in $D$ & for each $i$ in $\mathcal{P}(X, \succeq_z)$

  if

  $$x_T^i < y_T^i \;\&\; x_N^i = y_N^i$$

  $$x^k = y^k, k \neq i$$

  then

  $$X \prec_z Y$$

- Given (R) & (M)

  $\pi_z(x) > 0$ implies $D_j\pi_z(x) < 0$, for all $j$ in $T$

# Priority

- Recall problem 3 (priority)

- Priority (P):
  - for each $X$ in $D$,
  - for each $\varepsilon = (\varepsilon_T, \varepsilon_N)$, with $\varepsilon_T > 0$ and $\varepsilon_N = 0$,
  - for all $k,l$ in $\mathcal{P}(X, \succeq_z)$, with $\left(x^k, x^k, \ldots, x^k\right) \prec_z \left(x^l, x^l, \ldots, x^l\right)$

  we have

$$\left(\ldots, x^k + \varepsilon, \ldots, x^l, \ldots\right) \succ_z \left(\ldots, x^k, \ldots, x^l + \varepsilon, \ldots\right)$$

# Result

- Consider a poverty bundle $z$. A poverty ranking $\succeq_z$ satisfies R, F, M & P if and only if there exist
  - a vector $p_T >> 0$ (for the transferables in $T$)
  - a $C^1$-map $\psi$: $\mathcal{R}_+^{|N|} \to \mathcal{R}$ (for the non-transferables in $N$)
  - a $C^1$-map $\varphi$: $\mathcal{R} \to \mathcal{R}$, $a \to \varphi(a)$
    - strictly decreasing & strictly convex at $a < p_T \cdot z_T + \psi(z_N)$
    - and $\varphi(a) = 0$ elsewhere

  such that, for each $X$ and $Y$ in $D$, we have

$$X \succeq_z Y \Leftrightarrow \sum_{i \in I} \varphi\left(p_T \cdot x_T^i + \psi\left(x_N^i\right)\right) \le \sum_{i \in I} \varphi\left(p_T \cdot y_T^i + \psi\left(y_N^i\right)\right)$$

# Discussion

$$X \succeq_z Y \Leftrightarrow \sum_{i \in I} \varphi\left(p_T \cdot x_T^i + \psi\left(x_N^i\right)\right) \leq \sum_{i \in I} \varphi\left(p_T \cdot y_T^i + \psi\left(y_N^i\right)\right)$$

- $|T|>1$ & $|N|=0$ : budget (dominance) & zonoids
- $|T|=1=|N|$ : equivalence scales (& B89 dominance)
- Related impossibility results:
  - Sen's weak equity principle
  - Ebert's conflict
  - impossibility of a Paretian egalitarian

  …
- "fundamental difficulty to work in two separate spaces"
- role of differentiability …