Multidimensional poverty and inequality

Jean-Yves Duclos
Université Laval (Québec) and Institut d'Anàlisi Econòmica (Barcelona)

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Multidimensional poverty

Consensus *vs* practice

- There is a widespread agreement that poverty is a multidimensional issue, including a number of monetary and non-monetary deprivations.
- Given:
  - the presence of incomplete markets and of difficult-to-account externalities and public goods
  - and the fact that income is then imperfectly correlated with welfare
- it is also natural to be concerned with the joint distribution of various capabilities and outcomes of interest

Ethics

- Also: Two major non-welfarist approaches: the basic-needs approach and the capability approach.
- The first focuses on the need to attain some basic multidimensional outcomes.
- These outcomes are usually (explicitly or implicitly) linked with the concept of functionings

Functionings

”Living may be seen as consisting of a set of interrelated ’functionings’, consisting of beings and doings. A person’s achievement in this respect can be seen as the vector of his or her functionings. The relevant functionings can vary from such elementary things as being adequately nourished, being in good health, avoiding escapable morbidity and premature mortality, *etc.*, to more complex achievements such as being happy, having self-respect, taking part in the life of the community, and so on ” (Sen 1992).

Basic needs

”Basic needs may be interpreted in terms of minimum specified quantities of such things as food, shelter, water and sanitation that are necessary to prevent ill health, undernourishment and the like” (Streeten et al 1981).

Functionings

- Unlike functionings, the specification of basic needs depends on the characteristics of individuals and of the societies in which they live.

- For instance, the basic commodities required for someone to be in good health and not to be undernourished will depend on the climate and on the physiological characteristics of individuals.

- Human diversity is such that equality in the space of basic needs generally translates into inequality in the space in functionings.

Capabilities

- The capability approach is defined by the *capacity* to achieve functionings, as defined above.

  > ”the capability to function represents the various combinations of functionings (beings and doings) that the person can achieve. Capability is, thus, a set of vectors of functionings, reflecting the person's freedom to lead one type of life or another.” (Sen 1992)

- What matters for the capability approach:
  - the ability of an individual to function well in society;
  - *not* the functionings actually achieved by the person *per se*

Freedoms

- This distinction between outcomes and capabilities recognizes the importance of preference diversity and individuality in determining functioning choices.
- It is, for instance, not everyone's wish to be well-clothed or to participate in society, even if the capability is present.
- The difference between the capability and the functioning/basic needs approaches is in fact somewhat analogous to the difference between the use of income and consumption as indicators of living standards.
- Income shows the capability to consume, and ”consumption functioning” can be understood as the outcome of the exercise of that capability.

Illustrations

To illustrate the relationships between the main approaches to assessing poverty, consider the following figures

Illustration 1

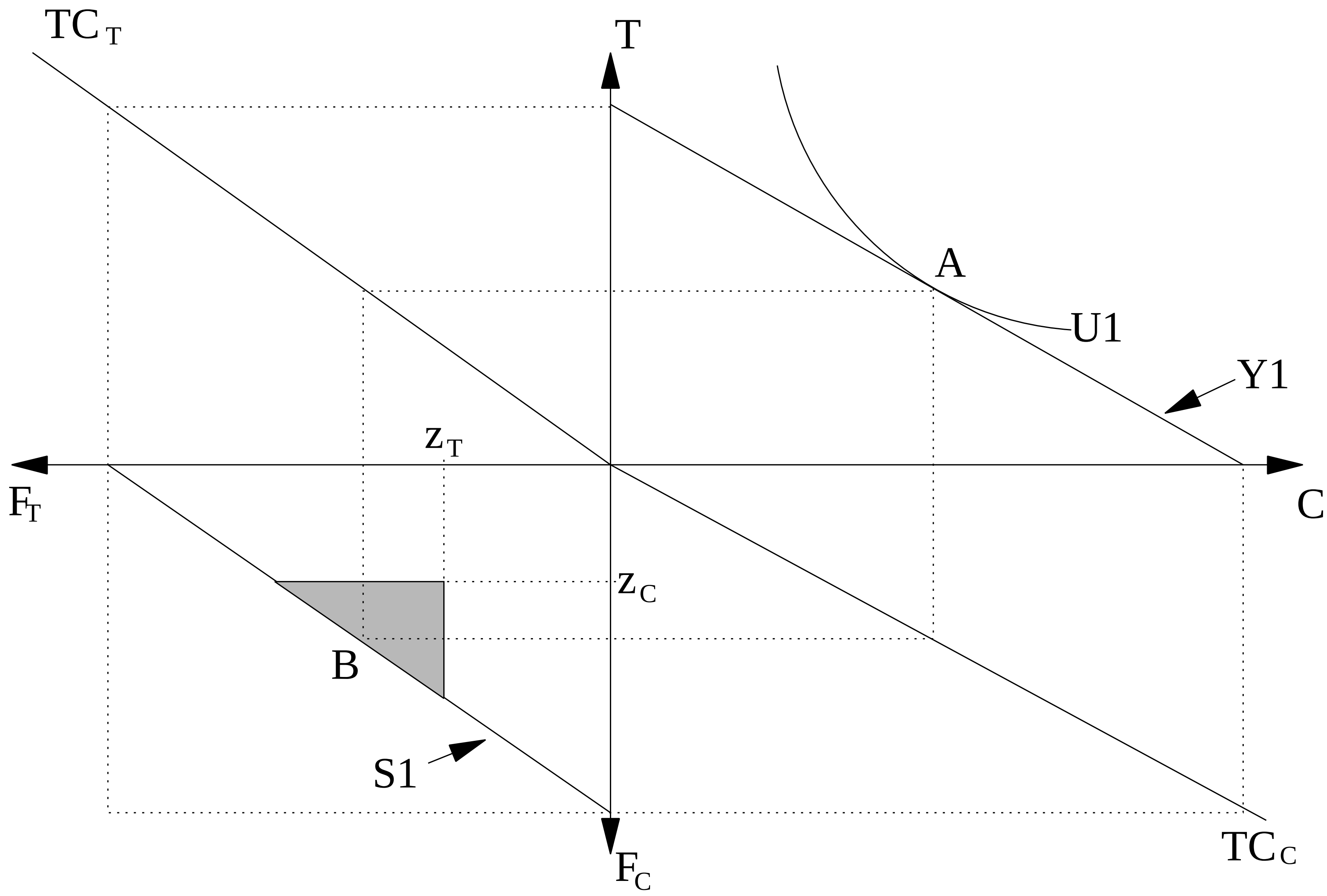


Illustration 2

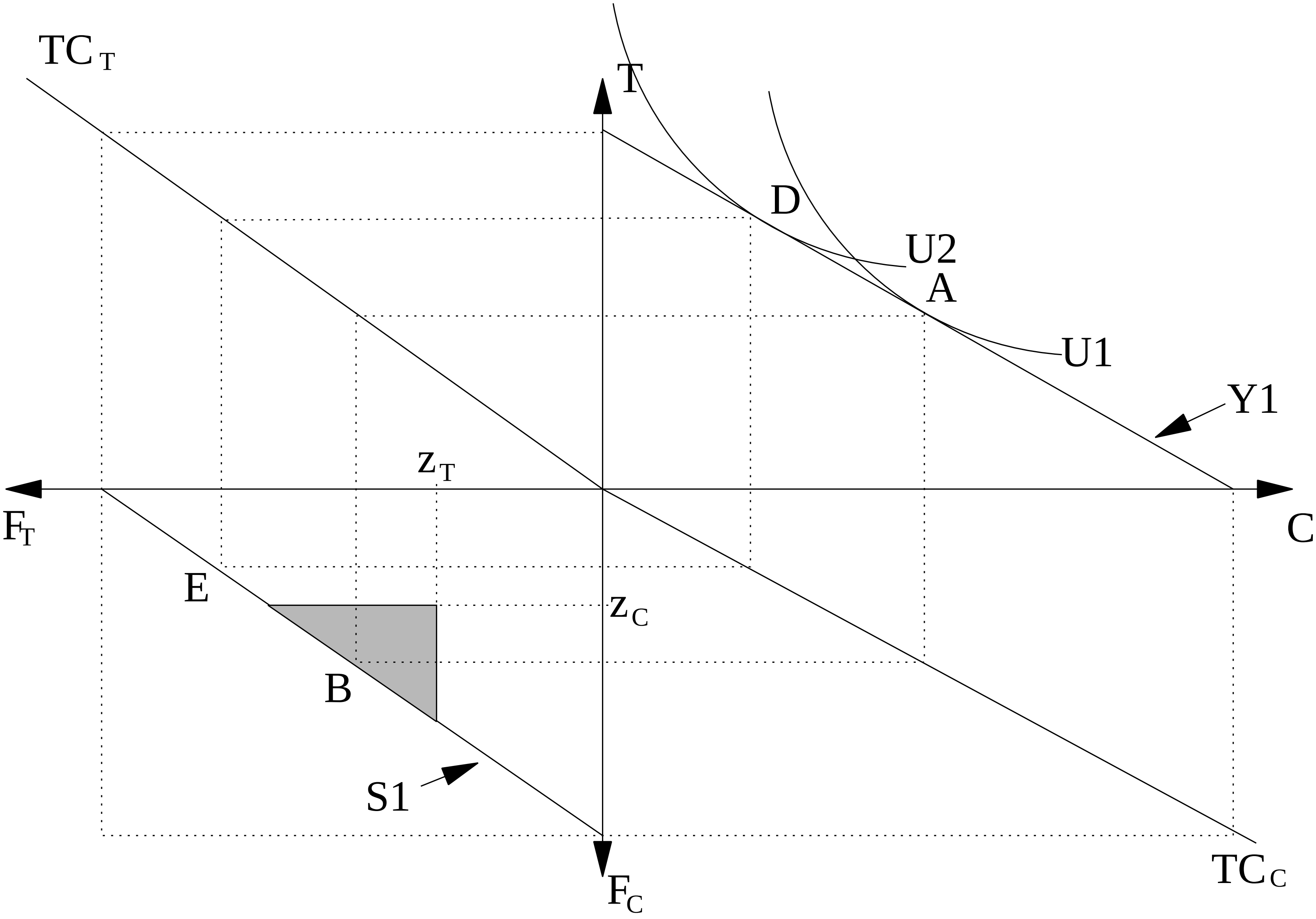


Illustration 3

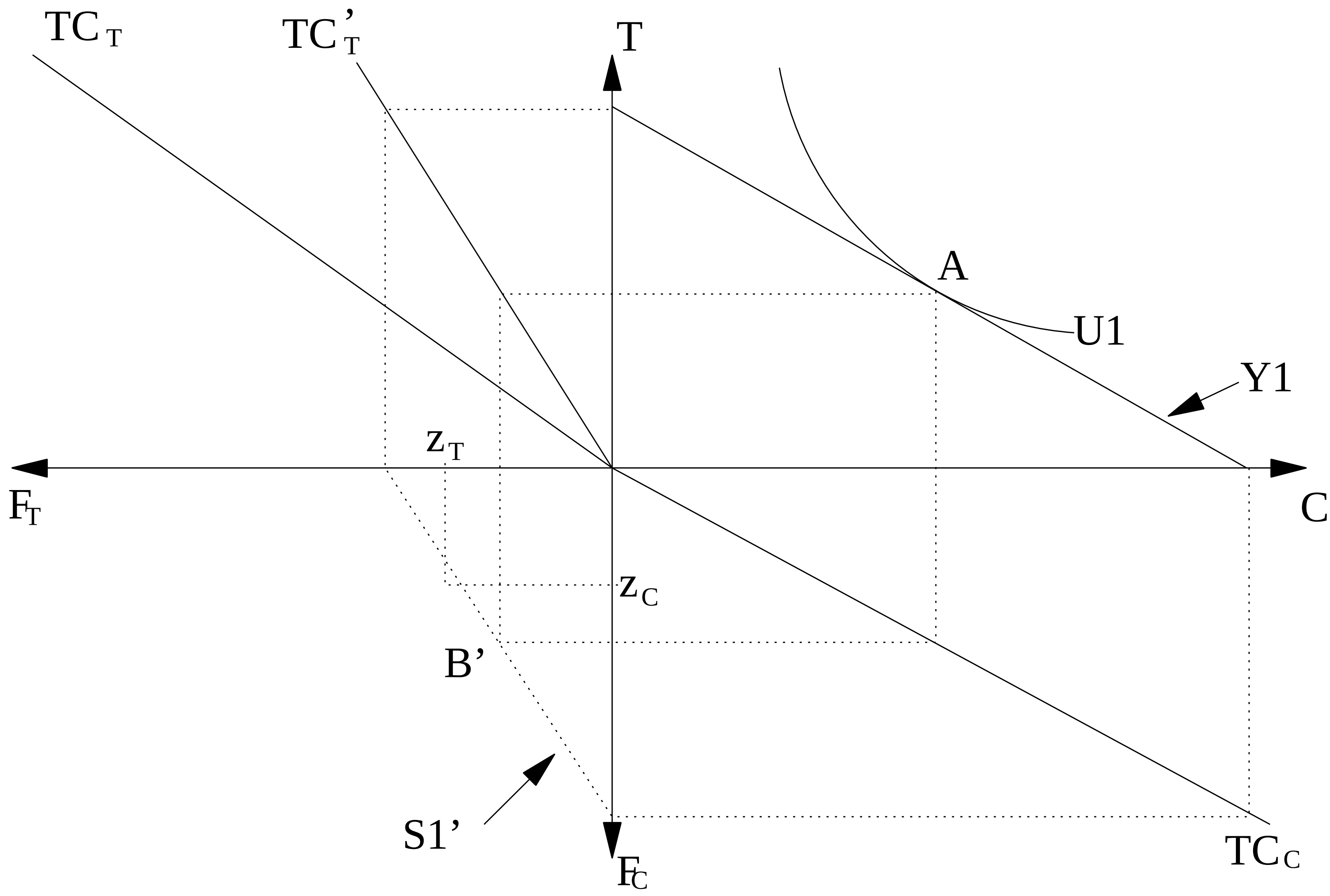
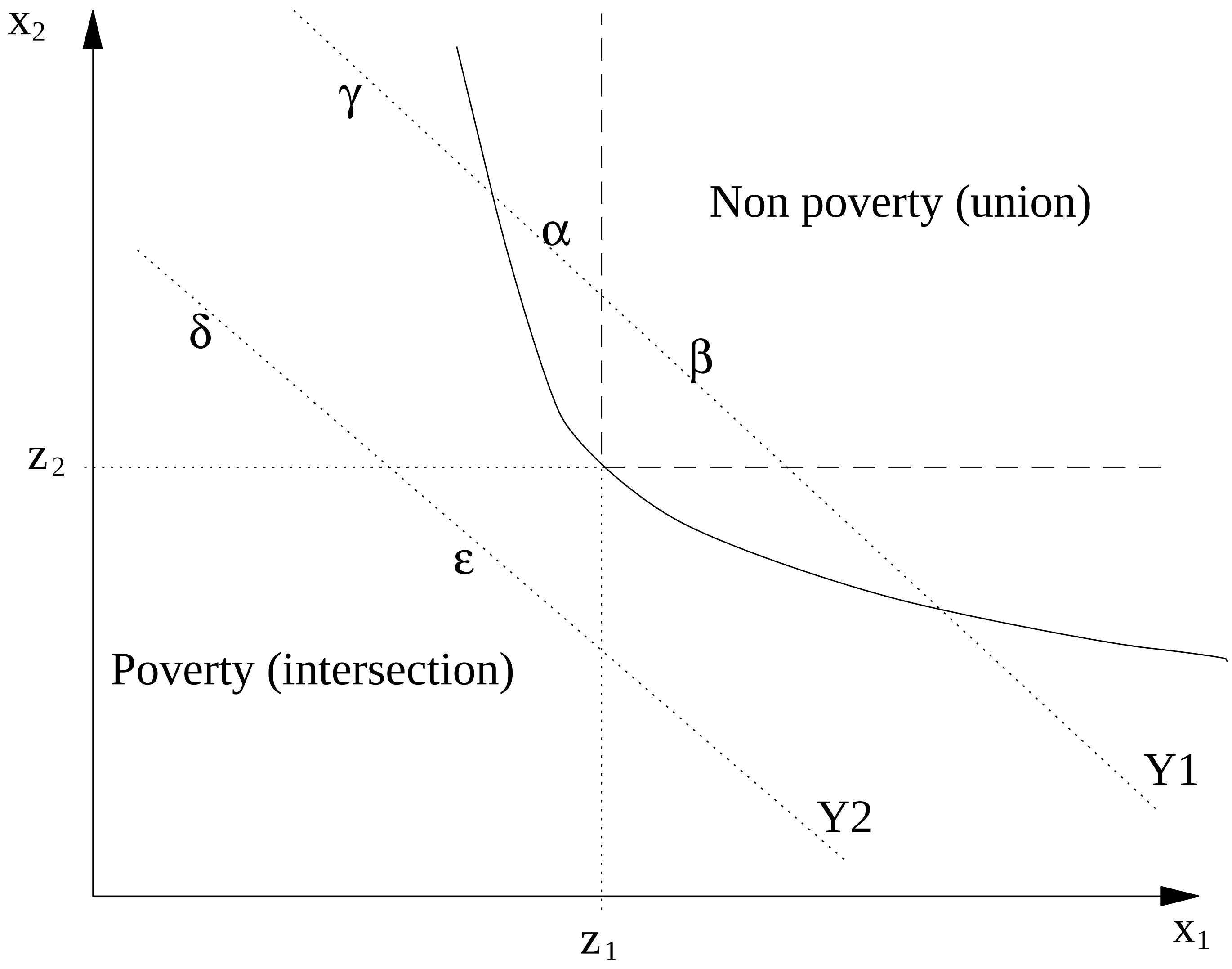


Illustration 4



Measurement

Aggregation

Main issues:

- How should the different attributes be integrated to yield a broader and fuller picture of poverty?
- Should we
  - ☐ focus on the situation of those who are poor according to all attributes simultaneously,
  - ☐ or also account for the deprivation of those who do not reach the required minimum for any one attribute?
- How are we to set multidimensional poverty lines?

Two methods

In considering the aggregation problem, a distinction is made between two different methods.

- The first
  - sums across individuals, to form a synthetic index for all individuals in one dimension,
  - and then combines all the one-dimensional indices to yield a multidimensional poverty measure.
  - An example of this is the *Human poverty Index* of the UNDP (1997).
- The second combines the multiple indicators of deprivation at the individual level first, and then aggregates them across individuals into an overall social index.

Aggregation at the individual level

- A simple way to study multidimensional poverty is to examine several poverty indices, separately for each dimension.
- For instance, Adams and Page (2001) find no clear relationship between monetary poverty and poverty in other welfare indicators in the Middle East and North Africa.
- This suggests aggregating the separate indicators into an overall index.

Human Development Report

- The Human Development Report published (UNDP) suggests adding, to a lack of income, indicators on longevity, good health, good nutrition, education, being well integrated into society, etc.

- This has led to a popular multidimensional poverty measure

Human Development indicators

1. An indicator that accounts for a short lifespan: the percentage of individuals whose life expectancy is less than 40 years, HPI_1
2. A measure which is related to the problem of access to education and communications: the proportion of the adult population that is illiterate, denoted HPI_2
3. HPI_3, is the arithmetic mean of three indicators:
   - the percentage of the population without access to health care (denoted $HPI_{3,1}$),
   - to safe water ($HPI_{3,2}$),
   - and the percentage of children under age five suffering from malnutrition ($HPI_{3,3}$).

Human Poverty Index

Anand and Sen (1997):

$$HPI = (w_1 HPI_1^\theta + w_2 HPI_2^\theta + w_3 HPI_3^\theta)^{\frac{1}{\theta}}, \tag{1}$$

with $w_1 + w_2 + w_3 = 1$ and $\theta \geq 1$.

- $\theta = 1$: the three elements of HPI are perfect substitutes.
- θ tends to infinity: index approaches maximum value of its three components, i.e. $\max(HPI_1, HPI_2, HPI_3)$.

- The HPI omits monetary deprivation.
- HPI does not account for the correlation that may exist between its components.
- Index does not provide information on how attributes are distributed among the population.
- Possible to have improvements in the HPI while large segments of society see a worsening of their situation.
- Ordinal and cardinal comparisons of poverty will be sensitive to the (arbitrary) values assigned to w_i and θ.

Axiomatic approach

Axiomatic approach to measurement

- Let x_i, $i = 1, 2, ..., n$, be a vector of k goods (or functionings, if we are able to observe them) of the ith person,
- X be a $(n \times k)$-matrix (whose ith row is x_i) summarizing the distribution of k attributes among n persons,
- $z = (z_1, ..., z_k)$ be a k-vector of basic needs (in units of x_i).

Axiomatic approach to measurement

A general definition of a multidimensional poverty measure P based on the distribution of individual poverty is given by

$$P(X, z) = F\left[\pi(x_i, z)\right], \tag{2}$$

- where $\pi(\cdot)$ is an individual poverty function that aggregates the many aspects of well-being
- and the function $F(\cdot)$ reflects the way in which individual poverty measures are aggregated to yield a global poverty index.

Union *vs* intersection

First distinction: *union vs intersection* of the various aspects of deprivation.

- *Union* definition:

$$\pi\left(x_i, z\right) \begin{cases} = 0, & \text{if } x_{i,j} \geq z_j, \quad \forall j = 1, 2, \ldots, k, \\ > 0, & \text{otherwise,} \end{cases} \quad . \tag{3}$$

- *Intersection* definition:

$$\pi\left(x_i, z\right) \begin{cases} > 0, & \text{if } x_{i,j} < z_j, \quad \forall j = 1, 2, \ldots, k, \\ = 0, & \text{otherwise,} \end{cases} \quad . \tag{4}$$

- A poverty measure should be continuous.
- Circumvents the problem of small errors of measurement causing draconian changes in poverty.

Axiom 1 *Continuity: the poverty measure must not be overly sensitive to a small variation in the quantity of an attribute.*

Anonymity

- An individual's "identity" should not affect the results of the analysis.

Axiom 2 *Symmetry (or anonymity): characteristics other than income do not enter poverty measurement.*

To compare populations of different sizes, we (may) have:

Axiom 3 *The population principle: if a matrix of attributes is replicated several times, total poverty remains unchanged.*

Homogeneity

- Poverty indices should be independent of measurement units for the different dimensions.

Axiom 4 *Scale invariance: the poverty measure is homogeneous of degree zero with respect to* X *and* z.

- This axiom will be fulfilled if any attribute is normalized by its corresponding poverty line.
- The individual poverty function will then have the following form:

$$\pi\left(x_i, z\right) = \pi\left(\frac{x_{i,1}}{z_1}, \ldots, \frac{x_{i,j}}{z_j}, \ldots, \frac{x_{i,k}}{z_k}\right). \qquad (5)$$

Focus

Axiom 5 *Focus: The poverty measure does not change if an attribute j increases for an individual i characterized by $x_{i,j} \geq z_j$.*

Using this axiom, we should find:

$$\frac{\partial \pi}{\partial x_{i,j}} = 0 \text{ if } x_{i,j} \geq z_j. \tag{6}$$

Axiom 6 *The poverty measure declines, or does not rise, following an improvement affecting any of a poor individual's attributes.*

Additivity

Foster and Shorrocks (1991) in a one-dimension perspective:

Axiom 7 *Subgroup decomposability: global poverty is a weighted mean of poverty levels within each subgroup:*

$$P(X, z) = \frac{1}{n}\sum_{i=1}^{n}\pi(x_i, z). \quad (7)$$

Decomposition by attribute

- In addition to decomposing the population by subgroup, Chakravarty *et al.* (1998) also support decomposition by attribute:

Axiom 8 *Factor decomposability: Global poverty is a weighted mean of poverty levels by attribute.*

- If the double decomposition is retained, then:

$$P(X, z) = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{k} \pi_j (x_{i,j}, z_j) . \tag{8}$$

- *Factor decomposability* leads to *union* poverty measures.

Indices

FGT extensions

$$\pi\left(x_i, z\right) = \sum_{j=1}^{k} a_j \left(\frac{z_j - x_{i,j}}{z_j}\right)^{\alpha}, \tag{9}$$

gives a multidimensional extension of the Foster, Greer and Thorbecke (1984) poverty measures suggested by Chakravarty *et al*. (1998).

- But this multiplicative extension

$$\pi(x_i, z) = \prod_{j=1}^{j} \left(\frac{z_j - x_{i,j}}{z_j}\right)^{\alpha_j}, \tag{10}$$

does not respect *additive factor decomposability* (it leads to *intersection* poverty).

Watts extension

Another form of $\pi\left(x_i, z\right)$ respecting *factor decomposability*:

$$\pi\left(x_i, z\right) = \sum_{j=1}^{k} a_j \ln\left[\frac{z_j}{\min\left(x_{i,j}, z_j\right)}\right], \tag{11}$$

in which case, we obtain a multidimensional extension of the Watts (1968) poverty index.

Tsui index

The index suggested by Tsui (2002) is not compatible with *additive factor decomposability* and is based on the *union* of the various dimensions of poverty:

$$\pi(x_i, z) = \prod_{j=1}^{j} \left[\frac{z_j}{\min(x_{i,j}, z_j)} \right]^{\beta_j} - 1. \tag{12}$$

Inequality

Inequality 1

- Sen (1976) suggests that poverty measures should be sensitive to inequalities within the less well-off members of society.

- Tsui (2002) introduced the following axiom:

Axiom 9 *Transfer: Poverty is not increased when matrix Y is obtained from matrix X by a redistribution of the attributes of the poor using a bistochastic transformation matrix.*

Inequality 2

- This property implies that the iso-poverty curves must be convex, or

$$\frac{\partial^2 \pi(x_i, z)}{\partial x_{ij} \partial x_{i,j}} \geq 0, \quad \forall x_{i,j} < z_j. \tag{13}$$

- The *transfer* axiom is satisfied by the Watts (1968) measure, the FGT measures when $\alpha > 1$, and the Tsui (2002) measures when $\beta_j > 0$.

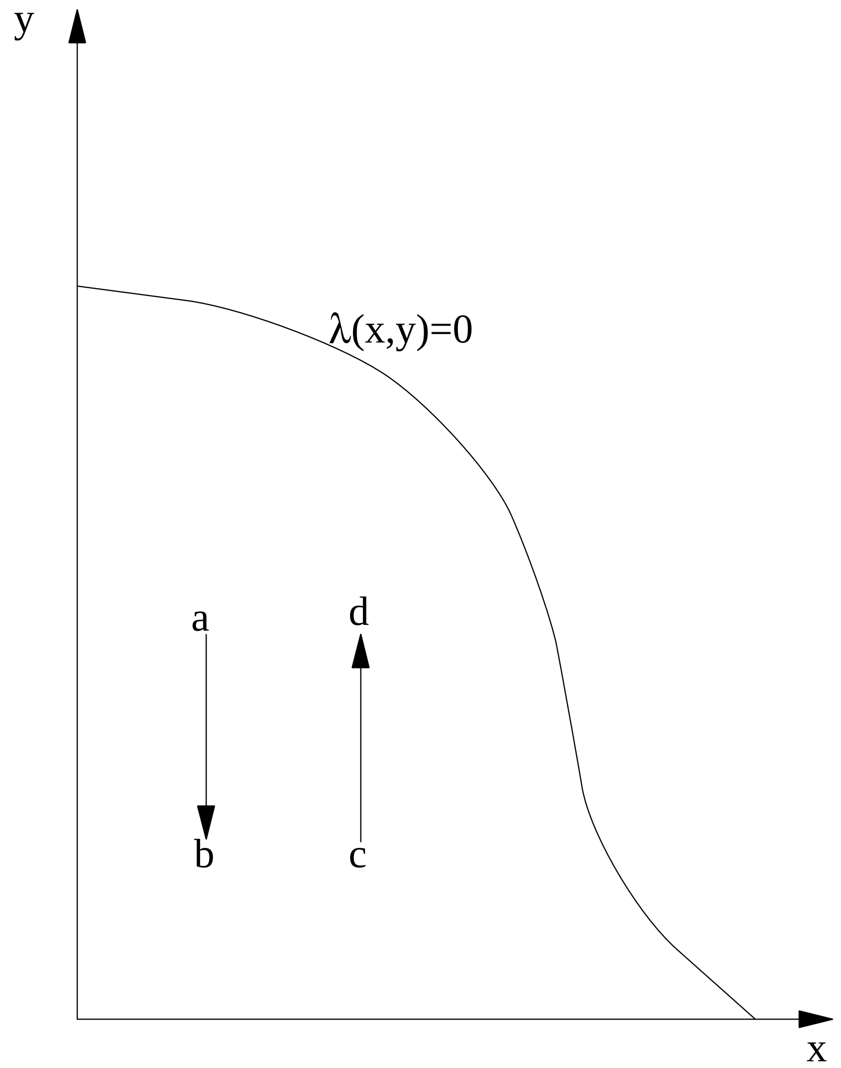
Correlation increases 1

- Let $k = 2$, $x_1 = (5, 3)$, $x_2 = (2, 7)$, $y_1 = (2, 3)$, and $y_2 = (5, 7)$.
- The marginal distributions are unchanged, but the correlation between the attributes among the poor is higher with y. Intuitively, poverty must then be larger.

Axiom 10 *Nondecreasing poverty under a correlation-increasing switch: let transfers leave the marginal distributions unchanged but increase the correlation between attributes of the poor, then*

$$P(Y, z) \geq P(X, z). \tag{14}$$

Correlation changes



Correlation increases 2

- Attributes are ”substitutable”
- We then have

$$\frac{\partial^2 \pi(x_i, z)}{\partial x_{ij} \partial x_{i,k}} \geq 0, \quad \forall x_{i,j} < z_j. \tag{15}$$

- The Tsui (2002) poverty measure will obey this if $\beta_j \beta_k > 0$.

Correlation increases 3

When the attributes are “complements”, we have instead:

$$\frac{\partial^2 \pi(x_i, z)}{\partial x_{ij} \partial x_{i,k}} \leq 0, \quad \forall x_{i,j} < z_j. \tag{16}$$

General index

Bourguignon and Chakravarty (2003), with $k = 2$:

$$P_{\alpha,\gamma}(X,z) = \frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{z_1 - x_{i,1}}{z_1}\right)^{\gamma} + b^{\frac{\gamma}{\alpha}}\left(\frac{z_2 - x_{i,2}}{z_2}\right)^{\gamma}\right]^{\frac{\alpha}{\gamma}} \tag{17}$$

where $\alpha \geq 1$, $\gamma \geq 1$, and $b > 0$.

$\alpha \geq 1$ ensures that the transfer principle for a single attribute is respected for poor people.

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$$P_{\alpha,\gamma}(X, z) = \frac{1}{n} \sum_{i=1}^{n} \left[\left(\frac{z_1 - x_{i,1}}{z_1} \right)^{\gamma} + b^{\frac{\gamma}{\alpha}} \left(\frac{z_2 - x_{i,2}}{z_2} \right)^{\gamma} \right]^{\frac{\alpha}{\gamma}} \tag{17}$$

When $\alpha \geq 1, \gamma \geq 1$ ensures that this principle extends to individuals who are poor in two attributes simultaneously.

General index

Bourguignon and Chakravarty (2003), with $k = 2$:

$$P_{\alpha,\gamma}(X, z) = \frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{z_1 - x_{i,1}}{z_1}\right)^{\gamma} + b^{\frac{\gamma}{\alpha}}\left(\frac{z_2 - x_{i,2}}{z_2}\right)^{\gamma}\right]^{\frac{\alpha}{\gamma}} \tag{17}$$

As the value of γ increases, the iso-poverty curve becomes more convex.

General index

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The elasticity of substitution between the two poverty deficits is $\frac{1}{\gamma - 1}$.

General index

Bourguignon and Chakravarty (2003), with $k = 2$:

$$P_{\alpha,\gamma}(X,z) = \frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{z_1 - x_{i,1}}{z_1}\right)^{\gamma} + b^{\frac{\gamma}{\alpha}}\left(\frac{z_2 - x_{i,2}}{z_2}\right)^{\gamma}\right]^{\frac{\alpha}{\gamma}} \tag{17}$$

The (positive) magnitude of b reflects the relative weight of the second attribute vis-à-vis the first.

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When $\alpha \geq \gamma \geq 1$, the two attributes are substitutes and $P_{\sigma,\gamma}\left(X,z\right)$ respects the property that *poverty is nondecreasing after an increase in correlation between the attributes*.

General index

Bourguignon and Chakravarty (2003), with $k=2$:

$$P_{\alpha,\gamma}(X,z)=\frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{z_1-x_{i,1}}{z_1}\right)^{\gamma}+b^{\frac{\gamma}{\alpha}}\left(\frac{z_2-x_{i,2}}{z_2}\right)^{\gamma}\right]^{\frac{\alpha}{\gamma}} \tag{17}$$

Conversely, when $\gamma \geq \alpha$, the two attributes are complements, and $P_{\alpha,\gamma}\left(X,z\right)$ satisfies the condition that *poverty is nonincreasing subsequent to a rise in the correlation between the two attributes*.

General index

Bourguignon and Chakravarty (2003), with $k = 2$:

$$P_{\alpha,\gamma}(X, z) = \frac{1}{n} \sum_{i=1}^{n} \left[\left(\frac{z_1 - x_{i,1}}{z_1} \right)^{\gamma} + b^{\frac{\gamma}{\alpha}} \left(\frac{z_2 - x_{i,2}}{z_2} \right)^{\gamma} \right]^{\frac{\alpha}{\gamma}} \tag{17}$$

When $\gamma = 1$, the iso-poverty curves are linear for these two attributes in the case of poor individuals.

General index

Bourguignon and Chakravarty (2003), with $k = 2$:

$$P_{\alpha,\gamma}(X,z) = \frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{z_1 - x_{i,1}}{z_1}\right)^{\gamma} + b^{\frac{\gamma}{\alpha}}\left(\frac{z_2 - x_{i,2}}{z_2}\right)^{\gamma}\right]^{\frac{\alpha}{\gamma}} \quad (17)$$

Finally, as the value of γ becomes very large, the measure becomes:

$$P_{\alpha,\infty}(X,z) = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \min\left(1, \frac{x_{i,1}}{z_1}, \frac{x_{i,2}}{z_2}\right)\right]^{\alpha}, \quad (18)$$

In this case, the two attributes are complementary and the iso-poverty curves assume the shape of Leontief curves.

Robustness Analysis

Sensitivity

- Poverty ranking can be reversed by a different choice of poverty lines.
- Poverty ranking can be reversed by a different choice of indicator-aggregating procedures.
- Poverty ranking can be reversed by a different choice of individual-aggregating procedures.
- Poverty ranking can be reversed by a different choice of samples.

Welfare and poverty

Duclos, Sahn and Younger (2006) define the individual welfare function as:

$$\lambda(x_1, x_2) : \Re^2 \rightarrow \Re \left| \frac{\partial \lambda(x_1, x_2)}{\partial x_1} \geq 0, \; \frac{\partial \lambda(x_1, x_2)}{\partial x_2} \geq 0. \right. \quad (19)$$

They assume that an unknown poverty frontier separates the poor from the non-poor population.

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This frontier is implicitly defined by $\lambda\left(x_1, x_2\right) = 0$.

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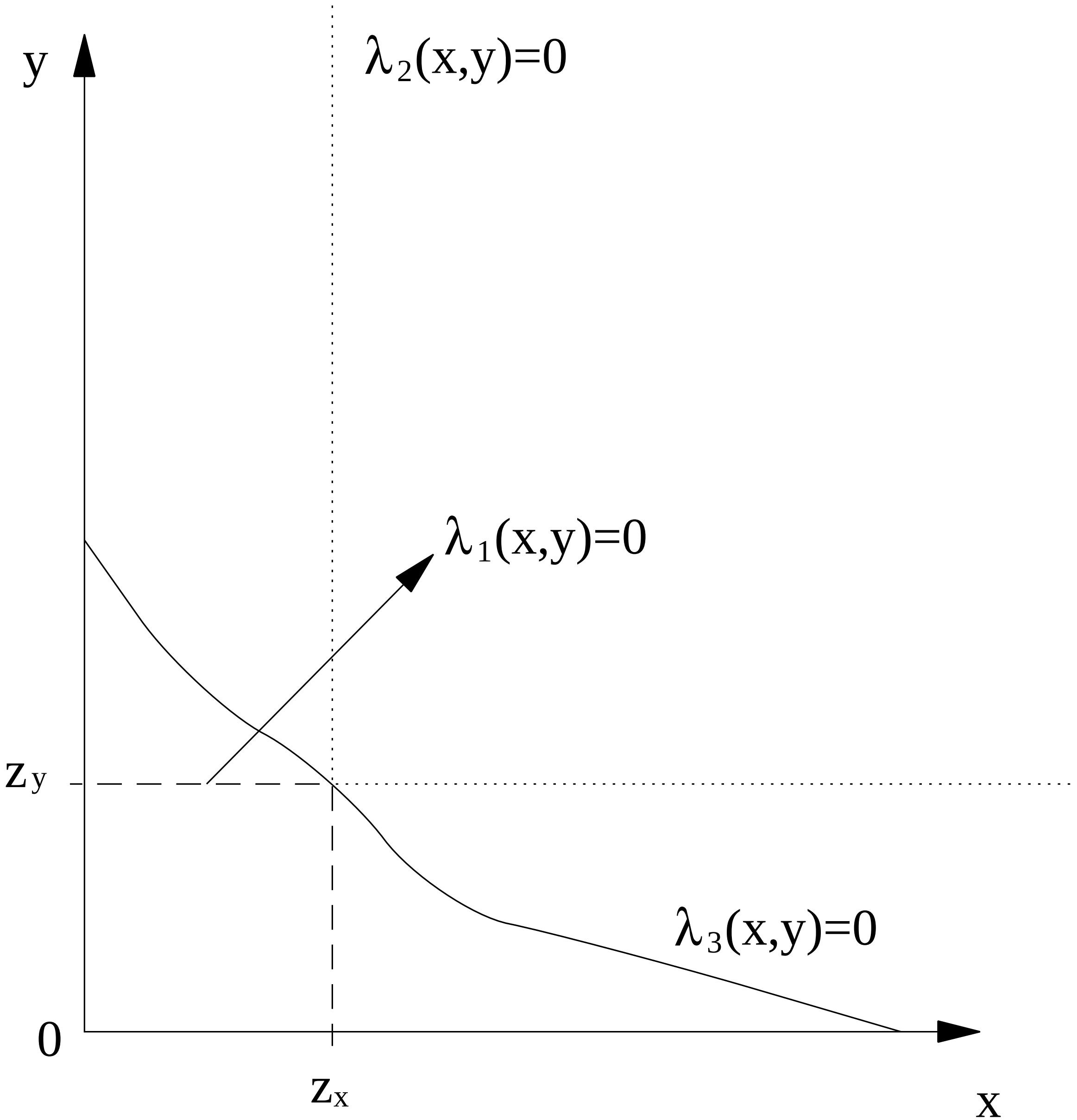
The set of the poor is then defined by:

$$\Lambda(\lambda) = \{(x_1, x_2) \,|\lambda(x_1, x_2) \leq 0\} \,. \quad (20)$$


y
$\lambda_2(x,y)=0$
$\lambda_1(x,y)=0$
z_y
$\lambda_3(x,y)=0$
0
z_x
x

Two dimensions

Two-dimensional poverty measure satisfying the *subgroup decomposability* axiom can be written as:

$$P(\lambda) = \int \int_{\Lambda(\lambda)} \pi(x_1, x_2, \lambda) dH(x_1, x_2), \tag{21}$$

where $\pi(x_1, x_2, \lambda)$ is the contribution of an individual characterized by the pair (x_1, x_2) to global poverty.

Two dimensions

Two-dimensional poverty measure satisfying the *subgroup decomposability* axiom can be written as:

$$P(\lambda) = \int\int_{\Lambda(\lambda)} \pi(x_1, x_2, \lambda) dH(x_1, x_2), \tag{21}$$

By the *focus* axiom, this function is

$$\pi(x_1, x_2, \lambda) \geq 0 \text{ if } \lambda(x_1, x_2) \leq 0, \tag{22}$$

$$= 0 \text{ otherwise.} \tag{23}$$

Two dimensions

Two-dimensional poverty measure satisfying the *subgroup decomposability* axiom can be written as:

$$P(\lambda) = \int \int_{\Lambda(\lambda)} \pi(x_1, x_2, \lambda) dH(x_1, x_2), \tag{21}$$

Depending on the analytical form chosen, the function $\pi\left(x_1, x_2, \lambda\right)$ measures poverty across the *intersection*, the *union*, or an intermediate combination of the two selected dimensions.

Consider the following multidimensional extension of the FGT class of measures:

$$P^{\alpha_1,\alpha_2}(x_1, x_2, z) = \int_0^{z_1} \int_0^{z_2} \left(\frac{z_1 - x_1}{z_1}\right)^{\alpha_1} \left(\frac{z_2 - x_2}{z_2}\right)^{\alpha_2} dH(x_1, x_2). \quad (22)$$

$P^{0,0}(x_1, x_2, z)$ is the bi-dimensional incidence of poverty, i.e. the proportion of the population that is poor in both of those attributes simultaneously.

Dominance testing

Consider the following multidimensional extension of the FGT class of measures:

$$P^{\alpha_1,\alpha_2}(x_1, x_2, z) = \int_0^{z_1} \int_0^{z_2} \left(\frac{z_1 - x_1}{z_1}\right)^{\alpha_1} \left(\frac{z_2 - x_2}{z_2}\right)^{\alpha_2} dH(x_1, x_2). \quad (22)$$

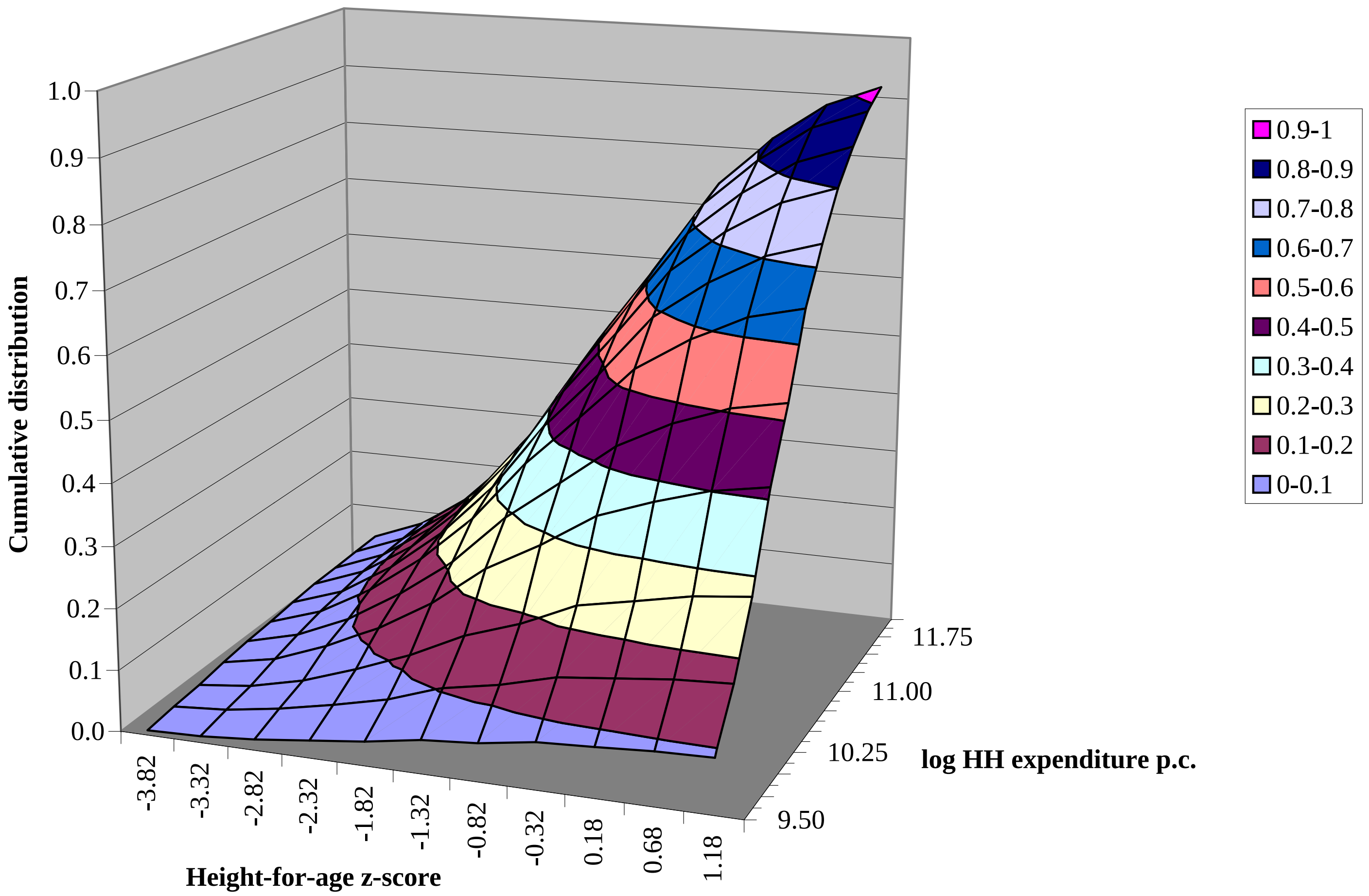
$P^{1,0}(x_1, x_2, z)$ aggregates the x_1 poverty deficit of those that are poor with respect to the second attribute.

Consider the following multidimensional extension of the FGT class of measures:

$$P^{\alpha_1,\alpha_2}(x_1,x_2,z) = \int_0^{z_1}\int_0^{z_2}\left(\frac{z_1-x_1}{z_1}\right)^{\alpha_1}\left(\frac{z_2-x_2}{z_2}\right)^{\alpha_2} dH(x_1,x_2). \quad (22)$$

$P^{1,1}(x_1,x_2,z)$ aggregates the products of the poverty deficits.

Bidimensional distribution



Classes of indices

Consider the following class of poverty measures:

$$\Pi^{1,1}(\lambda^+) = \left\{ P(\lambda) \;\middle|\; \begin{array}{c} \Lambda(\lambda) \subset \Lambda(\lambda^+) \\ \pi(x_1, x_2, \lambda) = 0 \text{ if } \lambda(x_1, x_2) = 0 \\ \pi^{x_j}(x_1, x_2) \leq 0, \quad j = 1, 2, \forall x_1, x_2 \\ \pi^{x_j x_k}(x_1, x_2) \geq 0, \quad j \neq k, \forall x_1, x_2, \end{array} \right\}, \tag{23}$$

where π^{x_j} $\left(\pi^{x_j,x_k}\right)$ corresponds to the first (cross) derivative of the function $\pi\left(x_1, x_2, \lambda\right)$ with respect to x_j (x_j and x_k).

The first row of equation (23) defines the upper limit of the poverty frontier.

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The second indicates that poverty measures of $\Pi^{1,1}\left(\lambda^*\right)$ are continuous all along the frontier separating the poor from the non-poor segments of the population.

Classes of indices

Consider the following class of poverty measures:

$$\Pi^{1,1}(\lambda^+) = \left\{ P(\lambda) \;\middle|\; \begin{array}{c} \Lambda(\lambda) \subset \Lambda(\lambda^+) \\ \pi(x_1, x_2, \lambda) = 0 \text{ if } \lambda(x_1, x_2) = 0 \\ \pi^{x_j}(x_1, x_2) \leq 0, \quad j = 1, 2, \forall x_1, x_2 \\ \pi^{x_j x_k}(x_1, x_2) \geq 0, \quad j \neq k, \forall x_1, x_2, \end{array} \right\}, \tag{23}$$

where π^{x_j} $\left(\pi^{x_j, x_k}\right)$ corresponds to the first (cross) derivative of the function $\pi\left(x_1, x_2, \lambda\right)$ with respect to x_j (x_j and x_k).

The third row stipulates that poverty measures in this class satisfy the *monotonicity* axiom.

Classes of indices

Consider the following class of poverty measures:

$$\Pi^{1,1}(\lambda^+) = \left\{ P(\lambda) \left| \begin{array}{c} \Lambda(\lambda) \subset \Lambda(\lambda^+) \\ \pi(x_1, x_2, \lambda) = 0 \text{ if } \lambda(x_1, x_2) = 0 \\ \pi^{x_j}(x_1, x_2) \leq 0, \quad j = 1, 2, \forall x_1, x_2 \\ \pi^{x_j x_k}(x_1, x_2) \geq 0, \quad j \neq k, \forall x_1, x_2, \end{array} \right. \right\}, \tag{23}$$

where π^{x_j} $\left(\pi^{x_j, x_k}\right)$ corresponds to the first (cross) derivative of the function $\pi\left(x_1, x_2, \lambda\right)$ with respect to x_j (x_j and x_k).

The fourth row says that attributes are substitutable.

Classes of indices

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Depending on the choice of functional form for $\pi\left(x_1, x_2, \lambda\right)$ and λ, this class may include poverty measures based on the *intersection*, the *union*, or any intermediate form of poverty.

We then have that

$$\Delta P(\lambda) > 0 \,\forall P(\lambda) \in \Pi^{1,1}(\lambda^{+}) \quad (24)$$

if and only if $\Delta P^{0,0}(x_1, x_2) > 0, \quad \forall (x_1, x_2) \in \Lambda\left(\lambda^{+}\right).$ (25)

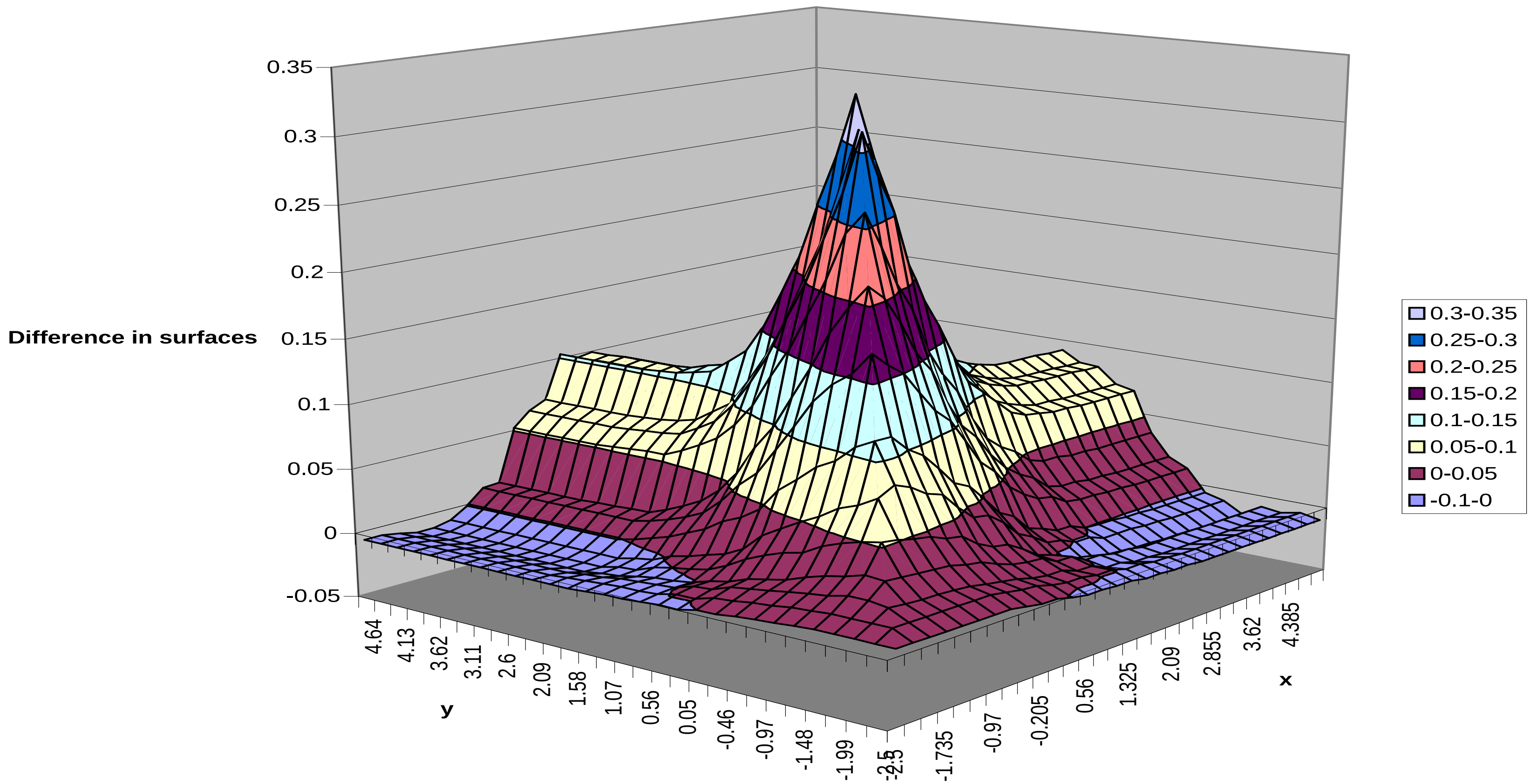
Generality of the methods

- The methods are more general than two other common ones:
  - One approach has been to combine many indicators of well-being into one, unidimensional index, and then compare that index across populations. The best-known example is the Human Development Index (UNDP, 1990).
    - Choosing to compare a single aggregate welfare index essentially reduces the domain for the test to a single line.

Generality

- A second approach is to compare many indicators of well-being independently: *i.e.* looking at the univariate dominance curve for each dimension of well-being.
  - It is possible that the univariate dominance curve for A lies above that for B, but that A is not above B at one or more interior points in the test domain.
    - Importance of capturing ”multiple” poverty
  - It is possible for the univariate dominance surfaces to cross but for A's surface to be above B's for a large area of interior points in the test domain. Consider next Figure.

No univariate but bivariate dominance



Examples

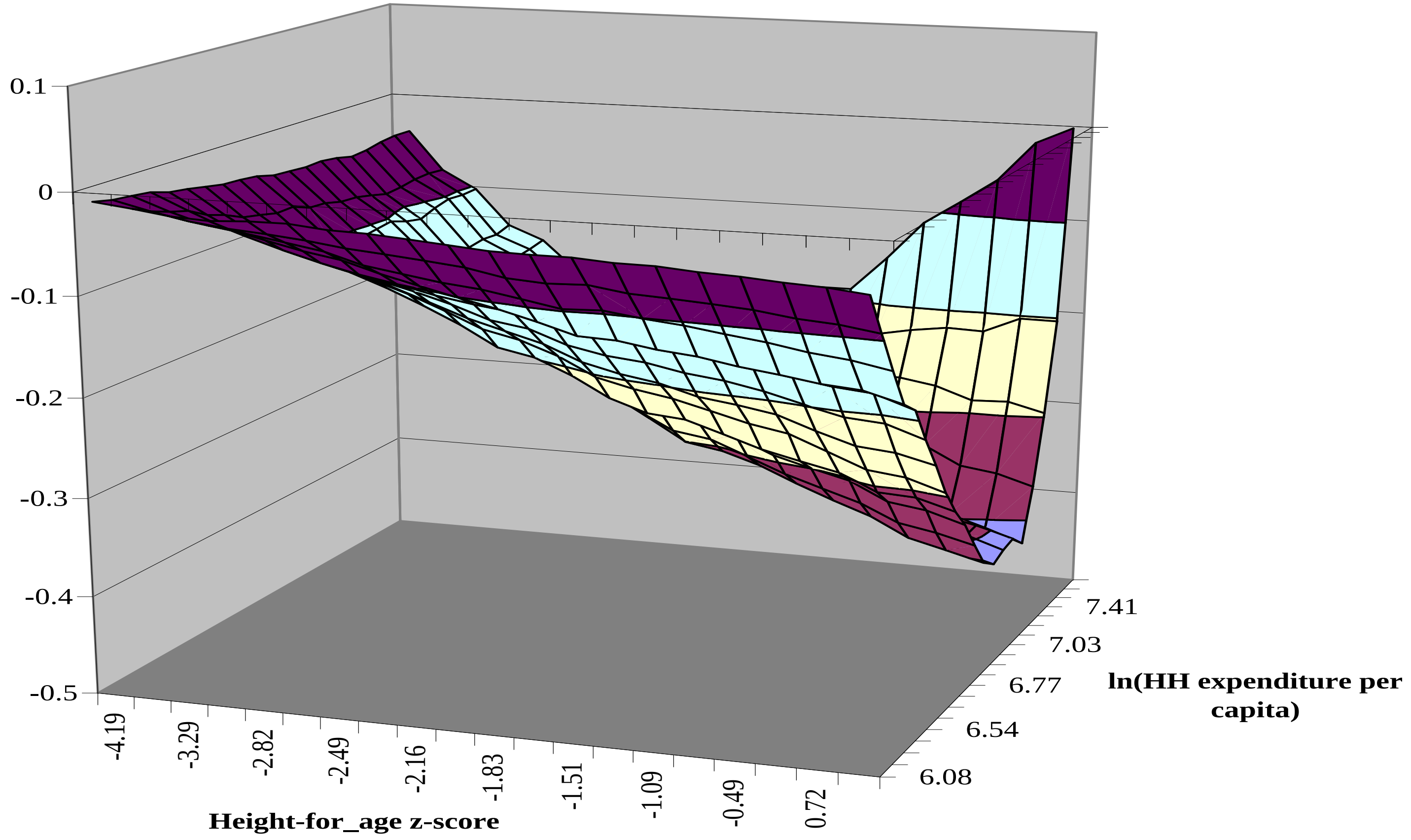
- Are rural people poorer than the urban ones in Viet Nam?
  - People living in rural areas tend to be poorer when judged by expenditures or income alone.
  - However: possible that people are better nourished in rural than urban areas, *ceteris paribus*, because they have tastes for foods that provide nutrients at a lower cost, or because unit prices of comparable food commodities are lower.

Viet Nam

- To test this, we measure welfare in two dimensions: *per capita* household expenditures and nutritional status, as measured by a child's gender and age standardized height, transformed into standard deviation or z-scores. (Use 1993 Viet Nam Living Standards Measurement Survey.)
  - y axis measures the height-for-age z-score (stunting)
  - x axis measures the *per capita* expenditures for the child's household
  - z axis measures the cumulative proportion of children that fall below the points defined in the (x, y) domain.

- We test for a significant difference in the dominance surface at each point of a grid, and reject the null of non-dominance of A by B only if all of the test statistics have the right sign and are significantly different from 0.

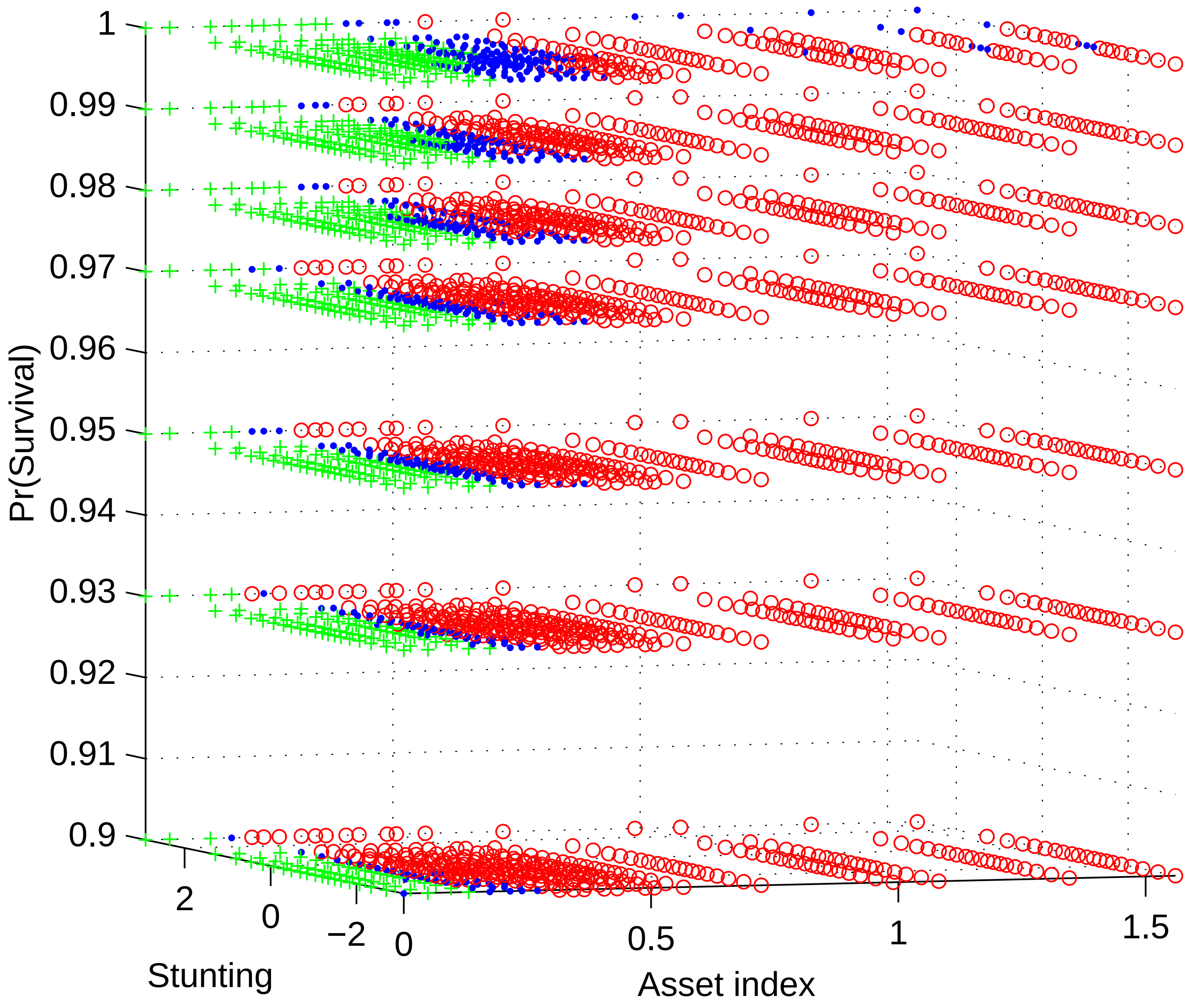
Urban minus Rural Dominance Surface for Viet Nam



Ghana, 1993-1998

- Second example tests for first-order poverty dominance in three dimensions: ”Did poverty decline in Ghana between 1993 and 1998?”
  - Three welfare variables for children under five years old: survival probability, height-for-age z-score (stunting), and index of household’s assets.

Difference between 1993 and 1998 surfaces for Ghanaian children



Extensions

Higher order: (2,1) class

$$\ddot{\Pi}^{2,1}(\lambda^+) = \left\{ P(\lambda) \left| \begin{array}{l} P(\lambda) \in \ddot{\Pi}^{1,1}(\lambda^+) \\ \pi^{x}(x,y;\lambda) = 0 \text{ whenever } \lambda(x,y) = 0 \\ \pi^{xx}(x,y;\lambda) \geq 0 \; \forall x,y \\ \text{and } \pi^{xxy}(x,y;\lambda) \leq 0, \; \forall x,y. \end{array} \right. \right\} \tag{26}$$

(2,1) test

We then have that

$$\Delta P(\lambda) > 0 \, \forall P(\lambda) \in \Pi^{2,1}(\lambda^{+}) \tag{27}$$

$$\text{if and only if } \Delta P^{1,0}(x_1, x_2) > 0, \quad \forall (x_1, x_2) \in \Lambda\left(\lambda^{+}\right). \tag{28}$$

(2,2) class

- Similar to Atkinson and Bourguignon (1982) for social welfare (2,2): they consider possibly different signs for $\pi^{xy}(x,y)$ and $\pi^{xxyy}(x,y)$ ("complement" and "substitute" classes)

$$\ddot{\Pi}^{2,2}(\lambda^+) = \left\{ P(\lambda) \left| \begin{array}{l} P(\lambda) \in \ddot{\Pi}^{2,1}(\lambda^+) \\ \pi^{xx}(x,y;\lambda) = 0 \text{ whenever } \lambda(x,y) = 0 \\ \pi^{yy}(x,y;\lambda) = 0 \text{ whenever } \lambda(x,y) = 0 \\ \pi^{xy}(x,y;\lambda) = 0 \text{ whenever } \lambda(x,y) = 0 \\ \pi^{yy}(x,y;\lambda) \geq 0 \; \forall x,y \\ \pi^{xyy}(x,y;\lambda) \leq 0, \; \forall x,y \\ \text{and } \pi^{xxyy}(x,y;\lambda) \geq 0, \; \forall x,y. \end{array} \right. \right\} \tag{29}$$

(2,2) test

We then have that

$$\Delta P(\lambda) > 0 \, \forall P(\lambda) \in \Pi^{2,2}(\lambda^{+}) \tag{30}$$

$$\text{if and only if } \Delta P^{1,1}\left(x_1, x_2\right) > 0, \quad \forall \left(x_1, x_2\right) \in \Lambda\left(\lambda^{+}\right). \tag{31}$$

Multidimensional inequality dominance

Extension of poverty dominance

- The above provides a framework for thinking about robust multidimensional comparisons.
- Can think of $\pi(x, y; \lambda)$ as individual contribution to total inequality
- Can think of

$$\Lambda(\lambda) = \{(x_1, x_2) \,|\lambda(x_1, x_2) \leq 0\} \tag{32}$$

as the set of those who can contribute to total inequality because of their lower welfare.

Normalization

- One problem: cannot use mean welfare for normalizing comparisons since individual welfare function $\lambda(x, y)$ is not known.
- Two types of normalizations can be made, one for each indicator of welfare.
- Use gaps between incomes and mean (absolute inequality) and between income shares and 1 (relative inequality):

$$y_\gamma = \gamma \left(\frac{y - \mu}{\mu} \right) + (1 - \gamma) (y - \mu) \tag{33}$$

with

$$F^\gamma(z) = F \left(\frac{\mu z + (1 - \gamma)\mu^2 + \gamma\mu}{(1 - \gamma)\mu + \gamma} \right) \tag{34}$$

Absolute/relative inequality

- Absolute inequality:

$$F^0(z) = F(z + \mu) \tag{35}$$

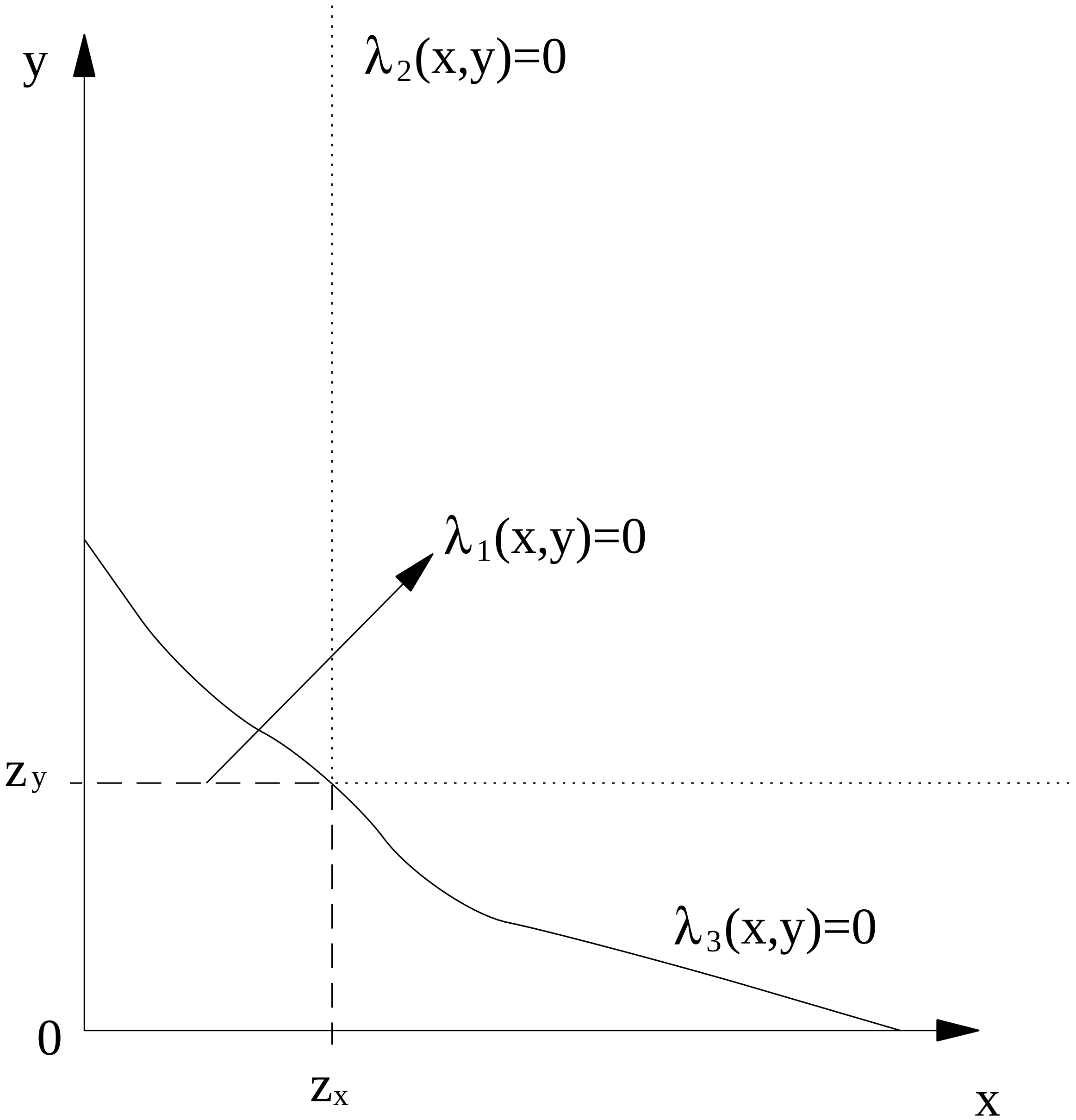
- Relative inequality:

$$F^1(z) = F(\mu z + \mu) \tag{36}$$

Interpretation

- Can then proceed to test inequality dominance using, for instance: (income, wealth); (income, health, education); (income, life expectancy); (income, nutrition); (income, access to public goods)

- If dominance is found, then inequality is ordered over a wide class of multidimensional inequality indices



y
λ2(x,y)=0
λ1(x,y)=0
zy
λ3(x,y)=0
0
zx
x

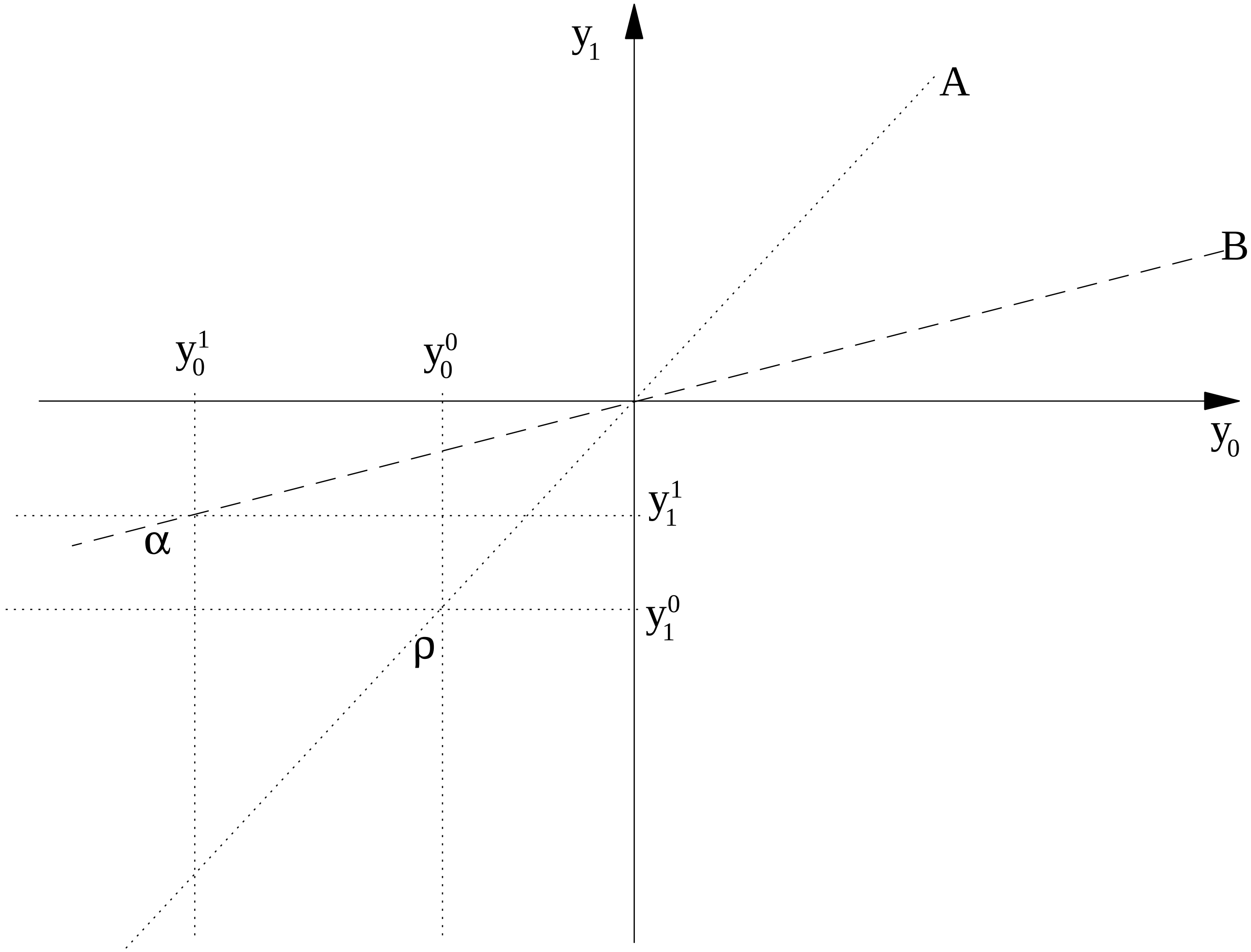
(2,1) inequality dominance

- (1,1) dominance can only be partial since normalization by mean
- (2,1) dominance: inequality in x_1 is more important when it affects groups with lower values of x_2
- Examples:
  - inequality within earlier periods of life more important than within later periods (*e.g.*, because of distinction between opportunities earlier in life and outcomes later in life; effect of luck or choices may be less important than the effect of opportunities);
  - inequality within underprivileged groups is more important (immigrants, "blacks", women, children, less healthy, vulnerable, fewer assets, facing greater uncertainty and capital market imperfections)

Joint dominance over absolute and relative inequality

- If bidimensional dominance over y_0 and y_1, then robust joint comparison of absolute and relative inequality
- Does not necessarily imply marginal dominance in absolute or in relative inequality over entire range of areas covered by bidimensional dominance
- If marginal dominance of absolute over $]-\infty, y_0^0]$ and of relative over $]-\infty, y_1^0]$, then we necessarily have bidimensional dominance over intersection region defined by $(y_0, y_1) \in]-\infty, y_0^0] \otimes]-\infty, y_1^0]$.

Absolute and relative inequality



Univariate and bivariate inequality dominance

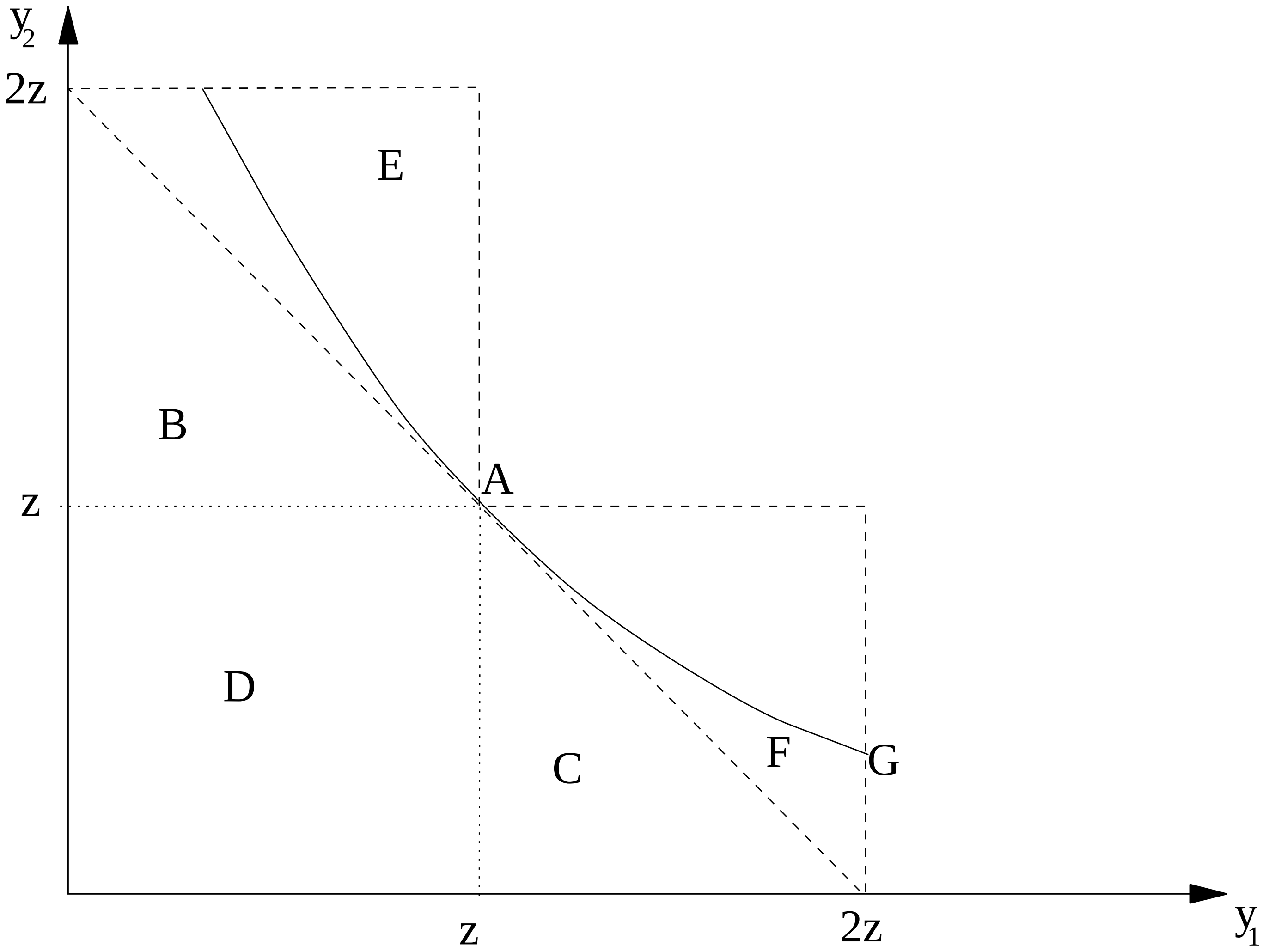
- If $\mu_A > \mu_B$, and y_0^A dominates y_0^B between $]-\infty, y_0^0]$, then y_1^A dominates y_1^B between $]-\infty, y_0^0/\mu_B]$ (with $y_0^0 < 0$) (see intersection area comprising point α in graph)
- If $\mu_A < \mu_B$, and y_1^B dominates y_1^A between $]-\infty, y_1^0]$, then y_0^B dominates y_0^A between $]-\infty, \mu_A y_1^0]$ (with $y_1^0 < 0$) (see intersection area comprising point ρ in graph)

Joint dominance over absolute and relative poverty

- Result similar to absolute and relative inequality: compare joint distribution of $(y, y/\mu)$ (for relative poverty line set to a proportion of the mean).
- But different from absolute and relative inequality:
  - consider absolute values of y, not absolute distances from mean;
  - relative poverty line can be a function of statistics other than mean.

Cardinally comparable indicators

Different types of temporal poverty



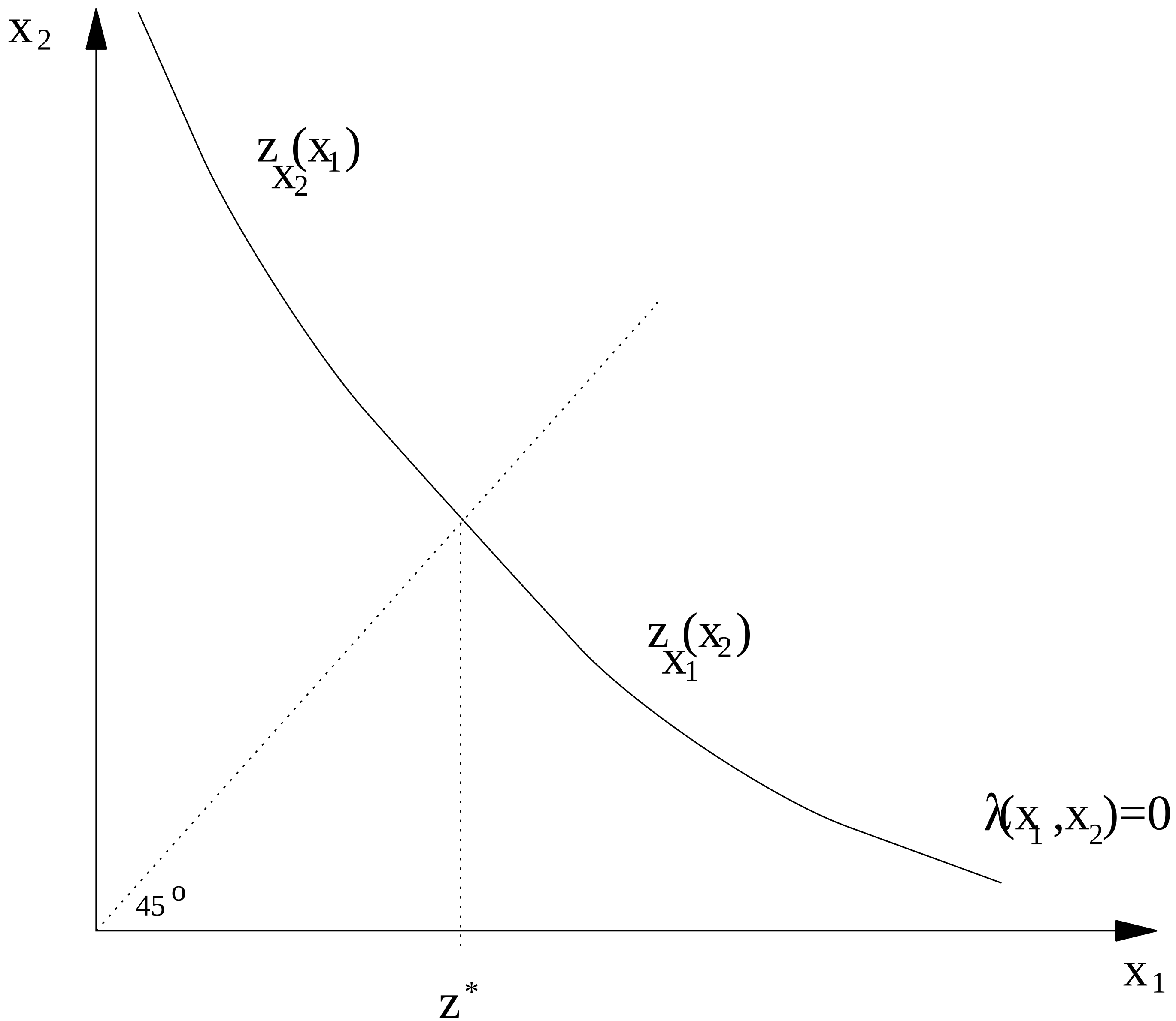
Symmetry and temporal poverty

- Let z^* be such that $\lambda(z^*, z^*) = 0$.
- Let $\lambda(x_1, z_{x_2}(x_1) = 0$ and $\lambda(z_{x_1}(x_2), x_2,) = 0$. Thus, $z_{x_1}(z_{x_2}(x_1)) = x_1$ (z_{x_1} is inverse of z_{x_2})
- Define two-period poverty function as sum of low x_1 and of low x_2 time poverty:

$$P(\lambda) = \int_0^{z^*} \int_{x_1}^{z_{x_2}(x_1)} \pi(x_1, x_2, \lambda) f(x_1, x_2) dx_2 dx_1 \tag{37}$$

$$+ \int_0^{z^*} \int_{x_2}^{z_{x_1}(x_2)} \pi(x_1, x_2, \lambda) f(x_1, x_2) dx_1 dx_2 \tag{38}$$

Symmetric temporal poverty



Symmetry and temporal poverty

- If $z_{x_1}^{(1)}(x_2) \longrightarrow 0$ for low x_2, then union measure for low x_2 (not possible to compensate low values of x_2 by higher values of x_1)
- If $z_{x_1}^{(1)}(x_2) \longrightarrow \infty$ as x_1 approaches z_{x_1}, then intersection measure for higher x_1 (high enough values of x_1 sufficient to exit time poverty)

Temporal poverty indices

- We can define the following class $\ddot{\Pi}^{1,1}(\lambda^+)$ of intertemporal poverty indices:

$$\ddot{\Pi}^{1,1}(\lambda^+) = \left\{ P(\lambda) \left| \begin{array}{l} \Lambda(\lambda) \subset \Lambda(\lambda^+) \\ \pi(x,y;\lambda) = 0, \text{ whenever } \lambda(x,y) = 0 \\ \pi^x(x,y;\lambda) \leq 0 \text{ and } \pi^y(x,y;\lambda) \leq 0 \; \forall x,y \\ \pi^{xy}(x,y;\lambda) \geq 0, \; \forall x,y \end{array} \right. \right\} \quad (39)$$

- If symmetry, we add the conditions:
  - $\pi^{x_1 x_2}(x_1, x_2; \lambda) = \pi^{x_1 x_2}(x_2, x_1; \lambda) \; \forall x_1, x_2$
  - $\pi^{x_1}(x_1, x_2; \lambda) = \pi^{x_2}(x_2, x_1; \lambda) \; \forall x_1, x_2$

Test for symmetric (1,1) indices

- Let:

$$\underline{P}^{0,0}(v,w) = \int_0^w \int_{x_2}^v f(x_1,x_2)dx_1dx_2 \quad (40)$$

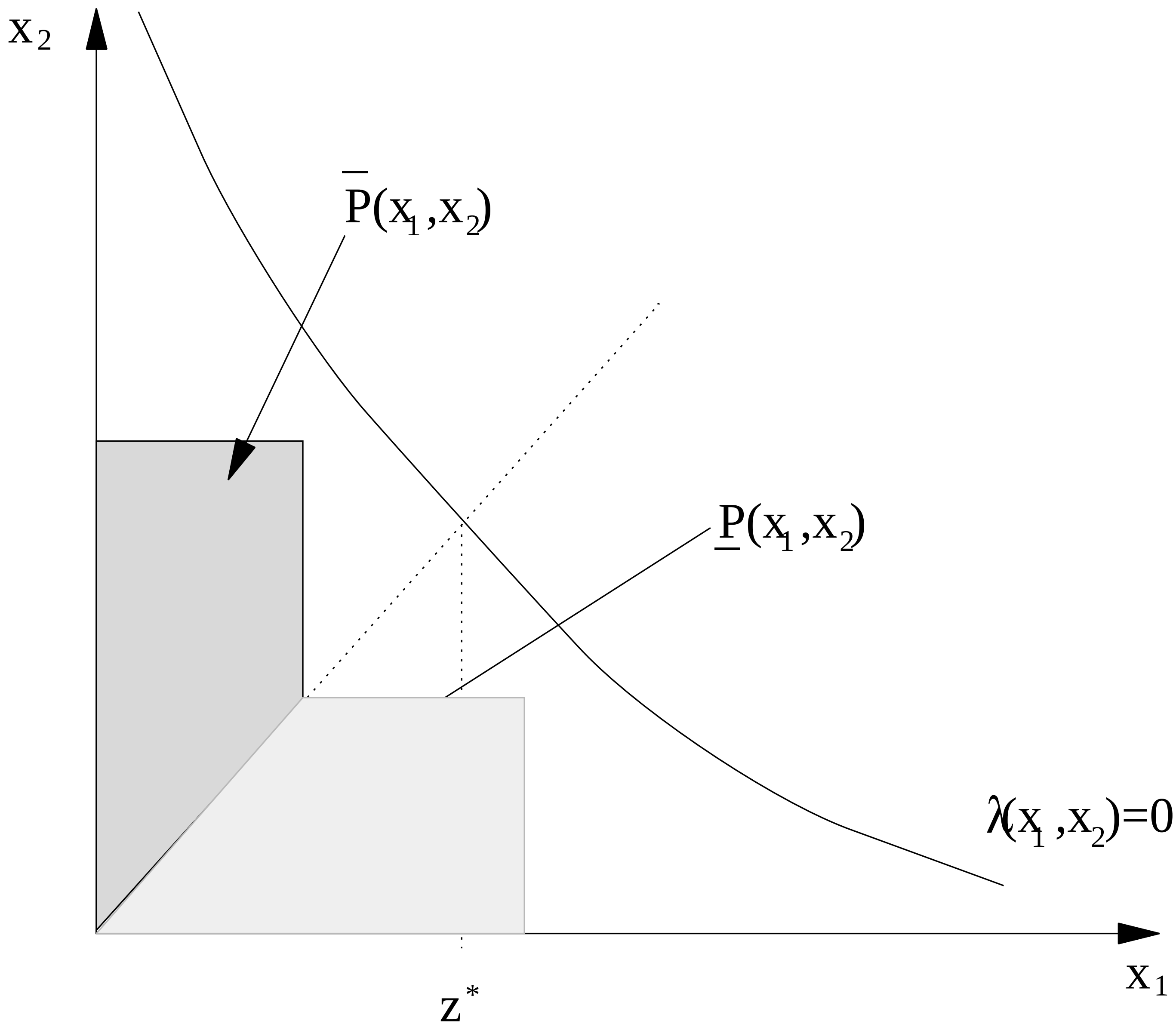
$$= \int_0^w \left[F(v|x_2) - F(x_1 = x_2|x_2)\right] f(x_2)dx_2 \quad (41)$$

and

$$\overline{P}^{0,0}(v,w) = \int_0^v \int_{x_1}^w f(x_1,x_2)dx_2dx_1 \quad (42)$$

$$= \int_0^v \left[F(w|x_1) - F(x_2 = x_1|x_1)\right] f(x_1)dx_1 \quad (43)$$

Symmetric temporal poverty



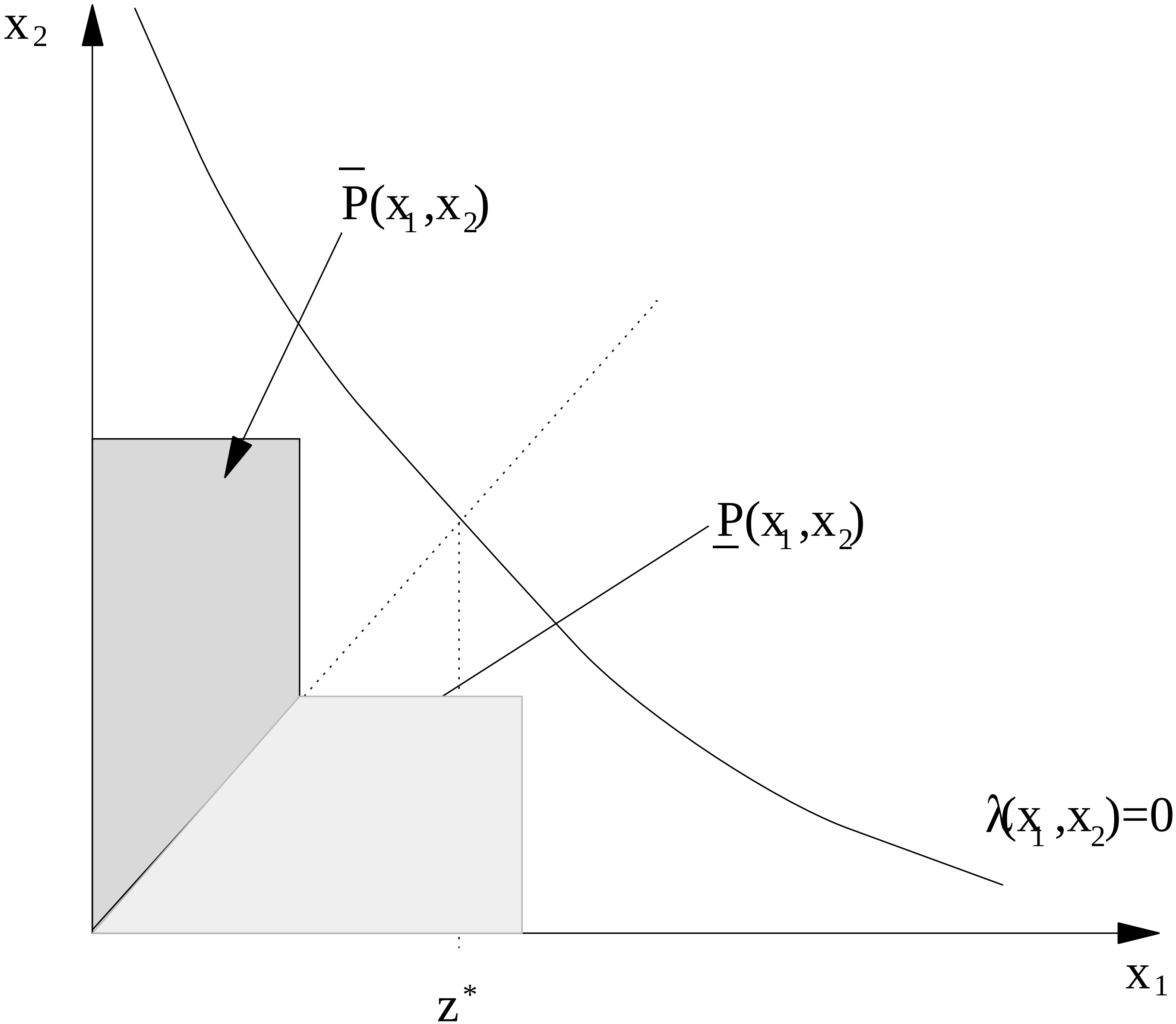
Test for symmetric (1,1) indices

- With symmetric indices, we then have:

$$\Delta P(\lambda) > 0,\ \forall P(\lambda) \in \ddot{\Pi}^{1,1}(\lambda^+)$$

$$\text{iff } \Delta \left[\underline{P}^{0,0}(x_1, x_2) + \overline{P}^{0,0}(x_2, x_1) \right] > 0,\ \forall (x_1, x_2) \in \Lambda(\lambda^+). ($$

Symmetric temporal poverty



Asymmetric indices

- For asymmetry, suppose that dimension x_1 is more important than x_2 (as for period 1 for welfare of young or vulnerable).

- This suggests that: $\pi^{x_1 x_2}(x_1, x_2; \lambda) > \pi^{x_1 x_2}(x_2, x_1; \lambda)$ if $x_1 < x_2$

  (changes in low values of x_1 have a greater impact on welfare than changes in low values of x_2)

- Also: $\left|\pi^{x_1}(x_2, x_2; \lambda)\right| > \left|\pi^{x_2}(x_2, x_2; \lambda)\right|, \forall x_2$

  (for equal values of x_1 and x_2, changes in x_1 have a greater impact on welfare than changes in x_2)

Test for asymmetry

With asymmetric functions, we then need to check for:

To be followed in the next sequel!

Asymmetric temporal poverty