Measuring Long-term Inequality of Opportunity

Rolf Aaberge, Magne Mogstad and Vito Peragine

Abstract

In this paper, we introduce and apply a general framework for evaluating *long-term* income distributions according to the Equality of Opportunity principle. Our framework allows for both an ex-ante and an ex-post approach to EOp. Our ex-post approach relies on a permanent income measure defined as the minimum annual expenditure an individual would need in order to be as well off as he could be by undertaking inter-period income transfers. There is long-term ex-post inequality of opportunity if individuals who exert the same effort have different permanent incomes. In comparison, the ex-ante approach focuses on the expected permanent income for individuals with identical circumstances. Hence, the ex-ante approach pays attention to inequalities in expected permanent income between different types of individuals. To demonstrate the empirical relevance of a long-run perspective on EOp, we exploit a unique panel data from Norway on individuals' incomes over their working lifespan.

Keywords: equality of opportunity, social welfare, inequality, permanent income, intertemporal choice, ex-ante, ex-post

JEL classification: D71, D91, I32

Acknowledgement: We would like to acknowledge the Norwegian Research Council for financial support.

1. Introduction

Over the last decades, increasing discontent has been expressed with distributional analysis based on observations of income for a single year. The line of reasoning is that high annual inequality might occur side by side with little or no inequality in long-term incomes, if individuals' positions in the annual income distributions change over time. This has led to a spur of research on inequality and social welfare in long-term or permanent income according to the Equality of Outcome (EO) principle.[1] At the same time, a large and growing literature has addressed the problem of evaluating income distributions according to equality of opportunity (EOp); in all the existing contributions, however, snapshots of income still form the basis of analysis.[2] The aim of this paper is to bridge these two strands of the literature by introducing and applying a general framework for evaluating *long-term* income distributions according to the EOp principle.[3]

We follow the basic idea of the EOp literature closely in assuming that individuals' outcomes arise from two classes of variable: Variables for which they should not be held responsible for (circumstances) and variables which belong to the sphere of individuals' responsibility (effort). Once this basic partition has been made, the concept of EOp can be decomposed into two distinct ethical principles: *The Compensation Principle*, which states that differences in outcomes due to circumstances are ethically inacceptable and should be compensated; and *the Reward Principle*, which states that differences due to effort are to be considered ethically acceptable and do not justify any intervention.

The existing literature on the measurement of opportunity inequality (see, among others, Bourguignon et al. 2003, Checchi and Peragine 2009, Dardanoni et al. 2006, Ferreira and Guignoux 2008, Lefranc et al. 2006a,b, Peragine 2002, 2004, Peragine and Serlenga 2008), has explored two main approaches to measure opportunity inequality, namely the ex-ante and the ex-post approach[4]. The two approaches give different definitions of EOp and embody the ethical principles illustrated above in different ways.

[1] See e.g. Shorrocks (1978), Chakravarty et al. (1985), Maasoumi and Trede (2001), and Aaberge and Mogstad (2009).

[2] A notable exception is Bourguignon et al. (2007), which will be discussed in detail below.

[3] We refer to the conceptualization of EOp proposed by philosophers such as Dworkin (1981a,b), Arneson (1989), Cohen (1989) and on the economic literature - initiated by Roemer (1993, 1998) - that has explored different ways in which the concept of equality of opportunity may be translated in formal economic models (see Fleurbaey 2008 for a survey).

[4] Notice that the ex ante and the ex post approaches can be incompatible (see Fleurbaey and Peragine 2009).

According to the ex-ante approach, there is equality of opportunity if the set of opportunities is the same for all individuals, regardless of their circumstances. Inequality of opportunity is reduced if inequality between individual opportunity sets decreases. This approach partitions the population into different types, where each type is formed by individuals endowed with the same set of circumstances. The type-specific outcome distribution is interpreted as the opportunity set of individuals with the same circumstances. Accordingly, it focuses on inequality between types, and is neutral with respect to inequality within types.[5]

In comparison, the ex-post approach states that there is equality of opportunity if all those who exert the same effort have the same outcome. Inequality of opportunity decreases if outcome inequality decreases among the individuals at the same degree of effort. The ex post approach emphasizes inequalities within groups of individuals at the same effort levels, so-called tranches. Differences between tranches are interpreted as due to individual effort, and are considered as fair.[6]

Our framework for evaluating long-term income distributions allows for both an ex-ante and an ex-post approach to EOp. Our ex-post approach relies on a permanent income measure defined as the minimum annual expenditure an individual would need in order to be as well off as he could be by undertaking inter-period income transfers. There is long-term ex-post inequality of opportunity if individuals who exert the same effort have different permanent incomes. In comparison, the ex-ante approach focuses on the expected permanent income for individuals with identical circumstances. Hence, the ex-ante approach pays attention to inequalities in expected permanent income between different types of individuals.

Our EOp framework requires aggregation in two steps. The first step consists of aggregating the income stream of each individual into an interpersonal comparable measure of permanent income. To this end, we draw on intertemporal choice theory and use a measure of permanent income which incorporates the costs of and constraints on making inter-period income transfers. The second step consists of aggregating individuals' permanent incomes into EOp measures of social welfare and inequality. Specifically, we employ an axiomatic approach to justify the introduction of a generalized family of rank-dependent measures of ex post as well as ex ante opportunity inequality and social welfare. Our long-term perspective serves to highlight that inequality opportunities may accumulate over time, if (dis)advantages beget (dis)advantages. Moreover, it allows for the possibility that groups viewed as (dis)advantaged based on snapshots of income may change positions over time. In such

[5] This is the approach proposed, in different framework, by Van de Gaer (1993), Kranich (1996), Peragine (2004a).

[6] This is the approach proposed by Roemer (1993, 1998) and used by Roemer et al. (2003), Peragine (2004b), Aaberge and Colombino (2009) and Checchi and Peragine (2009)

cases, policies that minimize snapshots of opportunity inequality may deviate from those optimizing equality of opportunity in long-term income.

To demonstrate the empirical relevance of a long-run perspective on EOp, we exploit a unique panel data from Norway on individuals' incomes over their working lifespan. This allows us to examine how well analyses of inequality and social welfare based on snapshots of income approximate the results based on long-term income. Further, we investigate how sensitive distributional comparisons between the EOp and the EO principle of justice are to the choice of accounting period of income. Finally, we assess to what extent the ex-ante and the ex-post approaches yield different pictures of opportunity inequality and social welfare.

Our paper proceeds by clarifying the differences between our framework and the method for measuring long-term EOp proposed by Bourguignon et al. (2007). Section 2 describes our method for aggregating the income streams of individuals into comparable measures of permanent income. Section 3 deals with the problem of aggregating permanent incomes across individuals into measures of long-term inequality and social welfare. Section 4 introduces a new family of rank-dependent EOp measures of inequality and social welfare. Section 5 reports the empirical results, before Section 6 concludes.

1.1 Long-term equality of opportunity: the order of aggregation

To model long-term EOp we can think of two different approaches. The first, which is more in line with an ex ante perspective, is based on the following idea: First divide individuals into types who are homogeneous with respect to exogenous circumstances at a given point in time; the income distributions of a type i, at a given time t, call it $F_{it}(x)$, represents the opportunity set open to any individual belonging to type i at time t. Then aggregate the type evaluations across time periods, obtaining a long-term expression of the opportunity sets. Finally, evaluate the distribution of long-term opportunity sets.

This is essentially the approach proposed by Bourguignon et al. (2007): They first aggregate individuals in order to evaluate the opportunity set at a given time t. Successively they introduce some criteria to rank different distributions of time streams of opportunity sets in terms of equality of opportunity. This approach is in fact declined in two different variants.

In a first approach, Bourguignon et al. (2007, p. 243) define EOp as follow: There is EOp if, in each period, there is not dominance between types. Hence, in each period of time, there should be equality or, as a weaker definition, non-dominance of opportunity sets. This can be seen as the natural extension of the approach used by Lefranc et al (2006) and Peragine and Serlenga (2008) to a long-term perspective.

Alternatively, Bourguignon et al. (2007) propose a long-term extension of the utilitarian version of EOp, according to which there is EOp if the long-term expected value of all types are equal. Hence, they first introduce a type evaluation function, according to which each type distribution (i.e., opportunity set) at a given period t is evaluated by the mean μ_{it} (this is the utilitarian evaluation); successively, they introduce *time aggregator* in order to obtain an expression of the long-term value of the opportunity set: For instance, $\phi_t(\mu_{it})$ where ϕ_t is a selected aggregator over time. Finally, they propose to maximize the minimum of the distribution of long.term type means. Hence, they use the following social welfare function:

$$W(F) = \min_i \int_T \phi_t \mu_{it} dt$$

This approach is coherent with an ex ante view of opportunity inequality and can be used if one is mainly interested in the inequality between types of individuals with the same circumstances.
On the other hand, their methodology, given the order of aggregation, is unable to model in a satisfactory way the individual distribution of income over time. In fact, in their methodology the time aggregation is performed over opportunity sets, rather than on individual outcomes.

As discussed in detail below, we take a different approach to long-term EOp. In our methodology, first the aggregation over time is performed for each individual. This allows us to be specific with respect to the underlying model of intertemporal choice. Successively, we obtain a distribution of long-term incomes: To such distribution we apply a partition into types and, hence, propose different models of equality of opportunity.

Our approach, in addition to being more satisfactory with respect to modelling the individual valuation of long-term income, is more general in that it allows to adopt either an ex ante or an ex post approach to long-term EOp. In the ex ante (utilitarian) approach, we define EOp as equality in the expected permanent income for individuals with identical circumstances, which coincides with the approach of Bourguignon et al. (2007) if and only if the mean permanent income for each type can be expressed as a weighted average of the period-specific mean incomes. In the ex post approach, we define EOp as equality of outcomes for individuals with the same effort, where both the outcome and the effort refer to a long-term perspective rather than to a snapshot. Moreover, both in the ex ante and

in the ex post approach, our framework allows for any position when it regards attitude towards inequality aversion.

2. Measurement of permanent income

This section outlines the permanent income introduced by Aaberge and Mogstad (2009), which forms the basis for our EOp measures. This permanent income measure had the advantage of conforming to the basic structure of intertemporal choice theory and justifies comparison of permanent incomes across individuals. We will first consider the case of a perfect credit market, before allowing for credit market imperfections.

2.1 Permanent income definition

Aaberge and Mogstad (2009) defines permanent income as the minimum annual expenditure an individual would need in order to be as well off as he could be by undertaking inter-period income transfers. To justify interpersonal comparison of individuals' income, they follow standard practice in assuming that inter-period income transfers are carried out in accordance with an intertemporal utility function that is common to all individuals. The common utility function is to be determined by the social planner based on his ethical value judgement, contains within it interpersonal comparability of both welfare levels and welfare differences, and can be viewed as a normative standard where individuals are treated symmetrically.[7]

Specifically, the social planner is assumed to employ the conventional discounted utility model with perfect foresight, where preferences are intertemporal separable and additive. Let $(C_1,C_2,..,C_T)$ and $(Y_1,Y_2,..,Y_T)$ be an individual's stream of consumption levels and exogenous real disposable incomes net of interests for an individual. Under the assumption of a perfect credit market, the real interest rates on savings and borrowing are equal across the population, though they may vary over time. Let r_t denote the real interest rates on income-transfers from period t-1 to t. From the viewpoint of the social planner, the individual's preferred consumption profile $(C_1^*, C_2^*,..., C_T^*)$ is defined as the solution of

$$U = \max_{C_{i1},...,C_{iT}} \sum_{t=1}^{T} u(C_t)(1+\delta)^{1-t} \tag{2.1}$$

subject to the budget constraint[8]

[7] The use of a common utility function is well-established in the public economic literature and has e.g. been proposed by Deaton and Muellbauer (1980) and Hammond (1991). It also forms the basis for the definition and measurement of a money-metric measure of utility in for example King (1983) and Aaberge et al. (2004).

[8] It is straightforward to extent the budget constraint to account for wealth, e.g. by assuming that the income in the first period Y_{i1} in (2.2) includes the initial stock of wealth.

(2.2) $$\sum_{t=1}^{T} C_t \prod_{j=1+t}^{T}(1+r_j) + C_T = \sum_{t=1}^{T} Y_t \prod_{j=1+t}^{T}(1+r_j) + Y_T.$$

The preferred consumption level in period *t*, C_t^*, can be expressed as a function f_t of $\delta, Y_1, Y_2, .., Y_T$, and $r_2, r_3, .., r_T$,

(2.3) $$C_t^* = f_t(\delta, Y_1, Y_2, ..., Y_T, r_2, r_3, ..., r_T) \text{ for all } t=1, 2, \ldots, T.$$

By inserting for (2.3) in (2.1) the maximum utility level (U) is given by

(2.4) $$U \equiv \sum_{t=1}^{T} u(C_t^*)(1+\delta)^{1-t}.$$

The permanent income measure is then defined as

(2.5) $$Z = u^{-1}(\Delta^{-1}U),$$

where $u^{-1}(t) = \inf\{x : u(x) \geq t\}$ is the left inverse of u and Δ is defined by

(2.6) $$\Delta = \sum_{t=1}^{T}(1+\delta)^{1-t} = \frac{1+\delta}{\delta}\left(1-\left(1+\delta\right)^{-T}\right).$$

Because the individual-specific Zs can be considered to be comparable money-metric measure of the well-being level associated with the income stream for a given individual, the distribution of the permanent incomes forms an appropriate basis for studying long-term inequality and social welfare. In particular, the social planner considers the income stream ($Y_{i1}, Y_{i2}, .., Y_{iT}$) of individual i to be preferable to the income stream ($Y_{j1}, Y_{j2}, .., Y_{jT}$) of individual j if and only if Z_i exceeds Z_j.

As shown in Aaberge and Mogstad (2009), Z encompasses standard measures of permanent income, depending on the assumptions made by the social planner about the intertemporal preferences of individuals and the functioning of the credit market. By assuming that the rates of time preferences and the real interest rates are equal to zero in each period, that is, $r_2 = r_3 = \cdots = r_T = \delta = 0$, we get that

Z is equal to the average (real) income, $(1/\mathrm{T})\sum_{\mathrm{t=1}}^{\mathrm{T}} \mathrm{Y_t}$. In Shorrocks (1978) as well as in most subsequent empirical studies of long-term inequality and income mobility according to the EOp principle, the average income is used as an approximation for permanent income. When $r_2 = r_3 = \cdots = r_T = \delta$, it follows directly that Z coincides with the annuity of the income stream, $\sum_{t=1}^{T}(1+\delta)^{T-t} Y_t / \sum_{t=1}^{T}(1+\delta)^{T-t}$ (when T is the basis for the annuity calculations). A benchmark case in intertemporal choice theory uses the annuity of the income stream as measure of permanent income (see e.g. Meghir, 2004). Further, Z coincides with the utility-equivalent annuity introduced by Nordhaus (1973) if

$$u(x) = \begin{cases} \dfrac{1}{1-\varepsilon}\left(x^{1-\varepsilon} - 1\right), & \varepsilon \neq 1 \\ \log x, & \varepsilon = 1. \end{cases} \tag{2.7}$$

That is, u is specified as a member of the Bergson family, which is a much used specification of the instantaneous utility function in intertemporal choice theory (Davies and Shorrocks, 2000).

2.2. Imperfect credit marked

In practice, the interest rates on borrowing and savings differ, which implies that (2.2) is no longer a valid representation of the budget constraints. In this case, the preferred consumption levels given by (2.3) will not form an appropriate basis for defining and measuring permanent income. Instead, we can apply the Kuhn-Tucker method to derive the conditional and preferred consumption profiles.

Suppose that each individual is in each period faced with a single borrowing interest rate and a single savings interest rate (but different individuals may face different interest rates on borrowing and/or savings). If there are no liquidity constraints, the preferred consumption profile ($C_1^*, C_2^*, ..., C_T^*$) is defined as the solution of (2.1) subject to the budget constraints

$$\begin{aligned} \mathrm{S_0} &= 0 \\ \mathrm{S_t} &= (1 + \mathrm{r}\gamma_\mathrm{t})\mathrm{S_{t-1}} + \mathrm{Y_t} - \mathrm{C_t}, \\ \mathrm{S_T} &= (1 + \mathrm{r}\gamma_\mathrm{t})\mathrm{S_{T-1}} + \mathrm{Y_T} - \mathrm{C_T} = 0 \end{aligned} \tag{2.8}$$

where S_t represents the assets at the end of period t earning an interest rate $r\gamma_{t+1}$, and

(2.9) $$r\gamma_t = \begin{cases} rs_t & if\ S_{t-1} \geq 0 \\ rb_t & if\ S_{t-1} < 0 \end{cases}, -1 < r\gamma_t < \infty, \quad t = 2,3,...,T,$$

where rs_t and rb_t denote the saving and borrowing rates. Solving this maximization problem requires comparison of 3^{T-1} conditional consumption profiles for each individual. The consumption profiles are distinctive in terms of whether individual *i* in the various periods is considered to be a saver, a borrower, or locate at the kink and thereby consume all his assets. Each of these conditional consumption profiles is a candidate for the individual's preferred consumption profile provided that the budget constraints are satisfied for the given values of Y_t and $r\gamma_t$. The optimal consumption profile is determined as the utility maximising choice among the conditional consumption profiles satisfying their respective budget constraints. By inserting the consumption levels of the optimal consumption profiles into (2.4), the corresponding *Z* is obtained from (2.5).[9]

3 Rank-dependent measures of inequality

Empirical analyses of inequality in income distributions are conventionally based on the Lorenz curve and associated summary measures of inequality, which means that concern is directed towards income shares without taking account of differences in mean incomes. To summarize the information content of the Lorenz curve and to achieve rankings of intersecting Lorenz curves the standard approach is to employ the Gini coefficient in combination with one or two inequality measures from the Atkinson family or the Theil family. However, since the Gini coefficient and Atkinson's and Theil's measures of inequality have distinct theoretical foundations it is difficult to evaluate their capacity as complementary measures of inequality. An alternative approach, which will be chosen in this paper, is to use measures of inequality which can be shown to have a common theoretical justification as criteria for ranking Lorenz curves.

The Lorenz curve L for a cumulative income distribution *F* with mean μ is defined by

(3.1) $$L(u) = \frac{1}{\mu}\int_0^u F^{-1}(t)dt, \quad 0 \leq u \leq 1,$$

where $F^{-1}(t) = \inf\{x : F(x) \geq t\}$ is the left inverse of *F*. Thus, the Lorenz curve *L(u)* shows the share of total income received by the poorest 100 *u* per cent of the population. Note that *F* can either be a

[9] Presence of liquidity constraints will reduce the number of available conditional consumption profiles that have to be compared. For example, the case where borrowing in each period is prohibited corresponds to reducing the number of conditional consumption profiles to those satisfying $S_t \geq 0$. Thus, deriving Z subject to liquidity constraints is straightforward and can be considered as a special case of the method outlined above.

discrete or a continuous distribution function. Although the former is what we actually observe, the latter often allows simpler derivation of theoretical results and is a valid large sample approximation. Below F will be assumed to be a continuous distribution function.

Under the restriction of equal mean incomes the problem of ranking Lorenz curves formally corresponds to the problem of choosing between uncertain prospects. This relationship has been utilized by e.g. Atkinson (1970) to characterize the criterion of non-intersecting Lorenz curves in the case of distributions with equal mean incomes. This was motivated by the fact that in cases of equal mean incomes the criterion of non-intersecting Lorenz curves is equivalent to second-degree stochastic dominance, which means that the criterion of non-intersecting Lorenz curves obeys the Pigou-Dalton principle of transfers. To perform inequality comparisons with Lorenz curves we can deal with distributions with equal means, or alternatively simply abandon the assumption of equal means and consider distributions of relative incomes.[10] The latter approach normally forms the basis of empirical studies of income inequality and will be used in this paper.

Let $\boldsymbol{L}$ denote the family of Lorenz curves, and let a social planner's ranking of members of $\boldsymbol{L}$ be represented by a preference ordering $\succeq$, which will be assumed to satisfy the following basic axioms:

Axiom 1 (Order). *$\succeq$ is a transitive and complete ordering on $\boldsymbol{L}$.*

Axiom 2 (Dominance). *Let $L_1, L_2 \in \boldsymbol{L}$. If $L_1(u) \geq L_2(u)$ for all $u \in [0,1]$ then $L_1 \succeq L_2$.*

Axiom 3 (Continuity). *For each $L \in \boldsymbol{L}$, the sets $\left\{L^* \in \boldsymbol{L} : L \succeq L^*\right\}$ and $\left\{L^* \in \boldsymbol{L} : L^* \succeq L\right\}$ are closed (w.r.t. L_1-norm).*

Given the above continuity and dominance assumptions for the ordering $\succeq$, Aaberge (2001) demonstrated that the following axiom,

Axiom 4 (Independence). *Let L_1, L_2 and L_3 be members of $\boldsymbol{L}$ and let $\alpha \in [0,1]$. Then $L_1 \succeq L_2$ implies $\alpha L_1 + (1-\alpha)L_3 \succeq \alpha L_2 + (1-\alpha)L_3$,*

characterizes the rank-dependent family of inequality measures $\tilde{J}_p$ defined by[11]

[10] See e.g. Smith's (1979) discussion of necessities and Sen's (1992) discussion of relative deprivation.

[11] Mehran (1976) introduced an alternative version of (3.2) based on descriptive arguments. For alternative motivations of the J_P-family and various subfamilies we refer to Donaldson and Weymark (1980, 1983), Weymark (1981), Yitzhaki (1983), Ben Porath and Gilboa (1994), and Aaberge (2000, 2007).

(3.2) $$J_p(L) = 1 + \int_0^1 L(v)dp(v) = 1 - \frac{1}{\mu}\int_0^1 p(v)F^{-1}(v)dv\,,$$

where p is a positive and non-increasing weight-function defined on the unit interval such that $\int vdp(v) = -1$. Note that p can be interpreted as a preference function of a social planner that assigns weights to the incomes of the individuals in accordance with their rank in the income distribution. Therefore, the functional form of p reveals the attitude towards inequality of a social planner who employs J_p to judge between Lorenz curves.

As was recognized by Ebert (1987) the justification of the social welfare function associated with J_p can be made in terms of value judgement of the trade-off between the mean and (in)equality in the distribution of income

(3.3) $$W_p = \mu(1 - J_p) = \int_0^1 p(v)F^{-1}(v)dv\,.$$

Note that W_p can be interpreted as the equally distributed (equivalent) level of income. An alternative axiomatic justification for using the latter term of (3.3) as a criterion for ranking distribution functions F rather than as a trade-off between mean and equality was provided by Yaari (1987, 1988). Note that the normative justification for the family of rank-dependent measures of inequality J_p is analogous to the justification for Atkinson's expected utility type of inequality measures. The essential differences between these two approaches for measuring inequality and social welfare arise from the independence axioms. Whilst the expected utility independence axiom requires that the ordering of distributions of individual welfare is invariant with respect to identical mixing of the distributions being compared, the rank-dependent independence axiom requires that the ordering is invariant with respect to identical mixing of Lorenz curves (or identical mixing of the inverses of distributions) being compared. For further discussion, see Yaari (1988) and Aaberge (2001).

By specifying $p(v) = 2(1 - v)$ we find that J_p becomes equal to the Gini coefficient. To complement the information provided by the Gini coefficient it appears attractive to use members of the rank-dependent family $C_k = J_{p_k}, k = 1,2,...,$ where

(3.4) $$p_k(v) = \begin{cases} -\log v, & k = 1 \\ \dfrac{k}{k-1}\left(1 - v^{k-1}\right), & k = 2,3,.... \end{cases}$$

As indicated by Aaberge (2000, 2007) the family $\{C_k : k = 1,2,...\}$ uniquely determines the Lorenz curve L. Thus, by restricting to this family of inequality measures no information is lost. However, in applied work it is for practical reasons convenient to employ a few measures of inequality. To this end, Aaberge (2007) draws on standard statistical practice to justify the use of C_1 (the Bonferroni coefficient), C_2 (the Gini coefficient) and C_3 as a basis for summarizing the inequality information in an income distribution and the associated social welfare functions W_1, W_2 and W_3 to assess the trade-off between efficiency and (in)equality. Moreover, these three measures of inequality also prove to supplement each other with regard to sensitivity to transfers at the lower, the central and the upper part of the income distribution.

4 An analytical framework for measuring inequality of opportunity

This section introduces the generalized family of rank-dependent measures of ex post as well as ex ante opportunity inequality and social welfare. The proposed framework are valid for measuring short-term as well as long-term EOp, depending on whether we let snapshots of income or permanent income form the basis.

We have a continuum of individuals. Each individual is completely described by a list of traits, which can be partitioned into two different classes: traits or factors beyond the individual control, represented by a person's vector of circumstances c belonging to an abstract and finite set $\Omega = \{\mathrm{c}^1, \ldots, \mathrm{c}^n\}$; and factors for which the individual is fully responsible, represented by a scalar variable[12] e (effort), $e \in \Theta \subseteq \Re_+$. We assume that the circumstances of an individual are constant over time: this implies that the type partition is constant over time.[13]

Income is generated by a function $g : \Omega \times \Theta \to \Re_+$:

$$g = (c, e) \tag{4.1}$$

Hence g is a "production function" that relates an individual's future income to her effort and her circumstances. We do not know the form of the function g; we know, however, that it is increasing in *e* and the same for all individuals. A society income distribution is represented by a cumulative distribution function $F : \Re_+ \to [0,1]$.

[12] In the analysis it could equally well be represented as a vector.

[13] The same assumption is made by Bourguignon et al. (2007).

We can partition any given population into n subpopulations, each representing a class identified by the variable c. For $c_i \in \Omega$, we call "type" i the set of individuals whose set of circumstances is c_i. Within type i there will be a cumulative distribution of income F_i and a population share denoted by q_i so that $F(x) = \sum_{i=1}^{m} q_i F_i(x)$. A second partition is based on the responsibility e: For all degrees of responsibility $e \in \Theta$ we call *tranche* e the set of individuals whose responsibility level is equal to e. We can observe c but not e. Given $F_i(x),\ i = 1,\ldots n$, with $F_i^{-1} : [0,1] \to \Re_+$ we denote $F_i^{-1}(s) = \inf\{x : F_i(x) \geq s\}$, which represents the income of the person whose rank in the distribution F_i is *s*, the left continuous inverse distribution of F_i.

Roemer (1993, 1998) suggests to take, as an inter-type comparable measure of effort, the quantile of the effort distribution in the type at which an individual sits; this, given the monotonicity of the income function, will correspond to the quantile in the income distribution of the type. We adopt this solution and hence we say that all individuals at the *s* quantile of their respective type income distributions have the same effort. Thus, considering types $1,\ldots,n$, we define the *tranche s* as the subset of individuals whose incomes are at the s^{th} quantile of their respective type income distributions.

4.1 Ex post approach

Let $\tilde{F}_1^{-1}(s) \leq \tilde{F}_2^{-1}(s) \leq \ldots \leq \tilde{F}_m^{-1}(s)$ be the ordering of incomes $F_1^{-1}(s), F_2^{-1}(s), \ldots, F_m^{-1}(s)$ across types at trance (quantile) *s*. Since the type-specific income distributions might intersect note that the type ordering by income might change across quantiles; i.e. $\tilde{F}_k^{-1}(s)$ and $\tilde{F}_k^{-1}(v)$ might represent different types. Accordingly, the proportion of people associated with the lowest income, the second lowest income, etc might change across quantiles. Thus, let $q_i(s)$ be the population share associated with the individual having rank *i* at trance *s*. Moreover, let $a_j(s) = \sum_{i=1}^{j} q_i(s)$ and $b_j(s) = 1 - a_j(s)$.

The quantile-specific mean and Lorenz curve are defined by

$$\mu(s) = \sum_{i=1}^{m} q_i(s)\tilde{F}_i^{-1}(s) = \sum_{i=1}^{m} q_i F_i^{-1}(s) \tag{4.2}$$

and

(4.3) $$L(a_j(s);s)=\frac{\sum_{i=1}^{j} q_i(s)\tilde{F}_i^{-1}(s)}{\mu(s)}, \quad j=1,2,...,m, \quad 0\le s\le 1.$$

Inserting (4.2) and (4.3) in (3.2) yields the following family of *quantile-specific rank-dependent measures of inequality of opportunity*

(4.4) $$J_p(s)=1-\frac{1}{h(s)\mu(s)}\sum_{j=1}^{m} q_j(s)p(a_j(s))\tilde{F}_j^{-1}(s),$$

where $h(s)=\sum_{j=1}^{m} q_j(s)p(a_j(s))$, $p(1)=1$ and $\frac{1}{m-1}\sum_{j=1}^{m} p(a_j)=1$.

Thus, $J_p(s)$ measures income inequality due to different circumstances for those who belong to tranche *s* and thus have exerted the same degree of effort. According to the *Compensation Principle*, $J_p(s)$ can be interpreted as the inequality of opportunity in tranche *s*. In order to define an overall measure of inequality of opportunity a method for aggregating the tranche-specific inequality measures is required. To this end, we rely on the *Principle of Utilitarian Reward* introduced by Fleurbaey (2008), which states the social irrelevance of inequalities due to differences in effort. This amounts to say that any "equalizing transfer" between tranches should not change income inequality or social welfare, whatever the effort level of the persons involved in it. That is, an equal weight should be assigned to each tranche-specific inequality measure, whatever degree of effort has been exerted. Accordingly, the average of the tranche-specific inequality of opportunity measures

(4.5) $$J_p=\int_0^1 J_p(s)ds$$

defines a family of rank-dependent measures of *overall inequality of opportunity*. Thus, inequality of opportunity decreases if outcome inequality decreases among the individuals with the same degree of effort.

The associated social welfare function is given by

(4.6) $$W_p=\mu(1-J_p)$$

where $\mu_i = \int_0^1 F_i^{-1}(t)dt$ and $\mu = \int_0^1 \mu(s)ds = \sum_{i=1}^m q_i \mu_i$.

Note that p_e defined by

(4.7) $$p_e(a_j(s)) = \begin{cases} 1, & j = 1 \\ 0, & j = 1,2,...,m \end{cases}$$

represents the upper limit of inequality aversion exhibited by the family of non-increasing weight functions *p*. By inserting (4.7) in (4.4), (4.5) and (4.6) we get

(4.8) $$J_e(s) = 1 - \frac{\tilde{F}_1^{-1}(s)}{\mu(s)} = 1 - \frac{\min_{1 \le i \le m} F_i^{-1}(s)}{\mu(s)},$$

(4.9) $$J_e = 1 - \int_0^1 \frac{\min_{1 \le i \le m} F_i^{-1}(s)}{\mu(s)} ds$$

and

(4.10) $$W_e = \mu \int_0^1 \frac{\min_{1 \le i \le m} F_i^{-1}(s)}{\mu(s)} ds .$$

In the Gini case we get, since $\sum_{j=1}^m q_j(s)a_j(s) = \sum_{j=1}^m q_j a_j$ and $b_j = 1 - a_j$, that

(4.11) $$G(s) = 1 - \frac{\sum_{j=1}^m q_j(s) b_j(s) \tilde{F}_i^{-1}(s)}{\mu(s) \sum_{j=1}^m q_j b_j},$$

and

(4.12) $$G = \int_0^1 G(s)ds = 1 - \frac{\int_0^1 \left[\sum_{j=1}^m q_j(s) b_j(s) \frac{\tilde{F}_i^{-1}(s)}{\mu(s)} \right] ds}{\sum_{j=1}^m q_j b_j}$$

4.2 Ex ante approach

The ex ante approach puts special emphasis on the differences in the outcome prospects for *classes* of individuals with identical circumstances. Accordingly, it focuses on inequality between types, and is instead neutral with respect to inequality within types. Hence, the ex ante approach is more focused on *inequalities between social groups*. Referring to the Principle of Utilitarian Reward introduced by Fleurbaey (2008), which states the social irrelevance of inequalities due to differences in effort, this implies that any equalizing transfer with identical circumstances should not change income inequality or social welfare, whatever the effort level of the persons involved in it. This means that an ex ante approach for measuring inequality of opportunity will solely depend on the type-specific mean incomes.

Assume that $\mu_1 \leq \mu_2 \leq ... \leq \mu_m$ and that q_i is associated with μ_i. Moreover we assume that every individual of a sub-population receives the same income equal to the sub-group mean. Thus, in this case the group-specific distributions are defined by

$$F_i^*(x) = \begin{cases} 0, x < \mu_i \\ 1, x \geq \mu_i \end{cases} \tag{4.13}$$

and the associated Lorenz curve[14] by

$$L^*(a_j) = \frac{\sum_{i=1}^{j} q_i \mu_i}{\sum_{i=1}^{m} q_i \mu_i}, \; j = 1,2,...,m. \tag{4.14}$$

Moreover, the associated rank-dependent family of inequality measures is defined by

$$J_p^* = 1 - \frac{\sum_{j=1}^{m} q_j p(a_j) \mu_j}{\mu \sum_{j=1}^{m} q_j p(a_j)} \; . \tag{4.15}$$

[14] The opportunity partial ordering based on the Lorenz curve below was characterized in Peragine (2002).

The J_p^* measure captures the inequality between types: this inequality, according to the *Compensation Principle*, can be interpreted as ex ante inequality of opportunity. The welfare function associated with J_p^* is defined by

$$W_p^* = \mu(1 - J_p^*) = \frac{\sum_{j=1}^{m} q_j p(a_j)\mu_j}{\sum_{j=1}^{m} q_j p(a_j)} \tag{4.16}$$

By inserting (4.7) in (4.16) and (4.17) we get

$$J_e^* = 1 - \frac{\min_{1 \le i \le m} \mu_i}{\mu} \tag{4.17}$$

and

$$W_e^* = \min_{1 \le i \le m} \mu_i \,. \tag{4.18}$$

Note that W_e^* corresponds to the EOp welfare function introduced by Roemer (1993, 1998) when considering snapshots of income. When μ_i represents the mean permanent income (defined by (2.5)) of type *i*, then W_e^* coincides with the EOp welfare function introduced by Bourguignon et al. (2007) if and only if μ_i can be expressed as weighted average of the period-specific type means.

When *p* represents the Gini weight function then J_p^* is given by

$$G^* = 1 - \frac{\sum_{j=1}^{m} q_j b_j \mu_j}{\mu \sum_{j=1}^{m} q_j b_j} \,. \tag{4.19}$$

5 Empirical results

This section implements the proposed methods to measure inequality of opportunity and social welfare, using panel data from Norway on individual income over the working lifespan.

This allows us to examine how well analysis of inequality of opportunity and social welfare based on snapshots of income approximate the results based on long-term income. Further, we investigate how sensitive analyses based on EOp and EO (equality of outcome) is to the choice of accounting period of income. For brevity, we will throughout the empirical analysis only employ the Gini coefficient.

Data. Our empirical analysis utilizes several register databases provided by Statistics Norway. The data include a rich longitudinal dataset containing records for every Norwegian from 1967 to 2006. The variables captured in this dataset include individual demographic information (sex, year and municipality of birth, number of siblings) and socio-economic data (income and education). Importantly, the dataset includes personal identifiers, allowing us to link children to their parents. Our income measure is defined in close agreements with international recommendations (see Expert Group on Household Income Statistics, 2001), and incorporates annual wages, capital income, and all public cash transfers. We use the consumer price index to make earnings and incomes from different periods comparable. Throughout this paper we focus on individual income, because our family data does not allow us to link spouses living in cohabitation. Nor can we identify married spouses before the mid 1970s. Therefore, we cannot construct a measure of family or household income.

Our empirical analysis focuses on the 1942-1944 cohorts in order to ensure availability of data on income for more or less the entire working lifespan. In particular, these cohorts are between 23 and 25 years old in 1967 and between 62 and 64 in 2006.[15] Our analytical sample is restricted to males, given their role of breadwinner and primary wage-earner for these cohorts. Also, we exclude individuals whose information on annual income is missing. Finally, we drop observations where information on parents is missing. The final sample used in the analysis consists of 26 090 individuals.

Circumstances and types. How to define the exact set of individual circumstances is a controversial question. Besides, in empirical work, observing the entire set of circumstances is clearly out of reach. In this paper, we focus on the dependence of individual opportunity on a restricted set of circumstances relating to family background. The role of families in determining socio-economic outcomes has been extensively discussed in economics as well

[15] Note that people are eligible for early retirement when they have become 62 years old.

as from other scientific perspectives. Parents influence their children via several channels: investment in their children's education, transmission of cultural values and social skills, and genetic endowments. As argued by Roemer (1998) and others, equality of opportunity requires compensating the influence of family background on individual outcomes, as it lies beyond individual responsibility and choice.

A large body of empirical research shows a strong degree of intergenerational transmission, as parental characteristics like education and income are highly correlated with the outcomes of children along the same dimensions (see e.g. Solon 1999). As a starting point, we therefore split the sample by birth cohort and further divide it into three groups based on the highest educational attainment of the parents: compulsory school (grades 1-7), middle school (grades 8-10), and higher education (11 or more years of education). Unfortunately, we do not have information on income of the parents during individuals' childhood. Next, we partition the sample according to an indicator for urbanity, equal to one if the individual was born in one of the five bigger cities of Norway (Oslo, Bergen, Trondheim, Stavanger, Drammen). The idea is that urbanity may proxy for neighbourhood effects, which may arise from varying local resource bases, such as availability of institutions like schools and childcare, as well as through social interaction in peer-groups, like attitudes and preference formation as well as the existence and enforcement of social norms.[16] Finally, we partition the sample according to an indicator for being raised in a large family, equal to one if the individual has two or more siblings. Drawing on the quantity-quality model of Becker and Lewis (1973), a large body of empirical research has shown a fairly strong association between number of siblings and children's outcome. In particular, it appears to be a non-monotonic relationship between family size and children's outcomes, with substantial negative effects of large sibship size (see Mogstad and Wiswall, 2009). Thus, to measure EOp, the sample is divided into 36 mutually exclusive and collectively exhaustive types based on circumstances.

Descriptive statistics. Table 1 displays the composition of the sample according to the variables used to defined circumstances. We see that about 20 percent of the sample was born in urban areas, and about 45 percent come from a large family. Moreover, it is evident that about 53 percent has parents with only compulsory schooling, almost 30 percent has at least

[16] Raaum et al. (2006) show significant effects of childhood neighbourhood on children's long-run outcomes.

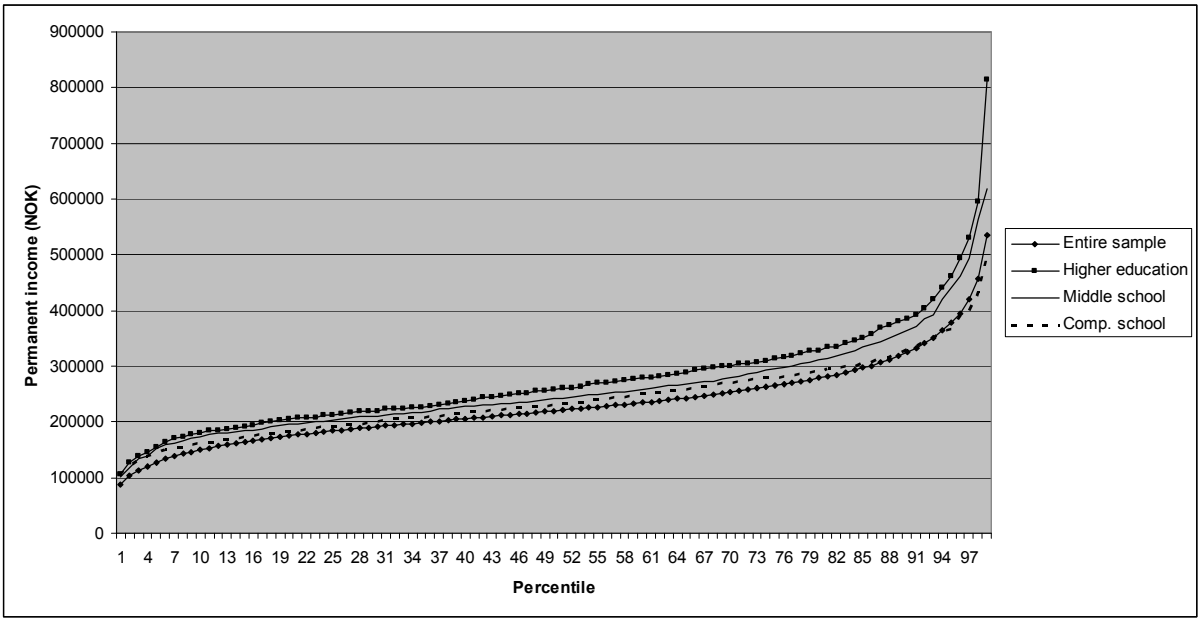
one parent who attended middle school, and less than 18 percent has at least one parent with higher education. It is further evident that the composition is fairly similar across cohorts. Table 1 also shows that the income of these cohorts increase over time. In particular, the period-specific incomes are, on average, lower than the mean income over the period 1967-2006 up until the early 1990s. It is also clear that the long-term mean income exceeds the permanent income measure, as only the latter takes into the costs of inter-period income transfers. As expected, the incomes of the older cohort are relatively high during the first years as they have more labor market experience, but are surpassed by the younger cohorts when they grow older. The mean and the permanent income are rather similar across cohorts. Figure 1 graphs the cumulative distribution of permanent income for the sample as a whole, and for different types according to parental education. As expected, permanent income is increasing in parent's education.

Table 1. Descriptive statistics

	Birth cohort			
	42	43	44	42-44
Period-specific income				
1967-1968	157,338	150,824	141,850	149,191
1975-1976	240,324	239,477	233,859	237,538
1985-1986	244,191	244,085	239,967	242,518
1995-1996	277,150	283,057	279,302	279,926
2005-2006	311,261	329,562	328,496	323,958
Mean income (1967-2006)	251,824	258,005	249,505	252,955
Permanent income (1967-2006)	234,976	237,163	230,058	233,787
Circumstances (populaton shares)				
Urban	18.18	20.09	21.06	19.92
Large family	46.02	44.51	43.69	44.62
Parental Education				
Comp. school only	53.48	52.59	51.39	52.38
Middle school	29.00	29.39	30.67	29.77
Higher education	17.52	18.02	17.95	17.85
Sample size	7,399	8,572	10,119	26,090

Notes: Period-specific income is defined as the average income over the two-year period. Permanent income is defined by (2.1), (2.7), (2.8) and (2..9) with $\varepsilon = 2$ and $\delta = .02$, and annual real interest rates on borrowing and savings given by Norwegian official statistics.

Figure 1. Marginal distribution of permanent income for entire sample and by parental education



Notes: Permanent income is defined by (2.1), (2.7), (2.8) and (2.9) with $\varepsilon = 2$ and $\delta = .02$, and annual real interest rates on borrowing and savings given by Norwegian official statistics. The distribution denoted entire sample includes the incomes of all individuals. The distributions denoted Higher education, Middle school and Compulsory school are graphed separately by parental education for the sub-sample of individuals from the 1944 cohort who were born in an urban area and living in a small family.

Equality of outcome.

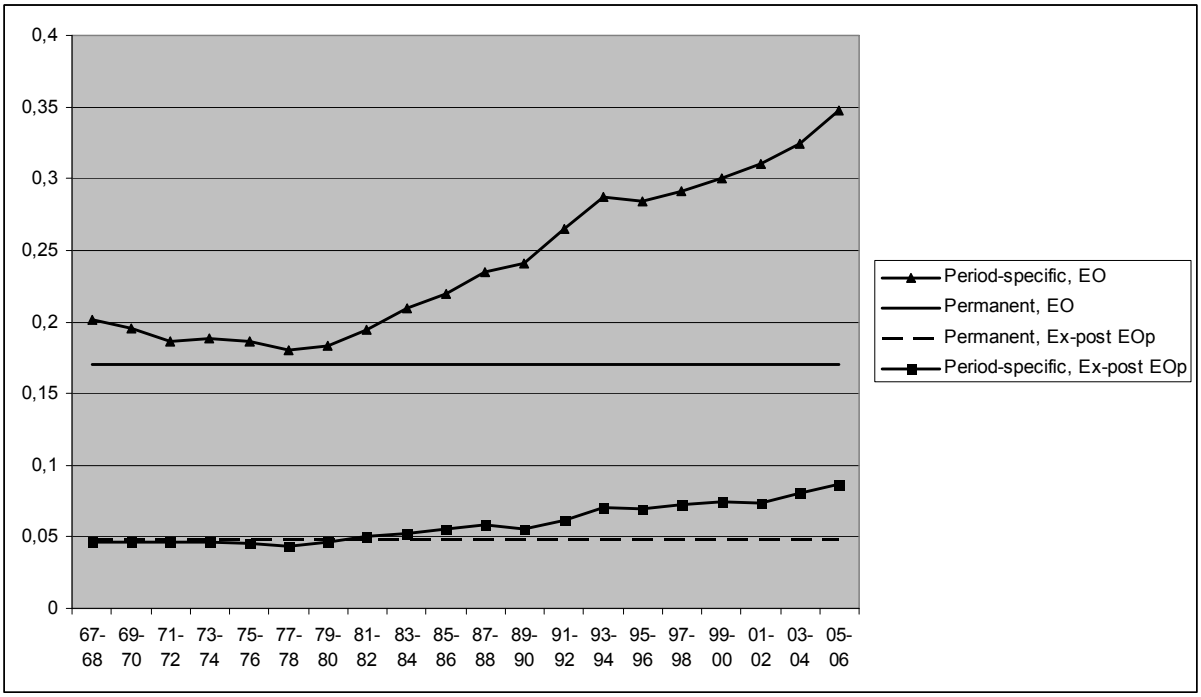
Figure 2 graphs the time-trend in inequality according to the EO principle based on period-specific (two years) income (cf. Period-specific, EO), and compare it to the EO measure of inequality based on permanent income (cf. Permanent, EO.) We see that inequality is fairly stable during the 1970s when the individuals are their mid 30s, with a Gini-coefficient ranging from .18 to .20. After this point in time, inequality is increasing steadily, almost doubling as the individuals reach the peak of the age-earnings profile.[17] In contrast, the Gini coefficient in permanent income is as low as .17, which suggests substantial income mobility. Moreover, our results indicate that snapshots of inequality based on income early in the working lifespan may provide a reasonable approximation of inequality in permanent income when the EO principle is employed.

Figure 3 reports the time-trend in social welfare according to the EO principle based on period-specific income (cf. Period-specific, EO), and compares it to EO measure of inequality in permanent income (cf. Permanent, EO). We see social welfare is increasing substantially during the 1970s, as mean incomes rise and inequality is stable. Over the next decade, social welfare declines as the welfare loss from increased inequality dominates the increase in mean incomes. During the economic boom in the 1990s, social welfare increased once again owing to a large increase in mean incomes,

[17] A possible explanation is that the age-earnings profile differs systematically across individuals, with relatively high earnings growth among individuals with high education. Another explanation is that skill-biased technical change has led to increased earnings inequality. Investigating these two explanations is beyond the scope of this paper.

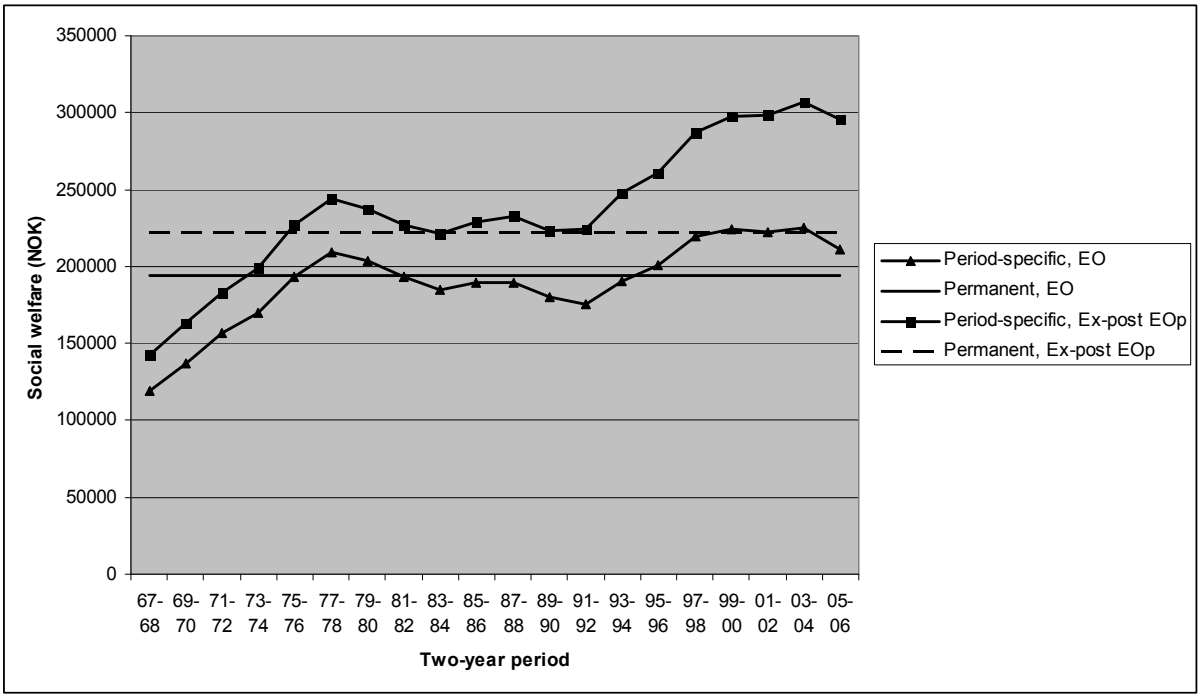
despite substantially higher income inequality. We also see that social welfare in permanent income mirrors social welfare in period-specific inequality fairly well when the individuals are between 30 and 50 years of age, in contrast to the inequality results.

Figure 2. Gini-inequality according to EO and ex-post EOp based on period-specific income and permanent income



Notes: Permanent income is defined by (2.1), (2.7), (2.8) and (2.9) with $\varepsilon = 2$ and $\delta = .02$, and annual real interest rates on borrowing and savings given by Norwegian official statistics. The ex-post Gini EOp measure of inequality is defined by (4.12).

Figure 3. Social welfare according to EO and ex-post EOp based on period-specific income and permanent income



Notes: Permanent income is defined by (2.1), (2.7), (2.8) and (2.9) with $\varepsilon = 2$ and $\delta = .02$, and annual real interest rates on borrowing and savings given by Norwegian official statistics. The ex-post EOp social welfare measure is defined by (4.6).

Ex-post equality of opportunity.

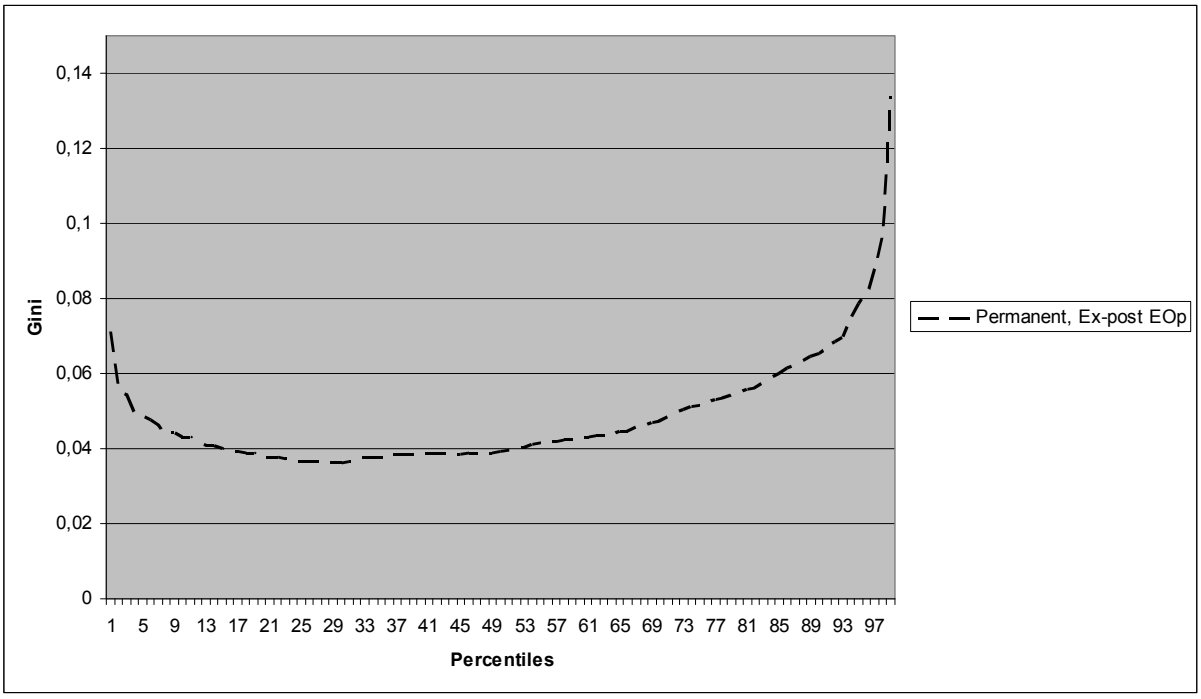
In order to estimate the Gini EOp coefficient defined by (4.12) and the associated social welfare function defined by (4.6) we first have to estimate the quantile-specific EOp Gini coefficient for an appropriate selection of quantiles. This study relies on estimates for all percentiles, which means that the overall EOp Gini is based on 100 quantile-specific EOp Gini estimates (see Figure 4 for permanent income).

Turning attention to our ex-post measures of EOp, Figure 2 shows the trend in inequality based on period-specific income (cf. Period-specific, ex-post EOp) and compares it to the EO measure of inequality in permanent income (cf. Permanent, ex-post EOp). We see immediately that the ex-post EOp measures of inequality are less than one third of the corresponding EO measures, suggesting that a large fraction of inequality of outcomes is attributable to effort. This is true both when permanent income and period-specific incomes form the basis for the analysis. Indeed, opportunities in long-term income appear to be very equally distributed. Figure 2 also reveals that the inequality time-trends in ex-post EOp is quite similar to that of EO. In addition, our results indicate that snapshots of inequality based on income early in the working lifespan may provide a reasonable approximation of inequality in permanent income, also when the EOp principle is employed.

Figure 4 breaks down the ex-post EOp measure of inequality into the different percentile-specific EOp measures. We see that inequality is remarkably stable across the percentiles. In particular, the

percentile-specific Gini coefficients vary between .04 and .06 for quantiles located between the 3 and 85 percentile, and somewhat higher at the tails of the distribution.

Figure 4. Percentile-specific Gini-inequality according to ex-post EOp based on permanent income



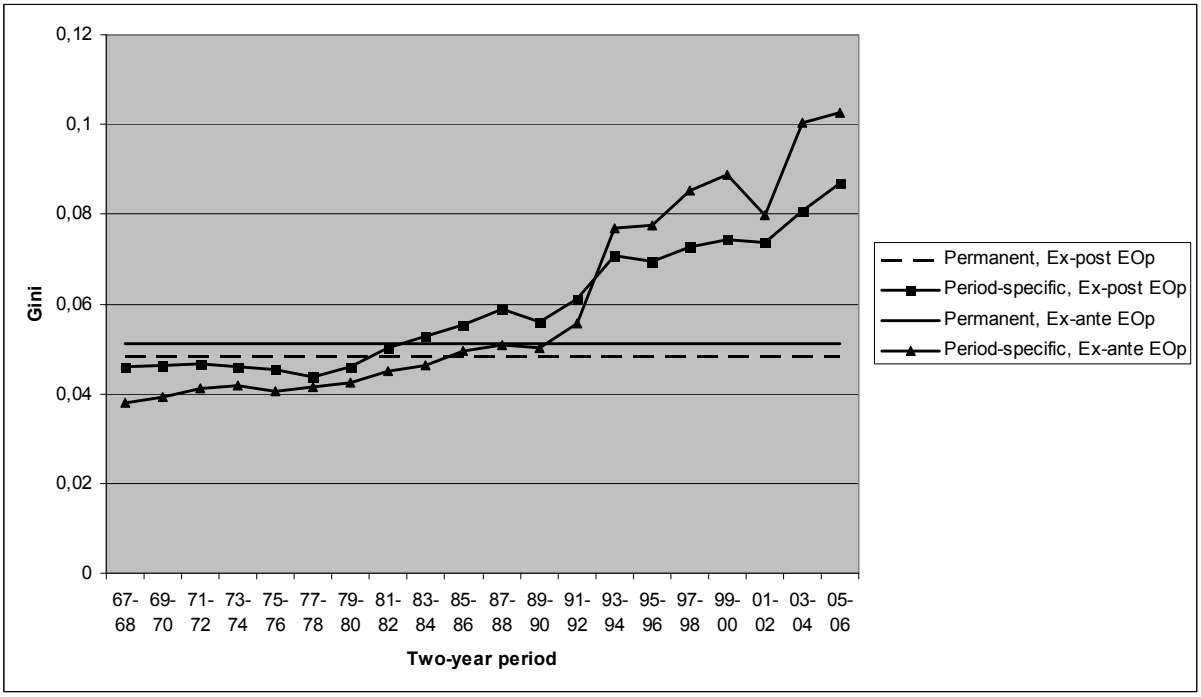
Notes: Permanent income is defined by (2.1), (2.7), (2.8) and (2.9) with $\varepsilon = 2$ and $\delta = .02$, and annual real interest rates on borrowing and savings given by Norwegian official statistics. The percentile-specific ex-post EOp measure is given by (4.4).

Figure 3 reports social welfare measures according to the ex-post EOp principle based on period-specific income (cf. Period-specific, EOp) and permanent income (cf. Permanent, EOp). Compared to the EO results, social welfare is considerably higher when the ex-post EOp principle forms the basis for the analysis, owing to lower inequality. Specifically, the ex-post EOp measure of social welfare in permanent income is almost 15 percent higher than the EO measure. Yet the time trend in social welfare based on period-specific incomes is quite similar across the two measures. In particular, we see that the ex-post EOp measure of social welfare based on snapshots of income during the period the individuals are between 30 and 50 years of age are fairly similar to the corresponding measures based on permanent income.

Ex-ante equality of opportunity.

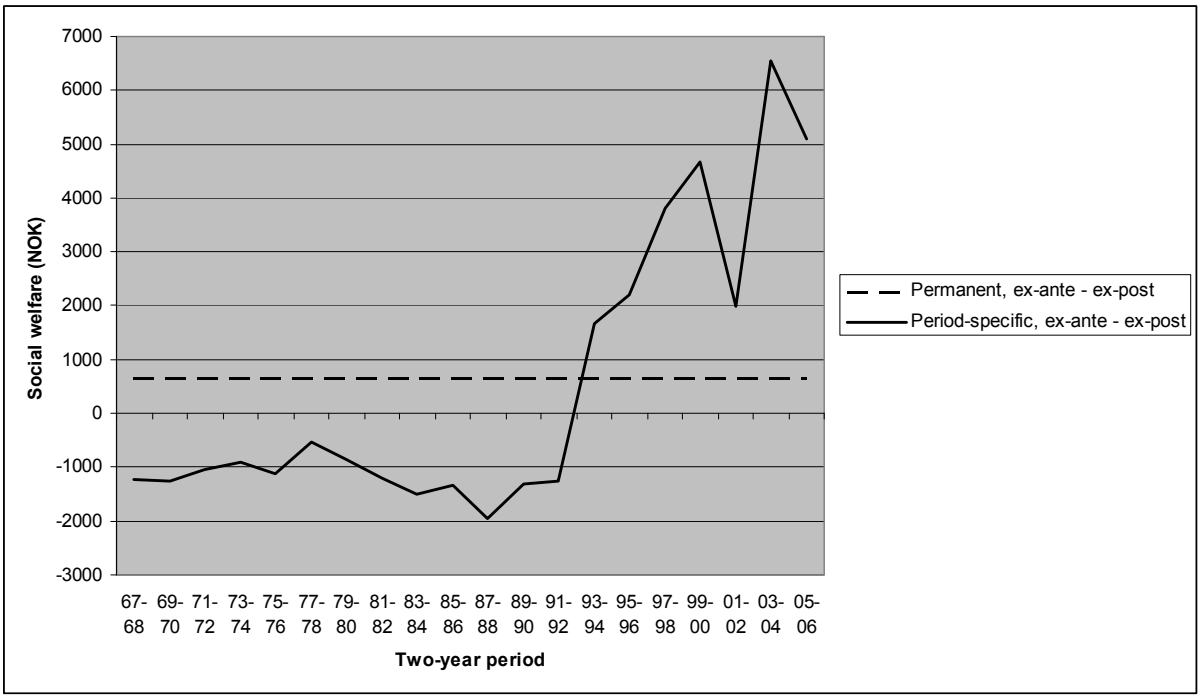
Figure 5 shows a strong similarity between ex-ante and ex-post EOp measures of inequality. This holds both for permanent income and the time-trend in period-specific income, except for the upper part. Figure 6 graphs the difference in social welfare according to the ex-ante and ex-post EOp measures. The results mirror the similarity in inequality across the two types of EOp measures.

Figure 5. Gini-inequality according to ex-ante and ex-post EOp based on period-specific income and permanent income



Notes: Permanent income is defined by (2.1), (2.7), (2.8) and (2.9) with $\varepsilon = 2$ and $\delta = .02$, and annual real interest rates on borrowing and savings given by Norwegian official statistics. The Gini ex-post and ex ante EOp measures of inequality are defined by (4.12) and (4.19).

Figure 6. Difference in social welfare according to ex-ante and ex-post EOp based on period-specific income and permanent income



Notes: Permanent income is defined by (2.1), (2.8) qnd (2.9) with $\varepsilon = 2$ and $\delta = .02$, and annual real interest rates on borrowing and savings given by Norwegian official statistics. The ex-post and ex ante EOp social welfare measures are defined by (4.6) and (4.16).

6 Summary and conclusion

More than half a century ago, Friedman (1957, p38) argued that "*the identification of low measured income with 'poor' and high measured income with 'rich' is justified only if measured income can be regarded as an estimate of expected income over a lifetime or a large fraction thereof*". His concern was that studies based on fluctuating annual incomes may provide a misleading picture of the consumption possibilities of individuals, and consequently, also the extent of inequality and social welfare in a society. Instead, methods used for distributional analysis should reflect that income can be, and regularly is, used for consumption not only within the year that it is obtained, but also in other years.[18]

In this paper, we introduce and apply a general framework for evaluating *long-term* income distributions according to the EOp principle. Specifically, we employ an axiomatic approach to justify the introduction of a generalized family of rank-dependent of opportunity inequality and social welfare. Our framework allows for both an ex-ante and an ex-post approach to EOp. Our ex-post approach relies on a permanent income measure defined as the minimum annual expenditure an individual would need in order to be as well off as he could be by undertaking inter-period income transfers. There is long-term ex-post inequality of opportunity if individuals who exert the same effort have different permanent incomes. In comparison, the ex-ante approach focuses on the expected permanent income for individuals with identical circumstances. Hence, the ex-ante approach pays attention to inequalities in expected permanent income between different types of individuals.

To demonstrate the empirical relevance of a long-run perspective on EOp, we have used a unique panel data from Norway on individuals' incomes over their working lifespan. The insights from our empirical results may be summarized with three conclusions. First, EOp measures of inequality are less than one third of the corresponding EO measures, suggesting that a large fraction of inequality of outcomes is attributable to effort. This is true both when permanent income and snapshots of income form the basis for the analysis. Hence, social welfare is much higher according to the EOp principle than the EO principle. Second, snapshots of income overstate inequality compared to analysis based on permanent income, suggesting substantial income mobility. However, our results indicate that snapshots of inequality based on income early in the working lifespan may provide a reasonable approximation of inequality in permanent income; in comparison, social welfare measures based on snapshots of income during the period the individuals are between 30 and 50 are fairly similar to the corresponding measures based on permanent income. This is true both when the EO and the EOp

[18] In this regard, it is of interest that most individuals in the OECD-countries actually carry out intertemporal income transfers to smooth consumption, largely by means of education loans, housing mortgage and pension schemes.See Borsch-Supan (2003) for a cross-country study of life-cycle savings.

principle form the basis for the analysis. Third, we find some similarity between ex-ante and ex-post EOp measures of inequality and social welfare. This holds both for permanent income and the time-trend in period-specific income, except for the latest 15 years of the participation of the labour market.

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