CANAZEI WINTER SCHOOL

Measuring inequality of opportunities

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Outline of the lecture

- Why to be interested in EOp?

- How to define EOp: compensation and reward, ex ante and ex post

Focus of the lecture: Inequality of opportunity: (i) testing for EOp; (ii) partial rankings; (iii) complete rankings

- EOp and education

Related literature: EOp and fairness (Marc Fleurbaey's lecture), EOp and social mobility (Arnaud Lefranc's lecture), EOp in an abstract setting (Ernesto Savaglio's lecture), EOp and optimal taxation (Roemer 1998, Roemer et al. 2003, Aaberge et al. 2003), EOp and education financing (Betts and Roemer, 1998).

Equality of Opportunity, by Dworkin, Arneson, Barry, Cohen, Roemer, Fleurbaey

- Equality of outcomes is too strong: limit the domain of equality requirement

- Critics of the consequentialism: the process generating a given distribution of outcomes is relevant

The principle of justice does not require equality of individuals' final achievements; once the opportunities to reach a valuable outcome have been equally split, which particular opportunity, from those open to her, the individual chooses, is outside the scope of justice.

See also John Goldthorpe, Michael Young's "The rise of Meritocracy" (1958)

The equality of opportunity model

$$outcome = g(C, E)$$

- *Circumstances* C: factors influencing a person's final outcome, for which she is not responsible (family background, race, talent, ...)

- *Effort* E: factors influencing a person's final outcome, for which she is responsible

- Principle of Compensation: outcome inequalities due to differences in circumstances are inequitable and to be compensated

- Principle of Reward: outcome inequalities due to effort are equitable and not to be compensated

The cut between circumstances and effort

- Dworkin: (extended) resources *versus* preferences

Example of education: The social origin affects school outcomes by a *Primary effect* (different resources and abilities) and a *Secondary effect* (expectations, ambitions, preferences)

- Arneson, Cohen, Roemer: effort variables are those on which the individual has control

The role of luck. Distinction between *option luck* (you take a bet: to ski off piste, to do ice climbing) and *brute luck* (not responsible)

Difficult to distinguish in practice.

Understanding opportunity inequality: an example

The role of education

$$\begin{aligned} Income &= g(Circumstances, Effort, Education) \\ Education &= f\,(Circumstances, Effort) \end{aligned}$$

Study the double role of circumstances. EOp for income requires:

1. EOp in education attainments

2. Strong relation between education and income

   - signaling effect of education

   - meritocracy in the labour market

Why is EOp relevant (in addition to normative reasons)?

Macro: Opportunity inequality and aggregate economic performance ("inequality traps": Bourguignon et al. 2007 and World Bank, 2006).

Example: EOp in the labour market and the private investement in education

Political economy: EOp and the support for redistribution. Social attitudes towards redistributive policies may be affected by the knowledge of the origin of income inequalities (Alesina and Glaeser, 2006).

Economic policy: Priorities of redistributive policies (EOp policies not necessarily redistributive policies).

Measuring inequality of opportunity

Ex ante versus ex post approaches

(see Flaurbaey 2008 - ch. 9, Goux D. and E. Maurin 2002, Ooghe, Schokkaert & Van de gaer, SCW 2007, Peragine JEI 2004)

Ex-ante approach: there is EOp iff the set of opportunities is the same for all individuals, regardless of the circumstances

$=>$ the Compensation Principle is defined in term of opportunity sets: compensate for different opportunity sets

$=>$ Reward is defined as neutrality wrt to the outcome "chosen" by the individual from his opportunity set

$=>$ focus on inequality of opportunity sets

- this is the problem studied by Kranich (JET, 1996) and Ok (JET, 1997) in an abstract setting: rank opportunity profiles of the form: $\mathbf{O} = (O_1, O_2, ...O_n)$

- Pattanaik and Xu (1990) and related literature: rank opportunity sets $O_{1,}O_2, ...$

Ex-ante approach: the model we use

$F(x)$ is the society outcome distribution

Partition the population into n types: *type i* $\left(T_{C_i}\right)$ is the set of individuals with circumstances C_i

$F_i(x) = F(x|C_i)$ is the distribution of outcome conditional to C_i (type i distribution);

$F_i(x)$ is the set of opportunities, expressed in levels of outcome, open to individuals with C_i.

Evaluate the distribution of opportunity sets $\Rightarrow$ evaluate $\{F_1(x),...,F_n(x)\}$

This model (Van de gaer 1993) is used for analyzing inequality of opportunity by

- Lefranc et al. (2006) to obtain test of the existence of EOp

- Peragine (1998, 2004) to obtain dominance conditions

- Bourguignon et al. (2003), Checchi and Peragine (2005), Dardanoni et al (2005), Pistolesi (2007) to obtain complete rankings

Ex-post approach: there is EOp iff all those who exert the same effort achieve the same outcome

=> the Compensation Principle is defined in terms of outcomes for individuals who exercise the same effort

=> Reward is defined as neutrality wrt to differences in the outcome distributions of groups of individuals with different effort levels

=> focus on outcome inequality among individuals with the same effort, for all levels of effort

Ex-post approach: the model (Roemer 1993)

$F(x)$ is the society outcome distribution

Partition the population into m tranches: *tranche p* $\left(T_{E_p}\right)$ is the set of individuals with effort E_p

$F_p(x) = F(x|E_p)$ is the distribution of outcome conditional to effort E_p (tranche p distribution)

Evaluating the entire distribution$\{F_1(x)$,..., $F_p(x)$, ...,$F_m(x)\}$

But effort may be unobservable.

Proposal (Roemer, 1993): *people in different types have exercised a comparable degree of responsibility if they are at the same percentile of their own type income distributions.*

$Tranche\, p$: set of individuals at the same quantile p of their own type outcome distribution

This approach has been first proposed by Roemer (1993) and used for optimal taxation problems (Roemer et al. 2006, Aaberge et al. 2003). It is used for opportunity inequality measurement by Peragine (MSS 2002, JEI 2004), Van de gaer et al (1998), Ruiz Castillo (2003), Villar (2005), Checchi and Peragine (2005).

Example

Society 1

types /effort level	low	high
type 1	10	20
type 2	20	30
type 3	30	40

Society 2

types /effort level	low	high
type 1	10	20
type 2	10	40
type 3	10	60

Measuring opportunity inequality

- **Testing for the existence of EOp**

  the ex ante approach

  the ex post approach?

- **Partial rankings based on opportunity egalitarian social evaluation function**

  the ex ante and the ex post approaches

- **Complete ranking**

Testing for the existence of EOp: the ex ante approach (Lefranc et al. 2006a,b, 2007)

In a distribution of opportunity sets $\{F_1(x), ..., F_n(x)\}$ *there is EOp iff, for any pair of opportunity sets* $\left(F_i(x), F_j(x)\right)$, *neither* F_i *is preferred to* F_j, *nor* F_j *is preferred to* F_i.

Introduce a *type* evaluation function:

$$\mathbf{V}(F_i) = \int u(x) f^i(x) dx$$

restrict the class of u functions:

(Property 1) $u' \geq 0$

(Property 2) Within-type inequality aversion $u'' \leq 0$

(Property 3) Within-type inequality neutrality, $u'' = 0$

Obtain dominance conditions:

$$
\begin{aligned}
\text{Property 1:} \quad & u' \geq 0 \Longleftrightarrow F_i \succeq_{FSD} F_j \\
\text{Properties 1 and 2:} \quad & u' \geq 0 \ \& \ u'' \leq 0 \Longleftrightarrow F_i \succeq_{SSD} F_j \\
\text{Properties 1 and 3:} \quad & u' \geq 0 \ \& \ u'' = 0 \Longleftrightarrow \mu_i \geq \mu_j.
\end{aligned}
$$

Criteria for (partial) rankings of opportunity sets.

Strong EOp *There is EOp iff, for all* (i, j),

$$F_i(x) = F_j(x), \forall x \in [0, z]$$

EOp1 (EOp of the first order) *There is EOp iff, for all* (i, j),

$$F_i \nsucc_{FSD} F_j \text{ and } F_j \nsucc_{FSD} F_i$$

EOp2 (EOp of the second order) *There is EOp iff, for all* (i, j),

$$F_i \nsucc_{SSD} F_j \text{ and } F_j \nsucc_{SSD} F_i$$

Weak EOp *There is EOp iff, for all* (i, j),

$$\mu_i = \mu_j$$

Ranking distributions of opportunity sets

The ex ante approach

Compare two distributions $F(x)$ and $G(x)$: first aggregate the welfare of each type, then aggregate the types (Van de gaer 1993, Peragine 1998, 2004):

$$W_{ex-ante}(F) = \sum_{i=1}^{n} q_i^F \mathbf{V}_i(F_i) = \sum_{i=1}^{n} q_i^F \int u^i(x) f^i(x) dx$$

This is a generalization of the **min of means** criterion (Fleurbaey 2008)

Peragine (JEI, 2004) generalizes to a non additive aggregation.

$$W(X) = \Phi(\mathbf{V}_1(F_1), ..., \mathbf{V}_n(F_n))$$

where Φ satisfies a property of inequality aversion between types.

SEF can be expressed in Harsanyi (1955)-type terms, to make clear the ex-ante interpretation:

$$W_{ex-ante}\left(F\right) = \sum_{i=1}^{n} \mathsf{Pr}\left\{k \in T_{C^i}\right\} E\left[u^i(x_k)\,\middle|\, k \in T_{C^i}\right]$$

$\mathsf{Pr}\left\{k \in T_{C^i}\right\}$ is the probability for an individual k of being endowed with the circumstances C^i (of facing the prospect $F_i\left(x\right)$)

$E\left[u^i(x_k)\,\middle|\, k \in T_{C^i}\right]$ is the expected utility associated to type i.

Impose ethical requirements by restricting the admissible profiles of utility functions $< u^1(x), ..., u^n(x) >$:

Property 1: Monotonicity $\frac{du^i(x)}{dx} \geq 0, \forall i = 1, ..., n$

Property 2: Reward, $\frac{d^2u^i(x)}{dx^2} = 0, \forall i = 1, ..., n$

Property 3: Compensation, $\frac{du^i(x)}{dx} \geq \frac{du^{i+1}(x)}{dx}, \forall i = 1, ..., n-1$

Property 4: $u^i(z) = u^j\,(z)\,, \forall i, j$.

In Peragine (JEI, 2004) these properties are obtained from basic transfer and symmetry axioms.

Theorem (Peragine, 1998): For all F,$G \in \mathbf{\Psi}$, $W(F) \geq W(G)$ for all $W \in \mathbf{W}_{1,2,3,4}$ if and only if $X \succcurlyeq_{GL\mu} Y \Leftrightarrow$

$$\sum_{i=1}^{k} q_i^F \mu_i^F \geq \sum_{i=1}^{k} q_i^G \mu_i^G, \forall k \in \{1, ..., n\}$$

- evaluate opportunity set by (weighted) mean: $q_i^F \mu_i^F$

- compare distributions of opportunity sets $\left(q_1^F \mu_1^F, ..., q_n^F \mu_n^F\right)$ by generalized Lorenz dominance

Note: this is a special case of Atkinson and Bourguinon (1987) characterization of the *sequential generalized Lorenz dominance.*

Extension

Theorem For all F,$G \in \Psi$, $W(F) \geq W(G)$ *for all* $W \in W_{1,3,4}$ *if and only if*

$$\sum_{i=1}^{k} q_i^G G_i(x) \geq \sum_{i=1}^{k} q_i^F F_i(x), \forall x \in [0, z], \forall k \in (1, ..., n)$$

- *evaluate opportunity set by* $q_i^F F_i(x)$

- *compare distributions* $\left(q_1^F F_1(x), ..., q_n^F F_n(x)\right)$ *by generalized Lorenz dominance*

The ex post approach

First aggregate the welfare of each tranche, then aggregate the tranches evaluations.

Introduce a *tranche-specific* evaluation function:$v_p(F_p(x))$

then aggregate the tranche evaluations to obtain the overall social evaluation function

$$W_{ex-post}(F) = \sum_{p=1}^{m} v_p(F_p(x))$$

Property 1: Monotonicity: v_p is increasing in x for all p

Property 2 Compensation: v_p is S-concave for all p

Theorem For all F,$G \in \Psi$, $W(F) \geq W(G)$ for all $W \in \mathbf{W}_{12}$ if and only if F generalized dominates G at every tranche ($F \succcurlyeq_{OGL} G$).

Empirically: you have to test for m different dominance conditions

Remark *For all* $F, G \in \Psi$, if $n = N$, *then the following statements are equivalent:* (i) $F \succcurlyeq_{GL} G$; (ii) $F \succcurlyeq_{GL\mu} G$; (iii) $F \succcurlyeq_{OGL} G$.

In empirical work can test for these dominance conditions

Extensions

Peragine (2002) uses a Yaari-type rank dependent social evalution function (see also Aaaberge 2007, and Zoli 2000 for a general treatment):

$$W_{yaari}(F) = \sum_{i=1}^{n} q_i^F \int_0^1 U_i(p) F_i^{-1}(p) dp$$

where $F_i^{-1}$is the left continuous inverse distribution of F_i : $F_i^{-1}(p) :=$ inf $\{x \in \Re_+ \,|F_i(x) \geq p\}$, $\forall p \in [0,1]$

U : $[0,1] \rightarrow \Re_+$ express the weight attached to any income at rank p of type i distribution

Impose ethical requirements by restricting the admissible profiles of functions $< U_1(p),\ ...,\ U_n(p) >$

Property 1 (Monotonicity) $\forall i \in \{1, ..., n\}, \ \forall p \in [0, 1] \ , U_i(p) \geq 0.$

Property 2 (Reward) $\forall i \in \{1, ..., n\}, \exists \beta_i > 0 : \ \forall p \in [0, 1] \ , U_i(p) = \beta_i.$

Property 3 (Compensation) $\forall p \in [0, 1], \forall i \in \{1, ..., n-1\}, U_i(p) > U_{i+1}(p)$

Property 4 (Kolm's principle of diminishing progressive transfer):

$\forall p \in [0, 1], \forall i \in \{1, ..., n-2\}, U_i(p) - U_{i+1}(p) > U_{i+1}(p) - U_{i+2}(p).$

The social concern for inequality in the endowment of circumstances decreases with the level of circumstances

Theorem (Peragine 2002) $For\ all\ F, G \in \boldsymbol{\Psi}$, $W(F) \geq W(G)$ for all $W \in \mathbf{W}$ if and only if

$(\mathbf{W_1})$: $[q_k^F F_k^{-1}(p) - q_k^G G_k^{-1}(p)] \geq 0, \forall k \forall p$

$(\mathbf{W_{13}})$: $\sum_{i=1}^{k} [q_i^F F_i^{-1}(p) - q_i^G G_i^{-1}(p)] \geq 0, \forall k \forall p$

$(\mathbf{W_{134}})$: $\sum\limits_{i=1}^{k} \sum\limits_{j=1}^{i} [q_j^F F_j^{-1}(p) - q_j^G G_j^{-1}(p)] \geq 0, \forall k \forall p$

$(\mathbf{W_{12}})$: $[q_k^F \mu_k^F - q_k^G \mu_k^G] \geq 0, \forall k$

$(\mathbf{W_{123}})$: $\sum_{i=1}^{k} [q_i^F \mu_i^F - q_i^G \mu_i^G] \geq 0, \forall k$

$(\mathbf{W_{1234}})$: $\sum\limits_{i=1}^{k} \sum\limits_{j=1}^{i} [q_j^F \mu_j^F - q_j^G \mu_j^G] \geq 0, \forall k$

Empirical application : EOp for higher education in Italy (Peragine and Serlenga, REI 2008)

Outcome: (i) final graduation marks;(ii) earnings and earnings after three years from graduation (return to education)

Circumstances: parental education

Populations defined by: (i) geographical location (North-Center and South) (ii)gender

Data

1. **Survey on the transition from college to work of a representative sample of university graduates**. Graduation marks and individual monthly income after 3 years from graduation. Source: Istat 2004, individuals who graduated in 2001 are interviewed 3 years after completion of degree. Drop-out information from *Survey on the transition from college to work of a representative sample of university graduates. secondary school graduates 2001.*

2. **SWIH** Annual individual disposal income. Source: Bank of Italy (Information from 2002 to 2004)

Methodology

We asses equality or stochastic dominance relationships by non-parametric tests (Beach and Davidson 1983, Davidson and Duclos 2000).

Test 1 **Weak EOp**: tests the null of equality of the means of the distribution of types i and j;

Test 2 **Strong EOp**: tests the null of equality of the distributions of types i and j;

Test 3 **EOp1**: tests the null of first order stochastic dominance of the distribution of type i over j and viceversa;

Test 4 **EOp2**: tests the null of second-order dominance of the distribution of type i over j and viceversa

Strategy

- we do not reject the null of Test (1) or Test (2) $\Rightarrow$ EOp is satisfied.

- Test (3) or (4) accepts dominance of one distribution over the other but not the other way around $\Rightarrow$ EOp is violated.

- Test (3) rejects dominance of each distribution over the other $\Rightarrow$ First order EOp is supported

- Test (3) and (4) conclude that the two distributions dominate each other we give priority to the results of Test (2).

Second step: ranking distribubution of opportunity sets

Test 5 **IOp 1** *Generalized Lorenz Dominance in the type means distribution*: sequentially tests the null of equality of the type weighted means

Test 6 **IOp 2** *Sequential First Order Stochastic Dominance*: sequentially tests the null of first order stochastic dominance of type weighted distributions.

Figure 1. Graduate final marks (including drop-out rates) c.d.f

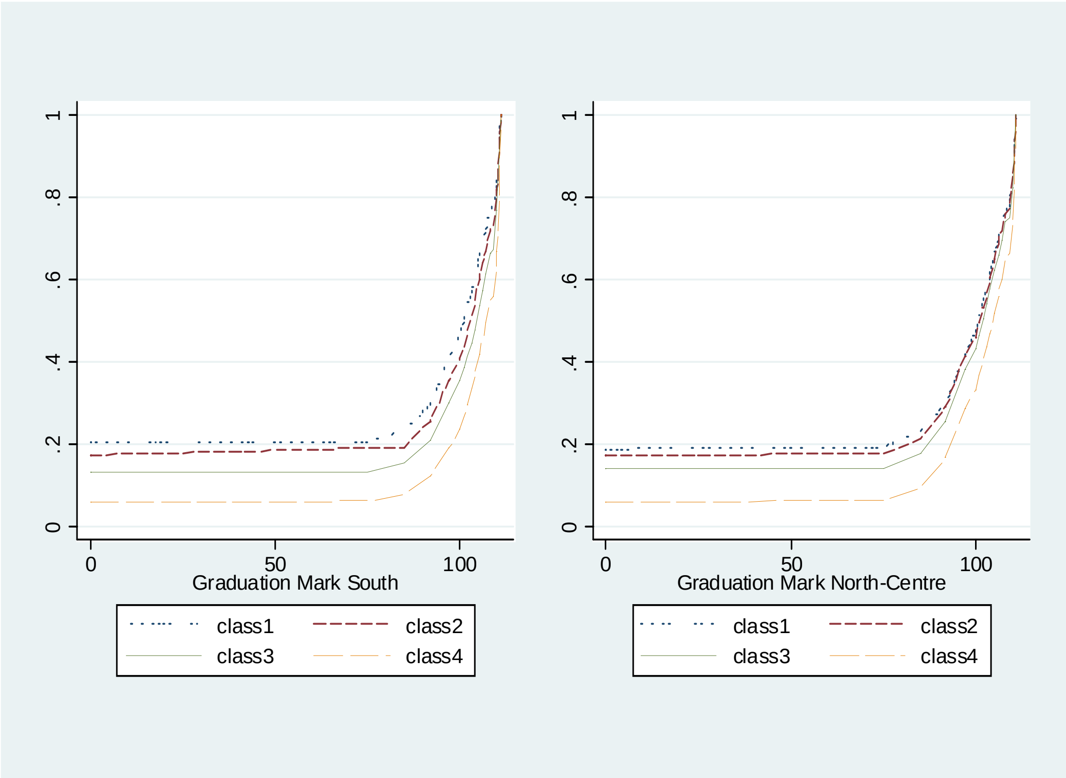


Figure 1bis. Graduate final marks c.d.f.
conditional to school type ("istituto")

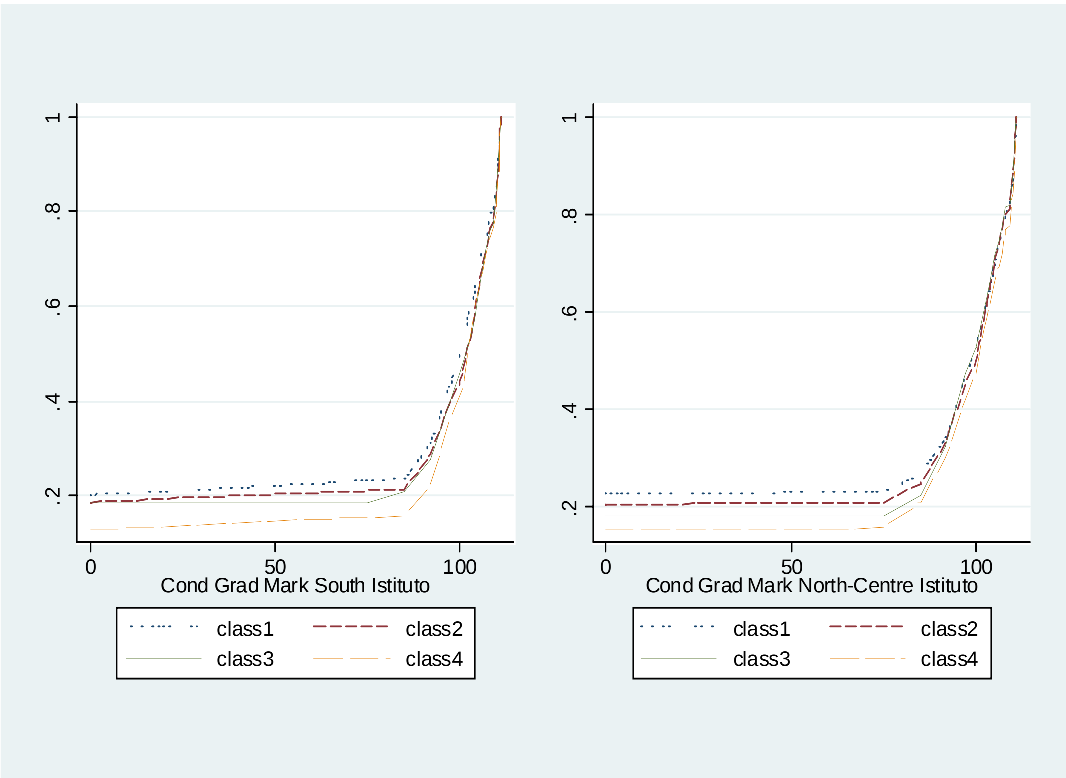


Figure 2. Income after 3 years from graduation c. d. f.

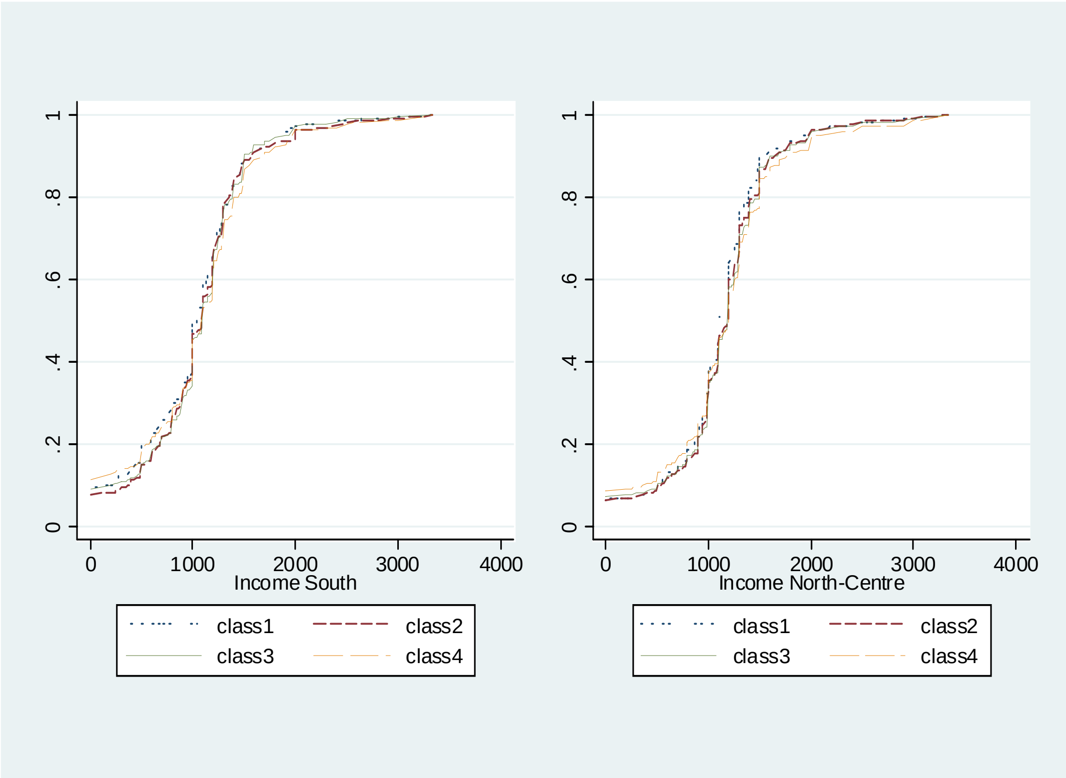


Figure 2 bis. Income after 3 years from graduation conditional to high marks c. d. f.

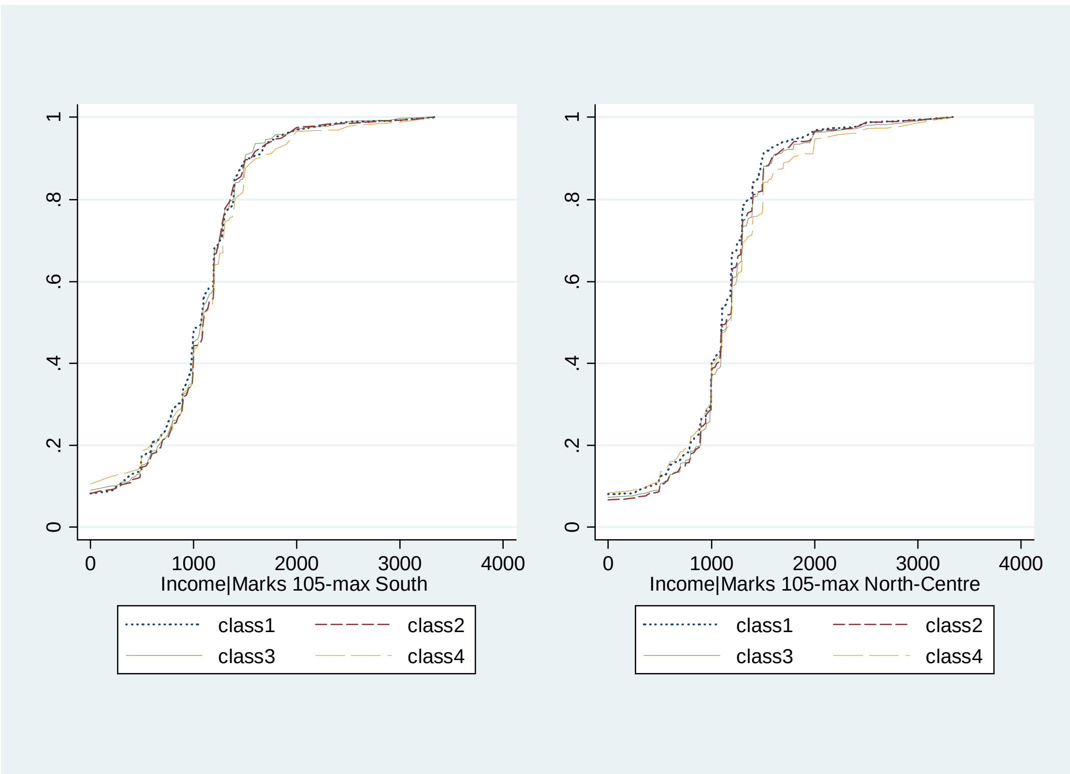
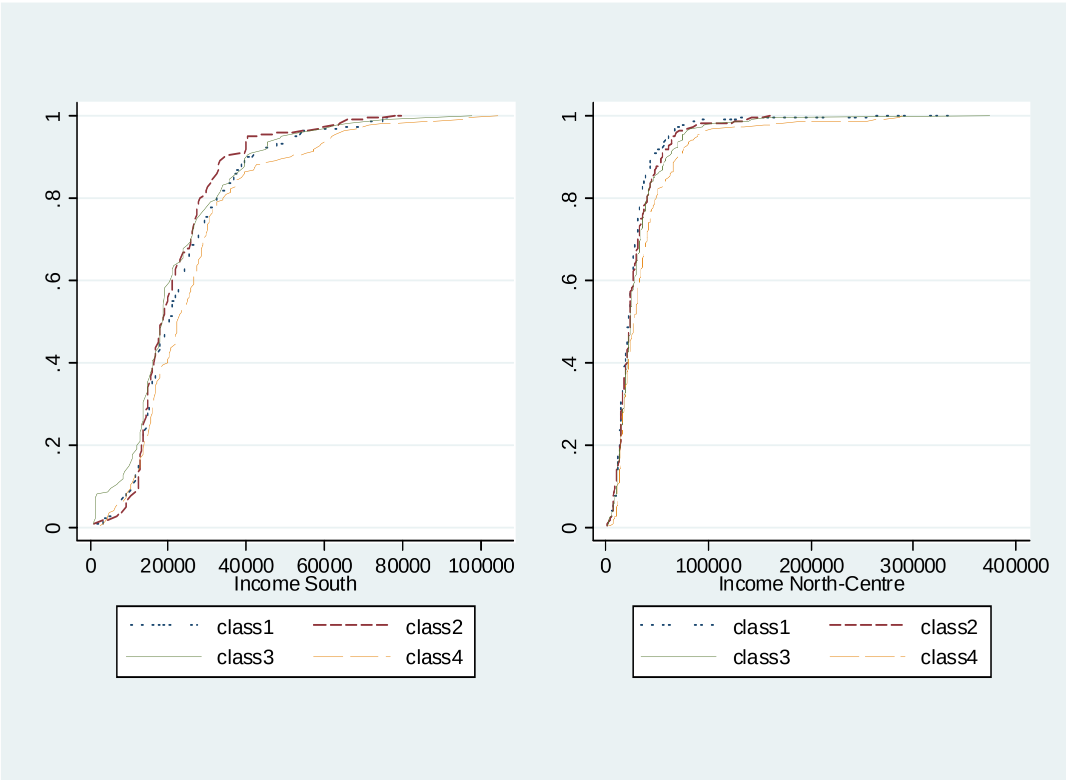


Figure 3. Income c.d.f.



Visual ranking confirmed by the results of the test.

Table 1. Test (1) Weak EOp (comparison of means)

Graduation mark								
	North-Centre				South			
	1	2	3	4	1	2	3	4
1	-	$<^{**}$	$<^{*}$	$<^{*}$	-	$<^{*}$	$<^{*}$	$<^{*}$
2		-	$<^{*}$	$<^{*}$		-	$<^{*}$	$<^{*}$
3			-	$<^{*}$			-	$<^{*}$
4				-				-
Income after 3 years from graduation								
1	-	$=^{*}$	$<^{*}$	$<^{*}$	-	$<^{*}$	$=^{*}$	$<^{*}$
2		-	$=^{*}$	$=^{*}$		-	$<^{**}$	$<^{*}$
3			-	$=^{*}$			-	$=^{*}$
4				-				-
Income								
1	-	$=^{*}$	$<^{*}$	$<^{*}$	-	$=^{*}$	$=^{*}$	$<^{**}$
2		-	$=^{*}$	$<^{*}$		-	$=^{*}$	$<^{*}$
3			-	$<^{*}$			-	$<^{*}$
4				-				-

Notes: *,**denote 5 and 10% level of significance, respectively. $>$ the mean of the distribution in the row is grater than the mean of the distribution in the column; $=$ the means are equal.

First Order Dominance									Second Order Dominance							
	North-Centre				South				North-Centre				South			
Graduation mark									Graduation mark							
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	-	$<^{*}$	$<^{*}$	$<^{*}$	-	$<^{*}$	$<^{*}$	$<^{*}$	-	$<^{**}$	$<^{*}$	$<^{*}$	-	$<^{*}$	$<^{*}$	$<^{*}$
2		-	$<^{*}$	$<^{*}$		-	$<^{*}$	$<^{*}$		-	$<^{*}$	$<^{*}$		-	$<^{*}$	$<^{*}$
3			-	$<^{*}$			-	$<^{*}$			-	$<^{*}$			-	$<^{*}$
4				-				-				-				-
Income 3 years after graduation									Income 3 years after graduation							
1	-	$<^{*}$	$<^{*}$	$<^{*}$	-	$\neq^{*}$	$<^{*}$	$<^{*}$	-	$<^{*}$	$<^{*}$	$<^{*}$	-	$\neq^{*}$	$<^{*}$	$<^{*}$
2		-	$\neq^{*}$	$\neq^{*}$		-	$<^{*}$	$<^{*}$		-	$\neq^{*}$	$\neq^{*}$		-	$<^{*}$	$<^{*}$
3			-	$\neq^{*}$			-	$<^{*}$			-	$\neq^{*}$			-	$<^{**}$
4				-				-				-				-
Income									Income							
1	-	$\neq^{*}$	$<^{*}$	$<^{*}$	-	$\neq^{*}$	$\neq^{*}$	$<^{**}$	-	$\neq^{*}$	$<^{*}$	$<^{*}$	-	$\neq^{*}$	$\neq^{*}$	$<^{**}$
2		-	$\neq^{*}$	$<^{*}$		-	$\neq^{*}$	$<^{*}$		-	$\neq^{*}$	$<^{*}$		-	$\neq^{*}$	$<^{*}$
3			-	$<^{*}$			-	$<^{*}$			-	$<^{*}$			-	$<^{*}$
4				-				-				-				-

Test **IOP 1**: $\sum_{i=1}^{k} q_i^F \mu_i^F \geq \sum_{i=1}^{k} q_i^G \mu_i^G, \forall k = 1, 2, 3, 4$

Table 3. Test **(**5**)** IOP 1

Graduation mark			
	North-Centre		South
1		<	
1+2		<	
1+2+3		>	
1+2+3+4		<	
Income after 3 years from graduation			
1		>	
1+2		>	
1+2+3		>	
1+2+3+4		>	
Income			
1		>	
1+2		>	
1+2+3		>	
1+2+3+4		>	

Notes: See notes to Table 1

Test **IOP 2**: $W(F) \geq W(G)$ iff $\sum_{i=1}^{k} q_i^F G_i(x) \geq \sum_{i=1}^{k} q_i^G F_i(x), \forall x \in [0, z], \forall k$

Table 4. Test (6) IOP2					
Graduation mark					
		North-Centre			
		1	1+2	1+2+3	1+2+3+4
South	1	$>^*$			
	1+2		$>^*$		
	1+2+3			$<^*$	
	1+2+3+4				$>^*$
Income after 3 years from graduation					
South	1	$<^*$			
	1+2		$<^*$		
	1+2+3			$<^*$	
	1+2+3+4				$<^*$
Income					
	1	$\neq^*$			
	1+2		$<^*$		
	1+2+3			$<^*$	
	1+2+3+4				$<^*$

Notes: See notes to Tables 1 and 2

Conclusions

Distributions of graduation marks:

- in South higher average marks and less overall inequality than in the North

- we reject EOp Hypotesis in both North and South: strong evidence of background effect on graduation marks, even conditioning for secondary school

- however, more evidence of equality of opportunities in the North than in the South.

- no dominance according to $\geq_{IOP1}$

- no dominance according to $\geq_{IOP2}$

Distributions of earnings (after 3 years and according to SWHI):

- in North higher average income, and less overall inequality than in the South

- we reject EOp Hypotesis in both North and South

- however, more evidence of equality of opportunities in the North than in the South.

- dominance of the North over the South according to the criterion $\geq_{IOP1}$

- *dominance* of the North over the South according to the criterion $\geq_{IOP2}$.

Complete rankings

The ex ante approach

Intuition: inequality between types is opportunity inequalty; inequality within types is effort inequality.

=> Measure opportunity inequality and decompose overall outcome inequality into opportunity inequality and effort inequality.

The ex post approach

Intuition: inequality within tranches is opportunity inequalty; inequality between tranches is effort inequality.

=> Measure opportunity inequality and decompose overall outcome inequality into opportunity inequality and effort inequality.

Different methods:

Non parametric (Checchi and Peragine 2005)**:** use well established results in inequality decomposition analysis

Parametric: need to impose a model of the relation $x = f(C, E)$

- linear models (Bourguignon et al. 2003)

- non linear models (Dardanoni et al. 2005, Pistolesi 2007)

Main advantage of parametric models: can study the partial effect of one (or a subset) of the circumstances variables, controling for the others.

Ferreira and Gignoux (2008) compare the two methods on data on six countries in Latin America.

The non parametric model: the ex ante approach

Overall income distribution X partitioned into n types according to C_i:

$$\begin{aligned} X &= (x_1, ..., x_N) \in \Re_+^N \\ X &= (\mathbf{x}_1, ..., \mathbf{x}_n) \in \Re_+^N \end{aligned}$$

where $\mathbf{x}_i$ is the type i income vector.

original distribution: $X = (\mathbf{x_1}, ..., \mathbf{x}_i, ..., \mathbf{x}_n) \in \Re^N_+$

smoothed distribution: $X^C_B = \left(\mu_{\mathbf{x_1}} \mathbf{1}_{N_1}, ..., \mu_{\mathbf{x}_i} \mathbf{1}_{N_i}, ..., \mu_{\mathbf{x}_n} \mathbf{1}_{N_n}\right) \in \Re^N_+$

standardized distribution: $X^C_W = (\tilde{\mathbf{x}}_\mathbf{1}, ..., \tilde{\mathbf{x}}_i, ..., \tilde{\mathbf{x}}_n) \in \Re^N_+$

-$\mu_{\mathbf{x}_i}$ is the average income of type i

-$\tilde{\mathbf{x}}_i$ is obtained by rescaling each type i income: $x^h_i \rightarrow \frac{\mu_X}{\mu_{\mathbf{x}_i}} x^h_i$

X^C_B eliminates within-types inequality $=>$ only opportunity inequality

X^C_W eliminates between-types inequality $=>$ only effort inequality

For a given measure of inequality $I : \mathbb{R}_+^N \rightarrow \mathbb{R}_+$, opportunity inequality is $I\left(X_B^C\right)$ or $OI_B^{ex-ante} = \frac{I\left(X_B^C\right)}{I(X)}$

Alternatively: opportunity inequality as a residual:$OI_W^{ex-ante} = 1 - \frac{I\left(X_W^C\right)}{I(X)}$

$OI_B^{ex-ante}$ and $OI_W^{ex-ante}$ can have different values: the result depends on the path of decomposition.

To have the same value, we need to use a "path independent" inequality measure (Foster and Shneyrov, 2000). Which measure?

The *mean logarithmic deviation* (I_0), the only index with a path-independent decomposition using the arithmetic mean as the group representative income.

Obtain: $OI_W^{ex-ante} = OI_B^{ex-ante}$

$$I_0(X) = I_0\left(X_W^C\right) + I_0\left(X_B^C\right)$$

Total income inequality = Effort inequality + Opportunity inequality.

The ex post approach

original distribution: (a) $X = \left(\boldsymbol{\chi}_1, ..., \boldsymbol{\chi}_p, ..., \boldsymbol{\chi}_m\right) \in \mathbb{R}_+^N$

smoothed distribution: (b) $X_B^E = \left(\mu_{\boldsymbol{\chi}_1^S} \mathbf{1}_{\frac{N}{m}}, ..., \mu_{\boldsymbol{\chi}_p^S} \mathbf{1}_{\frac{N}{m}}, ..., \mu_{\boldsymbol{\chi}_m^S} \mathbf{1}_{\frac{N}{m}}\right) \in \mathbb{R}_+^N$

standardized distribution: (c) $X_W^E = (\tilde{\mathbf{x}}_1, ..., \tilde{\mathbf{x}}_p, ..., \tilde{\mathbf{x}}_m) \in \mathbb{R}_+^N$

X_B^E eliminates within tranches inequality: $I\left(X_B^E\right)$ captures the inequality due effort.

X_W^E eliminates between tranches inequality: $I\left(X_W^E\right)$ captures the inequality due to circumstances, i.e., the inequality of opportunity.

$OI_W^{ex-post} = \frac{I\left(X_W^E\right)}{I(X)}$ or alternatively $OI_B^{ex-post} = 1 - \frac{I\left(X_B^E\right)}{I(X)}$

Use the *mean logarithmic deviation (MLD)* and obtain: $OI_W^{ex-post} = OI_B^{ex-post}$

$$I(X) = I\left(X_W^E\right) + I\left(X_B^E\right)$$

Extensions

Use ethical indeces based on social evaluation functions described before, both for ex ante and ex post approach.

Example 1: Use the utilitarian

$$W_F = \sum_{i=1}^{n} q_i^F \int u^i(x) f^i(x) dx$$

Use the specification $u^i(x) = \frac{x^{1-\epsilon_i}}{1-\epsilon_i}$ that satisfy the properties based on compensation and reward criteria.

Use an Atkinson-Kolm type inequality index and compute the between types inequality (see Fleurbaey 2008, Ch.9, and Li Donni, Peragine and Pignataro 2009 for an application to health and health care).

Example 2: Use the modified Yaari

$$W_{yaari}(F) = \sum_{i=1}^{n} q_i^F \int_0^1 U_i(p) F_i^{-1}(p) dp$$

and use Donaldson and Weymark (1980, 1983) specification $U_i(p) = \delta_i(1 - p)^{\delta_i - 1}$.

The problem of partial observability of circumstances

A "true" measure of opportunity inequality requires that all relevant circumstances be included in the vector C

This is unlikely to be the case in practice.

Consider the ex ante approach: the observed circumstances a subset of the true circumstances

The addition of a new circumstance $=>$ every type is further subdivided (at least two groups)

This cannot lower the between types inequality share.

$=>$ The estimates based on the decomposition procedure give lower bound estimates of inequality of opportunity.

The ex post approach: measure outcome inequality within tranches (effort classes).

Partial observability of circumstances does not affect the estimates of opportunity inequality.

Unless effort is proxied by the rank in the type outcome distribution.

The empirical application (Checchi and Peragine 2005)

Objective: separating circumstances and effort.
Data from SHIW (Bank of Italy), waves 1993, 1995, 1998 and 2000.

Individual outcome x: individual annual (gross) earnings

Circumstances c:
⇨ family backgrounds, measured as the highest educational attainment in the couple of parents

Populations defined by:
⇨ gender
⇨ location

Table 1 – Descriptive statistics – Gross earnings – Italy (SHIW) 1993–2000 – sample weights
first row: mean – second row: standard deviation – third row: observations

	North		Center–South		
Highest educational attainment among parents	man	woman	man	woman	Total
no formal education	19289.15	14189.12	14608.21	11156.24	14786.31
	6560.825	6340.921	6618.895	6544.096	7037.354
	329	185	1,313	514	2,341
primary school	19971.32	15037.77	17973.02	13821.12	17180.71
	8481.819	5689.796	9655.072	6161.756	8349.661
	2,138	1,664	3,140	1,702	8,644
lower secondary	21941.13	16457.4	19810.72	14915.97	18731.25
	9998.227	6244.776	9475.573	6019.18	8808.679
	808	703	829	577	2,917
upper secondary	23726.1	16703.06	21620.28	17624.39	20038.3
	13013.27	7562.451	12074.22	6931.528	10753.46
	497	441	478	472	1,888
bachelor	29017.76	22046.51	29536.29	16786.29	24080.08
	17783.94	9202.045	19780.29	6346.453	15268.1
	123	145	162	172	602
Total	20980.63	15780.95	18050.22	14274.43	17660.41
	9814.521	6455.006	10084.42	6583.299	9087.96
	3,895	3,138	5,922	3,437	16,392

Note: North includes Piemonte, Val d'Aosta, Liguria, Lombardia, Veneto, Friuli Venezia Giulia, Trentino Alto Adige and Emilia Romagna

Smoothing transformation $X \rightarrow X^S$: we partitioned the earnings distribution of each type into 10 quantiles, and replaced individual income with the average income of each cell (10 quantiles $\times$ 5 types of backgrounds $\times$ 2 gender, for each macroregion).

Table 2 – Mean earnings by “types” and “effort” and macro–regions – sample weights

first row: mean – second row: observations

region→	North										Centre-South									
gender→	men					women					men					women				
types→ quantiles↓	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	10166	10900	10042	8967	7892	5281	5591	6291	5170	6634	5024	6796	7790	7571	7259	3267	4169	4914	5020	6135
	33	214	82	50	14	19	167	72	45	15	134	317	85	49	17	54	172	58	48	18
2	14280	14065	14715	14647	15506	7797	9331	10353	9655	13138	8583	11871	12855	13204	15293	5044	6867	7686	10759	10191
	33	218	84	53	11	18	168	69	44	18	130	316	81	47	16	50	170	58	48	17
3	15289	15481	16205	16841	17384	10403	11751	12769	12825	15609	11241	13924	14746	15701	18549	6430	9926	10364	13765	14078
	34	210	77	47	12	19	175	72	46	12	131	311	83	48	16	58	169	58	46	17
4	16647	16661	17821	18832	18841	13117	13418	14473	14777	17791	13098	15169	16111	17421	21153	8189	12599	12713	15594	15918
	32	214	82	54	13	19	156	69	44	13	132	313	83	48	18	44	180	59	50	20
5	17712	17917	19103	20714	21106	14328	14583	15766	15997	19209	14475	16475	17564	18870	22989	10300	14260	14805	16773	16936
	35	215	79	49	12	20	172	70	42	15	130	319	83	50	14	51	173	56	44	16
6	19127	19180	20461	22345	26236	15316	15759	17022	17483	20342	15701	17755	19083	20773	26543	12523	15648	16163	18252	17945
	33	212	81	46	12	16	169	71	51	14	131	317	83	45	17	52	170	58	48	18
7	20292	20711	22476	24583	33093	16019	16812	18205	18443	22346	17164	19060	20702	23010	30507	13960	16948	17586	19654	19360
	31	214	81	49	13	21	160	74	39	17	134	305	84	49	16	51	167	57	55	15
8	21810	23116	25183	27805	42629	17213	18409	19633	20132	24280	18687	20777	22603	26412	38223	15508	18253	19250	21023	20539
	33	214	82	50	12	16	183	68	42	12	130	314	83	47	16	53	161	62	40	19
9	24356	27369	31343	34551	49441	18923	20007	21639	22844	30211	20889	23377	25845	31894	49251	17381	19780	20809	23017	22604
	33	214	83	50	12	19	148	68	44	16	130	315	82	48	16	50	171	54	46	16
10	33203	41053	44805	56949	68335	25358	25520	28583	32787	40792	27156	35277	40950	48004	85113	23541	24700	25143	29894	28072
	32	213	77	49	12	18	166	70	44	13	131	313	82	47	16	51	169	57	47	16

Rank ordering within each quantile is respected only starting from the third decile, in both regions and for both genders. Interpretations ?

The reduction in measured earnings inequality due to smoothing is limited.

Table 3 – Descriptive statistics: inequality measures

	entire sample	
Inequality measures	Actual gross earnings	Mean gross earnings (by region, sex, types and quantiles)
Relative mean deviation	0.16382	0.16344
Coefficient of variation	0.52627	0.47506
Standard deviation of logs	0.50935	0.47701
Gini coefficient	0.24517	0.24228
Theil index (GE(α),α=1)	0.11276	0.10233
Mean Log Deviation (GE(α),α=0)	0.11742	0.10692
Entropy index(GE(α), α=–1)	0.1678	0.13028
Half(Coeff.Var.squared)(GE(α),α=2)	0.13847	0.11283

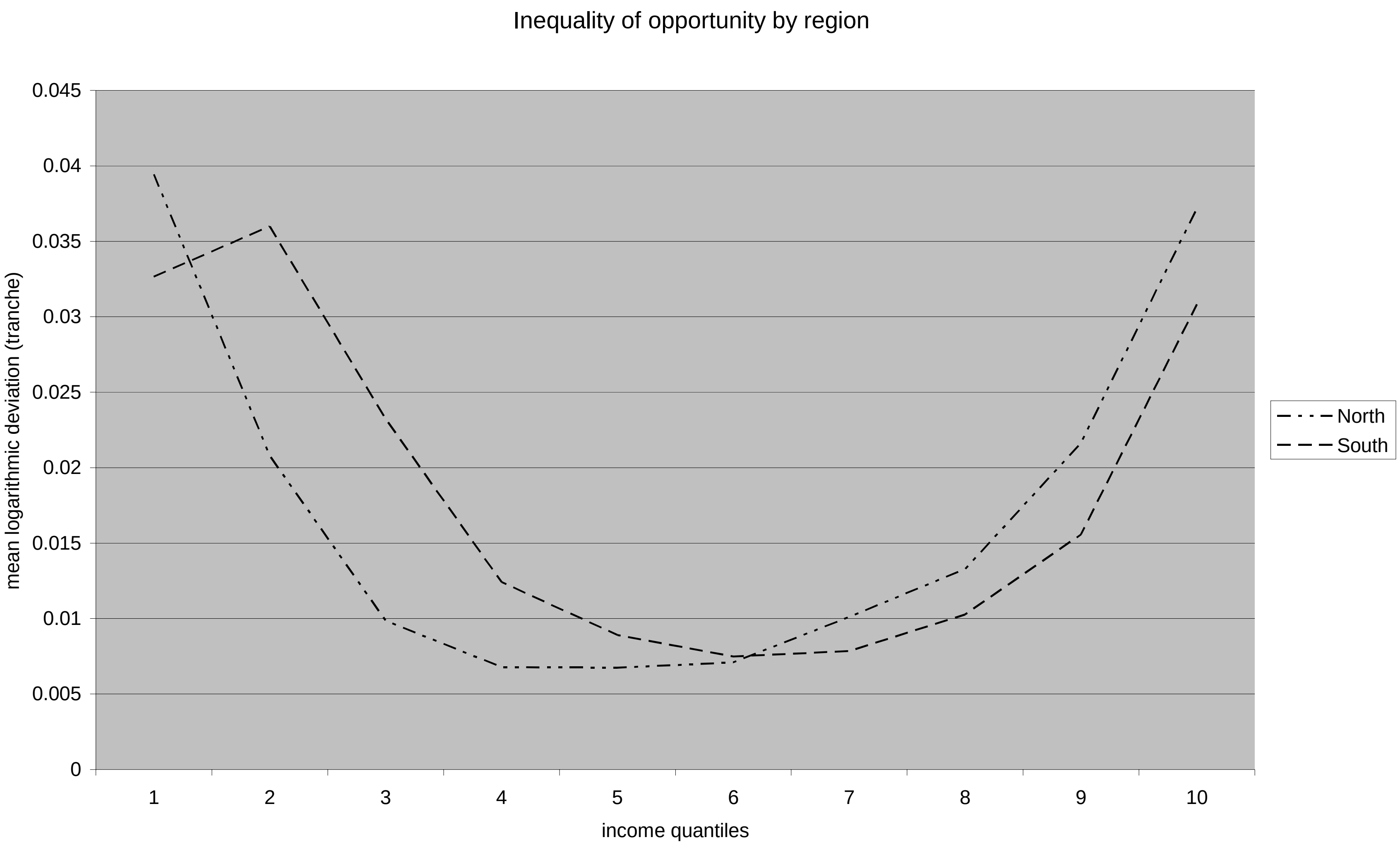
Measuring opportunity inequality

- opportunity inequality according to MLD: South is characterized by greater inequality of opportunity (even if lower in percentage terms), according to both the types and the tranches

Table 4 – Inequality decomposition, by macroegions – mean log deviation – "tranche" approach

	entire population				
	opportunity inequality	incidence % opportunity inequality	effort inequality	total inequality mean gross earnings (by region, sex, types and quantiles)	total inequality (actual gross earnings)
North	0.01729	18.0%	0.078869	0.096159	0.10669
Center-South	0.018518	16.6%	0.093169	0.111687	0.12218
Italy	0.035808	29.1%	0.087034	0.122841	0.11742

Figure 1 – Inequality of opportunity by macro regions – Italy (SHIW) 1993-2000



However there is a compositional effect: inequality of opportunities is much higher in South for both genders.

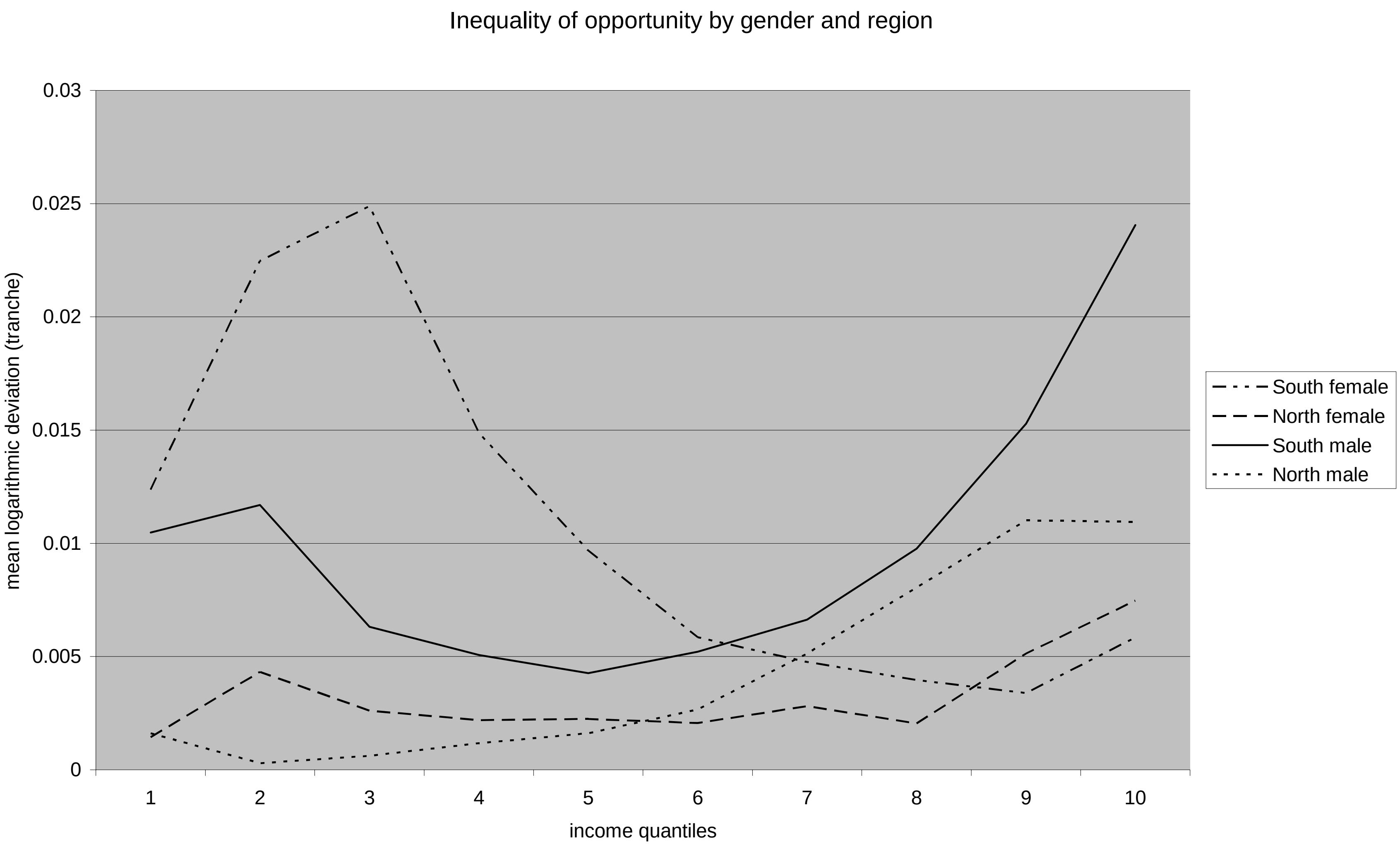
men

	opportunity inequality	incidence % opportunity inequality	effort inequality	total inequality mean gross earnings (by region, sex, types and quantiles)	total inequality (actual gross earnings)
North	0.004301	5.1%	0.080862	0.085163	0.09584
Center-South	0.009868	10.0%	0.088921	0.098789	0.10991
Italy	0.007659	8.2%	0.085723	0.093383	0.10795

women

	opportunity inequality	incidence % opportunity inequality	effort inequality	total inequality mean gross earnings (by region, sex, types and quantiles)	total inequality (actual gross earnings)
North	0.003213	3.9%	0.079776	0.082989	0.09335
Center-South	0.01087	9.3%	0.106614	0.117484	0.12685
Italy	0.007216	7.1%	0.093805	0.101021	0.11188

Figure 2 – Inequality of opportunity by gender and regions – Italy (SHIW) 1993-2000



The types approach is consistent with the findings.

Table 5 – Inequality decomposition, by macroegions – mean log deviation – "types" approach

	effort inequality	opportunity inequality	incidence % opportunity inequality	total inequality mean gross earnings (by region, sex, types and quantiles)
entire population				
North	0.091139	0.015549	14.57%	0.106688
Center-South	0.107079	0.015101	12.36%	0.12218
Italy	0.10024	0.015293	13.24%	0.115533
men				
North	0.091847	0.003993	4.17%	0.09584
Center-South	0.099836	0.010078	9.17%	0.109914
Italy	0.096666	0.007664	7.35%	0.10433
women				
North	0.09026	0.003091	3.31%	0.093351
Center-South	0.119559	0.007286	5.74%	0.126845
Italy	0.105576	0.005284	4.77%	0.11086

Standard regression gives consistent results and helps us to understand our findings:

⇨ while most of the parental background exerts its effect through favouring the educational attainment of the children in the North, it keeps on playing a role independently from education in the South.

⇨ gender penalty is lower in the North once we control for education (need to correct for self-selection into employment)

Table 6 – Determinants of earnings – Italy (SHIW) 1993–2000

OLS – robust t–statistics in brackets – * significant at 10%; ** significant at 5%; *** significant at 1%

	1	2	3	4
	north	centre–south	north	centre–south
female	-0.255	-0.22	-0.278	-0.277
	(20.09)***	(16.07)***	(22.89)***	(21.00)***
part-time	-0.814	-1.061	-0.746	-0.97
	(25.41)***	(33.08)***	(23.79)***	(31.02)***
potential experience	0.061	0.059	0.06	0.056
	(17.75)***	(18.18)***	(17.62)***	(17.52)***
potential experience squared	-0.001	-0.001	-0.001	-0.001
	(15.92)***	(16.19)***	(12.72)***	(11.78)***
parent primary (isced 1)	0.067	0.235	0.002	0.133
	(3.22)***	(13.19)***	-0.11	(8.09)***
parent lower secondary (isced 2)	0.181	0.368	0.049	0.172
	(7.47)***	(16.66)***	(2.09)**	(8.24)***
parent upper secondary (isced 3)	0.265	0.518	0.051	0.209
	(9.60)***	(20.24)***	(1.87)*	(8.22)***
parent tertiary (isced 4-5-6)	0.389	0.67	0.043	0.288
	(8.93)***	(17.27)***	-0.98	(7.67)***
completed years of education			0.058	0.065
			(28.53)***	(33.84)***
Observations	7033	9357	7033	9357
R^2	0.32	0.31	0.4	0.39

Note: constant and survey year dummies included.

Dependent variable is the log of gross labour earnings for dependent employees.