# Distributions in Motion: <br> A "disaggregated" approach to growth and the dynamics of poverty and inequality 

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## Plan of Lecture

1. An integrated framework for the analysis of growth and distribution dynamics.

- Based on growth incidence curves.

2. Understanding changes in distributions: statistical counterfactual decompositions.

- Generalized Oaxaca-Blinder decompositions

3. Understanding changes in distributions: towards economic decompositions?
4. Partial equilibrium approaches
5. General equilibrium approaches

## 1. A framework for the analysis of growth and distribution dynamics

Growth (in the mean), poverty dynamics and inequality dynamics are different ways of quantifying the movement of entire distributions over time.

Growth in Thailand, 1975-1992, seen as rightward shifts in the Cumulative Distribution Function.

EVERYONE'S MIRACLE? REVISTTING POVERTY AND INEQUALITY IN EAST ASIA
Figure 4.6 Thailand: Cumulative Distribution Functions, 1975-92


ACASE STUDY OF THAILAND, 1975-92
Figure 4.7 Thailand: Cumulative Distribution Functions, Detail of Low-Income Range, 1975-92


## The Pen Parade (or quantile function): $\mathrm{y}=\mathrm{F}^{-1}(\mathrm{p})$

Figura 1. Brasil 1981-1995: Paradas de Pen


## The Growth Incidence Curve: <br> An example from two different periods in Brazil

The Growth Incidence Curve was first formally described by Ravallion and Chen (2003). The version in discrete time is:

$$
g(p):=\frac{y_{t}(p)-y_{t-1}(p)}{y_{t-1}(p)}
$$

Figure 1: Growth Incidence Curves for Brazil


## Growth in mean incomes

- Growth in mean incomes is simply a weighted average of income growth along the distribution, with weights given by relative incomes.

$$
\begin{array}{r}
\mu=\int_{-\infty}^{\infty} y d F(y)=\int_{0}^{1} y(p) d p \\
\frac{\dot{\mu}}{\mu}=\int_{0}^{1} \frac{\dot{y}(p)}{y(p)} \frac{y(p)}{\mu} d p
\end{array}
$$

- This can be written in terms of the growth incidence curve (GIC):

$$
\frac{\dot{\mu}}{\mu}=\int_{0}^{1} g(p) \frac{y(p)}{\mu} d p
$$

- So growth (in the mean) is simply a particular aggregation of the percentilespecific growth rates in the GIC.

Which is, of course, just the proportional change in the Pen parade $F^{-1}(p)$, at every $p$.

## Changes in Poverty and Inequality Drawing (in part) on Kraay (2003)

Write a general poverty measure formulation as: $\quad P_{t}=\int_{0}^{F(z)} \pi\left(y_{t}(p), z\right) \cdot d p$ where $\pi\left(y_{t}(p), z, \theta\right)=\left(\frac{z-y_{t}(p)}{z}\right)^{\theta}$ gives you the FGT class, for instance, and $\pi\left(y_{t}(p), z\right)=\ln \left(\frac{z}{y_{t}(p)}\right)$ gives you the Watts index.

Differentiating with respect to time yields $\frac{d P_{t}}{d t}=\int_{0}^{F(z)} \eta_{t}(p) \cdot g_{t}(p) \cdot d p+\pi(z, z) \frac{d F(z)}{d t}$
with $\quad \eta_{t}(p) \equiv \frac{d \pi\left(y_{t}(p)\right)}{d y_{t}(p)} \cdot y_{t}(p) \quad$ and $\quad \mathrm{g}_{\mathrm{t}}(\mathrm{p}) \equiv \frac{\mathrm{dy}_{\mathrm{t}}(\mathrm{p})}{\mathrm{dt}} \cdot \frac{1}{\mathrm{y}_{\mathrm{t}}(\mathrm{p})}$

## Changes in Inequality

- Like poverty measures, many relative inequality indices can be written as functions of a sum of "individual relative income gaps":

$$
I_{t}=G\left[\int_{0}^{1} h\left(\frac{y_{t}(p)}{\mu}\right) \cdot d p\right]
$$

$G\left[\int_{0}^{1} h\left(\frac{y_{t}(p)}{\mu}\right) \cdot d p\right]=\frac{1}{\theta^{2}-\theta}\left[\int_{0}^{1}\left(\frac{y}{\mu}\right)^{\theta} d p-1\right]$ for example, gives the GE class.
$G\left[\int_{0}^{1} h\left(\frac{y_{t}(p)}{\mu}\right) \cdot d p\right]=1-\left[\int_{0}^{1}\left(\frac{y}{\mu}\right)^{1-\varepsilon} d p\right]^{\frac{1}{1-\varepsilon}}$ gives the Atkinson class.
Differentiating relative measures with respect to time yields:

$$
\frac{d I_{t}}{d t}=G^{\prime}() \int_{0}^{1} h^{\prime}\left(\frac{y(p)}{\mu}\right) \cdot \frac{\mu}{y(p)}\left[g_{t}(p)-\frac{\dot{\mu}}{\mu}\right] \cdot d p
$$

So poverty and inequality changes are also transformations of the information in the GIC.

## 2. Understanding Changes in Distributions:

 Statistical counterfactual decompositions.- To seek an understanding of changes in the distribution of incomes is to seek an understanding of why the GIC looks the way it does.
- To understand the nature and determinants of the incidence or distribution of economic growth.
- The first step is statistical:


$$
g(p)=\frac{y_{t}(p)-y^{s}(p)}{y_{t-1}(p)}+\frac{y^{s}(p)-y_{t-1}(p)}{y_{t-1}(p)}
$$

## Statistical counterfactual decompositions (continued)

- Of course, this is just another way of describing generalized Oaxaca-Blinder decompositions such as

$$
f_{t}(y)-f_{t-1}(y)=\left[f^{s}(y)-f_{t-1}(y)\right]+\left[f_{t}(y)-f^{s}(y)\right]
$$

- Where the counterfactual distribution is constructed from:

$$
\begin{gathered}
f_{t}(y)=\iiint \varpi_{t}(y, X) d X \\
f_{t}(y)=\int \ldots \int g_{t}(y \mid X) \not \phi_{t}(X) d X
\end{gathered}
$$

- By simulating a change in either the conditional distribution of $y$ on X , or on the joint distribution of X .
- For example:

$$
f^{s}(y)=\int \ldots \int g^{s}(y \mid X) \phi_{t}(X) d X
$$

## Statistical counterfactual decompositions (continued)

- There are a number of ways to implement such simulations in practice.
- They may be based simply on reweighting the sample, so as to reproduce the changes in the distribution of some exogenous characteristic, such as the age composition of the labor force, or the number of people receiving the minimum wage.
- DiNardo, Fortin and Lemieux (1996)
- Hyslop and Maré (2005)
- They may be based in importing parameters from models estimated in one year to the other.
- Juhn, Murphy and Pierce (1993)
- Bourguignon, Ferreira and Lustig (2005)


## The origins of statistical counterfactual decompositions

## a. The Oaxaca-Blinder Decomposition

- These approaches draw on the standard Oaxaca-Blinder Decompositions (Oaxaca, 1973; Blinder, 1973)
- Let there be two groups denoted by $r=w, b$.

$$
y_{i r}=X_{i r} \beta_{r}+\varepsilon_{i r}
$$

- Then $\mu_{y w}=\bar{X}_{i w} \beta_{w}$ and $\mu_{y b}=\bar{X}_{i b} \beta_{b}$
- So that $\mu_{y w}-\mu_{y b}=\bar{X}_{i w}\left(\beta_{w}-\beta_{b}\right)+\left(\bar{X}_{i w}-\bar{X}_{i b}\right) \beta_{b}$
- Or:

$$
\begin{aligned}
& \mu_{y w}-\mu_{y b}=\bar{X}_{i b}\left(\beta_{w}-\beta_{b}\right)+\left(\bar{X}_{i w}-\bar{X}_{i b}\right) \beta_{w} \\
& \text { "returns component" } \quad \text { "characteristics component" }
\end{aligned}
$$

- Caveats: (i) means only; (ii) path-dependence; (iii) statistical decomposition; not suitable for GE interpretation.

Modern applications: parametric method for wage distributions.
b. Juhn, Murphy and Pierce (1993)

$$
y_{i r}=X_{i r} \beta_{r}+\varepsilon_{i r} \quad \text { where } \quad \varepsilon_{i r}=F_{r}^{-1}\left(\theta_{i r} \mid X_{i r}\right)
$$

Define: $\quad y_{i}^{\prime}=X_{i 0} \beta_{1}+F_{0}^{-1}\left(\theta_{i 0} \mid X_{i 0}\right)$

$$
y_{i}^{\prime \prime}=X_{i 0} \beta_{1}+F_{1}^{-1}\left(\theta_{i 0} \mid X_{i 0}\right)
$$

Then: $\quad I\left(F\left(y_{i}^{\prime}\right)\right)-I\left(F\left(y_{i 0}\right)\right) \rightarrow \quad$ Returns component

$$
I\left(F\left(y_{i}^{\prime \prime}\right)\right)-I\left(F\left(y_{i}^{\prime}\right)\right) \quad \longrightarrow \quad \text { Unobserved charac. component }
$$

$$
I\left(F\left(y_{i 1}\right)\right)-I\left(F\left(y_{i}^{\prime \prime}\right)\right) \quad \longrightarrow \quad \text { Observed charac. Component. }
$$

The Juhn-Murphy-Pierce (1993) decomposition results: US


Fig. 7.-Ninetieth-tenth percentile log wage differential and components, 1963-89

## Modern applications: non-parametric method for wage

 distributions.c. DiNardo, Fortin and Lemieux (1996)

Essentially, DFL propose estimating a counterfactual income distribution such as:

$$
f^{s}(y)=\iiint g_{t}(y \mid X) \phi^{s}(X) d X
$$

By appropriately reweighing the sample, as follows.

$$
\begin{aligned}
& f^{s}(y)=\iiint g_{t}(y \mid X) \phi_{t}(X) \psi(X) d X \\
& \text { where } \quad \psi(X)=\frac{\phi(X \mid t=1)}{\phi(X \mid t=0)}
\end{aligned}
$$

A variant of this approach is applied to HPCY distributions by Hyslop and Maré. ${ }^{15}$

## Modern applications: parametric and semi-parametric mixed methods for HPCY distributions.

d. Bourguignon, Ferreira and Lustig (2005)

- Depart from

$$
f_{t}(y)=\int \ldots \int g_{t}(y \mid X) \phi_{t}(X) d X
$$

- Note that this can be written:

$$
f^{t}(y)=\iiint g^{t}(y \mid V, W) h_{1}^{t}\left(v_{1} \mid V_{-1}, W\right) h_{2}^{t}\left(v_{2} \mid V_{-1,2}, W\right) \ldots h_{v}^{t}\left(v_{v} \mid W\right) \psi^{t}(W) d V d W
$$

- For example
- $\mathrm{v}_{1}$ : number of children
- $\mathrm{V}_{2}$ : occupation
- $\mathrm{V}_{3}$ education

Let $k^{0}=\left\{g^{0}, \boldsymbol{h}^{0}\right\}$ and $k^{1}=\left\{g^{1}, \boldsymbol{h}^{1}\right\}$ be ordered sets of conditional distributions.
Define a counterfactual (ordered) set of conditional distributions $k^{s}$, the dimension of which is $v+1$, (like $k^{0}$ and $k^{l}$ ) whose elements are drawn either from $k^{0}$ or $k^{l}$.

Define a counterfactual distribution $\mathrm{f}_{0 \rightarrow 1}^{\mathrm{s}}\left(\mathrm{y} ; \mathrm{k}^{\mathrm{s}}, \psi^{0}\right)$

For example, the counterfactual distribution $f_{0 \rightarrow 1}^{3}\left(y ; g^{0}, h_{1}{ }^{1}, h_{-1}{ }^{0}, \psi^{0}\right)$ is given by:

$$
f_{0 \rightarrow 1}^{s}(y)=\iiint g^{0}(y \mid V, W) h_{1}^{1}\left(v_{1} \mid V_{-1}, W\right) h_{2}^{0}\left(v_{2} \mid V_{-1,2}, W\right) \ldots . . h_{v}^{0}\left(v_{v} \mid W\right) \psi^{0}(W) d V d W
$$

For each counterfactual distribution $f^{s}$, the difference between $f^{0}$ and $f^{1}$ can be decomposed as follows:

$$
\begin{gathered}
f^{1}(y)-f^{0}(y)=\left[f^{s}(y)-f^{0}(y)\right]+\left[f^{1}(y)-f^{s}(y)\right] \\
\mu_{q}^{1}(y)-\mu_{q}^{0}(y)=\left[\mu_{q}^{s}(y)-\mu_{q}^{0}(y)\right]+\left[\mu_{q}^{1}(y)-\mu_{q}^{s}(y)\right]
\end{gathered}
$$

The next step is to estimate those conditional distributions. We do so through a set of parametric models, built around three blocks:
(1) Earnings and self-employment equations:

$$
\log y_{h i}^{j}=X_{h i} \beta^{j}+\varepsilon_{h i}^{w j}
$$

(2) Occupational structure equations:

$$
\begin{aligned}
& I_{h i}^{s}=1 \text { if } Z_{h i} \Omega^{L s}+\varepsilon_{i}^{L s}>\operatorname{Max}\left(0, Z_{h i} \Omega^{L j}+\varepsilon_{i}^{L j}\right), j=1, \ldots J+1, \forall j \neq s \\
& I_{h i}^{s}=0 \text { for all } s=1, \ldots J+1 \text { if } Z_{h i} \Omega^{L s}+\varepsilon_{i}^{L s} \leq 0 \text { for all } s=1, \ldots J+1
\end{aligned}
$$

If $\varepsilon$ has a Weibull distribution, the probability of individual i choosing occupation s is given by:

$$
P_{i}^{s}=\frac{e^{z_{i} \lambda_{s}}}{e^{z_{i} x_{s}}+\sum_{j \neq s} e^{z_{i} \lambda_{j}}}
$$

which is estimated through a standard multinomial logit model.
(3) Conditional distributions of education and family size.

Education: $\quad \mathrm{ML}_{\mathrm{E}}\left(\mathrm{E} \mid \mathrm{A}, \mathrm{R}, \mathrm{r}, \mathrm{g}, \mathrm{n}_{\mathrm{ah}} ; \gamma\right)$
Fertility:

$$
\mathrm{ML}_{\mathrm{C}}\left(\mathrm{n}_{\mathrm{ch}} \mid \mathrm{E}, \mathrm{~A}, \mathrm{R}, \mathrm{r}, \mathrm{~g}, \mathrm{n}_{\mathrm{ab}} ; \psi\right)
$$

Other Incomes: $T\left(y_{0 h} \mid E, A, R, r, g, n_{a h} ; \xi\right)$
Household incomes are then aggregated as follows:

$$
y_{h}=\frac{1}{n_{h}}\left[\sum_{i=1}^{n_{h}} \sum_{j=1}^{J} \mathrm{I}_{h i}^{j} y_{h i}^{j}+y_{h}^{s e}+y_{0 h}\right]
$$

## In practice

- After one estimates those models for both $t=0$ and $t=1$, various counterfactual distributions are constructed by:
- Importing the relevant set of parameter estimates from $t=1$ to $t=0$ (or vice-versa).
- Importing the (actual or simulated) residual terms for each individual.
- Predicting the counterfactual income levels (and occupations or educations or family structures, as needed) for each individual.
- Computing the desired counterfactual statistics, such as inequality or poverty measures, for the resulting counterfactual distribution.
- Graphing changes in the distribution for each step of the decomposition.


## An Example: The Brazilian Slippery Slope, 1976-1996

| Table 7: Simulated Poverty and Inequality for 1976, Using 1996 coefficients. |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  | Mean | Inequality |  |  |  | Poverty |  |  |  |  |  |
|  | $\mathrm{p} / \mathrm{c}$ |  |  |  |  | $\mathrm{Z}=\mathrm{R} \$ 30 /$ month |  |  | $\mathrm{Z}=\mathrm{R} \$ 60 /$ month |  |  |
|  | Income | Gini | E(0) | E(1) | E(2) | $\mathrm{P}(0)$ | $\mathrm{P}(1)$ | P (2) | $\mathrm{P}(0)$ | $\mathrm{P}(1)$ | $\mathrm{P}(2)$ |
| 1976 observed | 265.101 | 0.595 | 0.648 | 0.760 | 2.657 | 0.0681 | 0.0211 | 0.0105 | 0.2209 | 0.0830 | 0.0428 |
| 1996 observed | 276.460 | 0.591 | 0.586 | 0.694 | 1.523 | 0.0922 | 0.0530 | 0.0434 | 0.2176 | 0.1029 | 0.0703 |
| Price Effects |  |  |  |  |  |  |  |  |  |  |  |
| $\alpha, \beta$ for wage earners | 218.786 | 0.598 | 0.656 | 0.752 | 2.161 | 0.0984 | 0.0304 | 0.0141 | 0.2876 | 0.1129 | 0.0596 |
| $\alpha, \beta$ for self-employed | 250.446 | 0.597 | 0.658 | 0.770 | 2.787 | 0.0788 | 0.0250 | 0.0121 | 0.2399 | 0.0932 | 0.0490 |
| $\alpha, \beta$ for both | 204.071 | 0.598 | 0.655 | 0.754 | 2.190 | 0.1114 | 0.0357 | 0.0169 | 0.3084 | 0.1249 | 0.0673 |
| $\alpha$ only, for both | 233.837 | 0.601 | 0.664 | 0.774 | 2.691 | 0.0897 | 0.0275 | 0.0129 | 0.2688 | 0.1040 | 0.0545 |
| All $\beta$ (but no $\alpha$ ) for both | 216.876 | 0.593 | 0.644 | 0.736 | 2.055 | 0.0972 | 0.0303 | 0.0143 | 0.2837 | 0.1114 | 0.0590 |
| Education $\beta$ for both | 232.830 | 0.593 | 0.639 | 0.759 | 2.691 | 0.0779 | 0.0234 | 0.0110 | 0.2531 | 0.0953 | 0.0488 |
| Experience $\beta$ for both | 240.618 | 0.600 | 0.664 | 0.771 | 2.694 | 0.0851 | 0.0265 | 0.0125 | 0.2592 | 0.1000 | 0.0525 |
| Gender $\beta$ for both | 270.259 | 0.595 | 0.649 | 0.751 | 2.590 | 0.0650 | 0.0191 | 0.0090 | 0.2160 | 0.0797 | 0.0404 |
| Occupational Choice Effects |  |  |  |  |  |  |  |  |  |  |  |
| $\gamma$ for both sectors (and both heads + others) | 260.323 | 0.609 | 0.650 | 0.788 | 2.633 | 0.0944 | 0.0451 | 0.0331 | 0.2471 | 0.1082 | 0.0671 |
| $\gamma$ for both sectors (only for other members) | 265.643 | 0.598 | 0.657 | 0.757 | 2.482 | 0.0721 | 0.0231 | 0.0119 | 0.2274 | 0.0867 | 0.0454 |
| $\gamma, \alpha, \beta$ for both sectors | 202.325 | 0.610 | 0.649 | 0.788 | 2.401 | 0.1352 | 0.0597 | 0.0402 | 0.3248 | 0.1466 | 0.0902 |
| Demographic Patterns |  |  |  |  |  |  |  |  |  |  |  |
| $\mu \mathrm{d}$ only, for all | 277.028 | 0.574 | 0.585 | 0.704 | 2.432 | 0.0365 | 0.0113 | 0.0063 | 0.1711 | 0.0554 | 0.0264 |
| $\mu \mathrm{d}, \gamma, \alpha, \beta$, for all | 210.995 | 0.587 | 0.577 | 0.727 | 2.177 | 0.0931 | 0.0433 | 0.0321 | 0.2724 | 0.1129 | 0.0677 |
| Education Endowment Effects |  |  |  |  |  |  |  |  |  |  |  |
| $\mu \mathrm{e}$ only, for all | 339.753 | 0.594 | 0.650 | 0.740 | 2.485 | 0.0424 | 0.0136 | 0.0073 | 0.1593 | 0.0567 | 0.0287 |
| $\mu \mathrm{d}, \mu \mathrm{e}$ for all | 353.248 | 0.571 | 0.584 | 0.688 | 2.320 | 0.0225 | 0.0078 | 0.0049 | 0.1131 | 0.0359 | 0.0173 |
| $\mu \mathrm{e}, \mu \mathrm{d}, \gamma, \alpha, \beta$, for all | 263.676 | 0.594 | 0.600 | 0.727 | 1.896 | 0.0735 | 0.0374 | 0.0296 | 0.2204 | 0.09181 | 0.0561 |

Source: Based on "Pesquisa Nacional por Amostra de Domicílios" (PNAD) of 1976 and 1996.

The Brazilian Slippery Slope: Price Effects

$\rightarrow$ alphas and betas $\rightarrow$ 1996-1976

The Brazilian Slippery Slope: Price and Occupational Effects

$\rightarrow$ alphas and betas $\rightarrow$ alphas, betas, gammas $\rightarrow$ 1996-1976

## The Brazilian Slippery Slope: Price, Occupation and Endowment Effects (Disaggregated into Education and "Fertility")

Figure 15: A Complete Decomposition


Source: "Pesquisa Nacional por Amostra de Domicilios" (PNAD), 1976 and 1996.
Source: Ferreira and Paes de Barros (1999)

## 3. Understanding Changes in Distributions: Towards economic counterfactual decompositions.

- Generalized Oaxaca-Blinder decompositions such as those discussed above, whether parametric or semiparametric, suffer from two shortcomings:
- Path-dependence
- The counterfactuals do not correspond to economic equilibria. There is no guarantee that those counterfactual incomes would be sustained after agents were allowed to respond and the economy reached a new equilibrium.
- I.e. These are statistical decomposition tools. They are not suitable for identifying causal impacts.
- Assessing the impact of a particular policy, for example, requires moving towards an economic decomposition.


### 3.1. Partial Equilibrium Approaches (A targeted intervention)

- The first steps towards economic decompositions, in which the counterfactual distributions may be interpreted as corresponding to a counterfactual economic equilibrium, are partial in nature.
- One example comes from attempts to simulate distributions after some transfer, in which household responses to the transfer (in terms of child schooling and labor supply) are incorporated.
- Bourguignon, Ferreira and Leite (2003)
- Todd and Wolpin (2006)
(These two papers differ considerably in how they model behavior. Todd and Wolpin are much more structural.)
- May be useful for simulating assigned programs before they are implemented, or for simulating alternative program designs.
- Should be combined with a credible ex-post evaluation strategy from the outset.


## An «ex-ante evaluation» of Bolsa Escola

 (Bourguignon, Ferreira and Leite, WBER, 2003)- How would the introduction of a conditional cash transfer perform with respect to its twin stated objectives: the reduction of current and future poverty?

1. Are the school enrollment incentives built into CCTs effective? (Do households change their behavior in response to the program?)
2. What is the impact of the program on current poverty and/or inequality?

## An « ex-ante evaluation » of Bolsa Escola

 (Bourguignon, Ferreira and Leite, WBER, 2003)- The Bolsa Escola Program (now morphed into Bolsa Familia):
- Means-test: income per capita less than R\$90 (50\% of the 1999 minimum wage)
- Conditionality : 6-15 year-olds must
- Be enrolled in school.
- Attend at least $85 \%$ of classes.
- Transfer : R\$15 per child in school
- Limit: R\$45 per household
- Monitoring at the local and federal levels
- Introduced in July 2001. No ex-post evaluations by 2003.


## Empirical strategy

1. Estimate discrete choice model for children's occupation on a preprogram cross-section.

$$
p_{i j}=\frac{\operatorname{Exp}\left[Z_{i} \cdot\left(\gamma_{j}-\gamma_{0}\right)+Y_{-i} \cdot\left(\alpha_{j}-\alpha_{0}\right)+w_{i}\left(\beta_{j}-\beta_{0}\right)\right]}{1+\sum_{j=1}^{2} \operatorname{Exp}\left[Z_{i} \cdot\left(\gamma_{j}-\gamma_{0}\right)+Y_{-i} \cdot\left(\alpha_{j}-\alpha_{0}\right)+w_{i}\left(\beta_{j}-\beta_{0}\right)\right]}
$$

2. Estimate earnings equation for children to predict counterfactual wages for all kids.

$$
\log w_{i}=X_{i} \cdot \delta+m * \operatorname{Ind}\left(S_{j}=1\right)+u_{i}
$$

3. Simulate effect of conditional transfers:

$$
\begin{array}{ll}
U_{i}(0)=Z_{i} \cdot \gamma_{0}+\alpha_{0} Y_{-1}+\beta_{0} w_{i}+v_{i 0} & \\
U_{i}(1)=Z_{i} \cdot \gamma_{1}+\alpha_{1}\left(Y_{-1}+T\right)+\beta_{1} w_{i}+v_{i 1} & \text { if } Y_{-I}+M w_{i} \leq Y^{\circ} \\
U_{i}(1)=Z_{i} \cdot \gamma_{1}+\alpha_{1} Y_{-1}+\beta_{1} w_{i}+v_{i 1} & \text { if } Y_{-I}+M w_{i}>Y^{\circ} \\
U_{i}(2)=Z_{i} \cdot \gamma_{2}+\alpha_{2}\left(Y_{-1}+T\right)+\beta_{2} w_{i}+v_{i 2} & \text { if } Y_{-I} \leq Y^{\circ} \\
U_{i}(2)=Z_{i} \cdot \gamma_{2}+\alpha_{2} Y_{-1}+\beta_{2} w_{i}+v_{i 2} & \text { if } Y_{-I}>Y^{\circ}
\end{array}
$$

## Counterfactual occupational choice

Table 6: Simulated effect of Bolsa Escola on schooling and working status (all children 10-15 years old)

|  | All Households |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Not going to school | Going to school and working | Going to school and not working | Total |
| Not going to school | $60.7 \%$ | $14.0 \%$ | $25.3 \%$ | $6.0 \%$ |
| Going to school and working | - | $97.8 \%$ | $2.2 \%$ | $16.9 \%$ |
| Going to school and not working | - | - | $100.0 \%$ | $77.1 \%$ |
|  |  |  |  | $79.0 \%$ |
| Total | $3.7 \%$ | $17.3 \%$ | $100.0 \%$ |  |

## Poor Households

|  | Not going to school | Going to school and working | Going to school and not working | Total |
| :--- | :---: | :---: | :---: | :---: |
| Not going to school | $41.3 \%$ | $21.7 \%$ | $37.0 \%$ | $8.9 \%$ |
| Going to school and working | - | $98.9 \%$ | $1.1 \%$ | $23.1 \%$ |
| Going to school and not working | - | - | $100.0 \%$ | $68.1 \%$ |
|  |  |  |  |  |
| Total | $3.7 \%$ | $24.7 \%$ | $71.6 \%$ | $100.0 \%$ |

Source: PNAD/IBGE 1999 and author's calculation

- $40 \%$ currently not enrolled would have the incentive to change status and enroll
-Impact on children currently working is smaller
-Impacts stronger for the poor (means test)

Table 7: Simulated effect on schooling and working status of alternative specifications of conditional cash transfer program (all children 10-15 years old)

|  | All Households |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Original | Bolsa escola's program | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 |
| Not going to school | 6.0\% | 3.7\% | 2.9\% | 2.2\% | 2.8\% | 3.2\% | 6.0\% |
| Going to school and working | 16.9\% | 17.3\% | 17.4\% | 17.4\% | 17.4\% | 17.5\% | 16.8\% |
| Going to school and not working | 77.1\% | 79.0\% | 79.7\% | 80.3\% | 79.8\% | 79.3\% | 77.2\% |
| Total | 100.0\% | 100.0\% | 100.0\% | 100.0\% | 100.0\% | 100.0\% | 100.0\% |
|  | Poor Households |  |  |  |  |  |  |
|  | Original | Bolsa escola's program | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 |
| Not going to school | 8.9\% | 3.7\% | 1.9\% | 0.6\% | 1.8\% | 3.6\% | 8.9\% |
| Going to school and working | 23.1\% | 24.7\% | 25.1\% | 25.4\% | 25.2\% | 24.9\% | 23.0\% |
| Going to school and not working | 68.1\% | 71.6\% | 72.9\% | 74.0\% | 73.0\% | 71.4\% | 68.2\% |
| Total | 100.0\% | 100.0\% | 100.0\% | 100.0\% | 100.0\% | 100.0\% | 100.0\% |

Source: PNAD/IBGE 1999 and author's calculation
note: Scenario 1: transfer equal R\$30, maximum per household $\mathrm{R} \$ 90$ and means test $\mathrm{R} \$ 90$
Scenario 2: transfer equal $\mathrm{R} \$ 60$, maximum per household $\mathrm{R} \$ 180$ and means test $\mathrm{R} \$ 90$
Scenario 3: diferent values for each age, no household ceiling and means test R\$90
Scenario 4: transfer equal R\$15, maximum per household R\$45 and means test R\$120
Scenario 5: Bolsa escola without conditionality
-No conditionality vs. Bolsa Escola: conditionality is key -Impact quite sensitive to changes in T, the transfer amount - Impact less sensitive to changes in the means test $\mathrm{Y}^{0}$

## Counterfactual income distribution (viewed through summary statistics)

Table 8. Simulated distributional effect of alternative specifications of the conditional cash transfer program

|  | Original | Bolsa escola's program | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mean Income per capita | 254.2 | 255.4 | 256.5 | 258.8 | 256.4 | 255.6 | 255.3 |
| Inequality measures |  |  |  |  |  |  |  |
| Gini coefficient | 0.591 | 0.586 | 0.581 | 0.570 | 0.581 | 0.585 | 0.586 |
| Mean of logarithmic deviation | 0.692 | 0.659 | 0.636 | 0.601 | 0.639 | 0.658 | 0.660 |
| Theil index | 0.704 | 0.693 | 0.682 | 0.663 | 0.684 | 0.691 | 0.693 |
| Square coeffcient of variation | 1.591 | 1.573 | 1.556 | 1.522 | 1.558 | 1.570 | 1.574 |
| Poverty measures |  |  |  |  |  |  |  |
| Poverty headcount | 30.1\% | 28.8\% | 27.5\% | 24.6\% | 27.7\% | 28.8\% | 28.9\% |
| Poverty gap | 13.2\% | 11.9\% | 10.8\% | 8.8\% | 10.9\% | 11.9\% | 12.0\% |
| Total square deviation from poverty line | 7.9\% | 6.8\% | 5.9\% | 4.6\% | 6.0\% | 6.8\% | 6.8\% |
| Annual cost of the program (million Reais) |  | 2076 | 4201 | 8487 | 3905 | 2549 | 2009 |

Source: PNAD/IBGE 1999 and author's calculation
note: Scenario 1: transfer equal $\mathrm{R} \$ 30$, maximum per household $\mathrm{R} \$ 90$ and means test $\mathrm{R} \$ 90$
Scenario 2: transfer equal $\mathbf{R} \$ 60$, maximum per household $\mathrm{R} \$ 180$ and means test $\mathrm{R} \$ 90$
Scenario 3: diferent values for each age, no household ceiling and means test $\mathrm{R} \$ 90$
Scenario 4: transfer equal $\mathrm{R} \$ 15$, maximum per household $\mathrm{R} \$ 45$ and means test $\mathrm{R} \$ 120$
Scenario 5: Bolsa escola without conditionality

### 3.2. General Equilibrium Approaches (Economy-wide policies)

- Trade liberalization may affect the wage distribution in many ways...
- Ferreira, Leite and Wai-Poi (2007) "Trade Liberalization, Employment Flows and Wage Inequality in Brazil" (WB PRWP\#4108)
- This paper asks whether trade liberalization contributed to the decline in wage inequality and, if so, how.
- Extends Goldberg \& Pavnick (2005) two-stage estimation of effects of changes in trade variables on industry and wagepremia to employment model; and combines it with JMP (1993)style simulations


## 1. Motivation

- Brazil's trade liberalization episode took place between 1988 and 1995, when tariff and non-tariff barriers were considerably reduced.

Figure 1: Protection and Import Penetration in Brazil, 1985-1999


## 1. Motivation

- During the same period, wage inequality declined. Is there a causal link?

Figure 3: Hourly Wage Inequality in Brazil, 1987-2004


$$
\square_{\text {Gini }} \text { Theil }
$$

## 2. Methodology

## 1. Estimation:

- First Stage:

$$
\begin{aligned}
& \ln w_{i j}=X_{i j} \beta+I_{i j} * w p_{j}+\left(I_{i j} * S_{i j}\right) s p_{j}+\varepsilon_{i j} \\
& \operatorname{Pr}\{j=s\}=P^{s}\left(Z_{i}, \lambda\right)=\frac{e^{Z_{i} \lambda_{s}}}{e^{Z_{i} \lambda_{s}}+\sum_{j \neq s} e^{Z_{i} \lambda_{j}}} \\
& -\quad \text { Second Stage: } \quad \Delta v_{j t}=\Delta T_{j t} \gamma+\eta_{j t}
\end{aligned}
$$

## 2. Methodology

2. Decomposition of changes in the wage distribution (using the two-stage trade effects framework):

$$
\begin{aligned}
& w_{i j}^{88}=\exp \left(X_{i j}^{88} \beta^{88}+I_{i j}^{88} * w p_{j}^{88}+\left(I_{i j}^{88} * S_{i j}^{88}\right) s p_{j}^{88}+F_{s 8}^{-1}\left(\theta_{i 88}\right)\right) \\
& w_{i j}^{1}=\exp \left(X_{i j}^{8} \beta^{88}+I_{i j}^{88} * w p_{j}^{s}+\left(I_{i j}^{88} * S_{i j}^{88}\right) s p_{j}^{88}+F_{s}^{-1}\left(\theta_{i 88}\right)\right) \quad w p_{j}^{s}=\left(T_{j}^{95}-T_{j}^{88}\right) \hat{\gamma}_{w p} \\
& w_{i j}^{2}=\exp \left(X_{i j}^{88} \beta^{88}+I_{i j}^{88} w_{1} p_{j}^{2}+\left(I_{i j}^{88} * S_{i j}^{88}\right) s p_{j}^{5}+F_{88}^{-1}\left(\theta_{i 88}\right)\right) \\
& s p_{j}^{s}=\left(T_{j}^{95}-T_{j}^{88}\right) \hat{\gamma}_{s p} \\
& w_{i j}^{3}=\exp \left(X_{i j}^{8} \beta^{88}+I_{i j}^{s} w p_{j}^{s}+\left(I_{i j}^{s} S_{i j}^{88}\right) s p_{j}^{s}+F_{s 8}^{-1}\left(\theta_{i 88}\right)\right) \\
& \lambda_{0 j}^{s}=\left(T_{j}^{95}-T_{j}^{88}\right) \hat{\gamma}_{\lambda_{0}} \\
& w_{i j}^{4}=\exp \left(X_{i j}^{88} \beta^{5}+I_{i j}^{5} w p_{j}^{95}+\left(I_{i j}^{5} * S_{i j}^{88}\right) s p_{j}^{95}+F_{8 s}^{-1}\left(\theta_{i B 8}\right)\right) \\
& \beta^{s}=\left\{\beta_{e d}^{95} ; \beta_{-e d}^{88}\right\} \\
& w_{i j}^{5}=\exp \left(X_{i j}^{83} \beta^{95}+I_{i j}^{5} * w p_{j}^{95}+\left(I_{i j}^{5} * S_{i j}^{88}\right) s p_{j}^{95}+F_{s 8}^{-1}\left(\theta_{i 88}\right)\right) \\
& w_{i j}^{6}=\exp \left(X_{i j}^{85} \beta^{95}+I_{i j}^{5} * w p_{j}^{95}+\left(I_{i j}^{5} * S_{i j}^{88}\right) s p_{j}^{95}+F_{95}^{-1}\left(\theta_{i 88}\right)\right) \\
& w_{i j}^{95}=\exp \left(X_{i j}^{95} \beta^{95}+I_{i j}^{95} * w p_{j}^{95}+\left(I_{i j}^{95} * S_{i j}^{95}\right) s p_{j}^{95}+F_{95}^{-1}\left(\theta_{i 95}\right)\right)
\end{aligned}
$$

## 3. Results (Wage Decomposition)

Table 8: Actual and Counterfactual Hourly Wage Distributions

|  | $\mathrm{P}_{90} / \mathrm{P}_{10}$ | GE (0) | GE (1) | Gini |
| :---: | :---: | :---: | :---: | :---: |
| $=\exp \left(X_{i j}^{88} \beta^{88}+I_{i j}^{88} * w p_{j}^{88}+\left(I_{i j}^{88} * S_{i j}^{88}\right) s p_{j}^{88}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 16.9 | 0.703 | 0.780 | 0.611 |
| $=\exp \left(X_{i j}^{88} \beta^{88}+I_{i j}^{88} * w p_{j}^{s}+\left(I_{i j}^{88} * S_{i j}^{88}\right) s p_{j}^{88}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 16.9 | 0.705 | 0.784 | 0.611 |
| $\exp \left(X_{i j}^{88} \beta^{88}+I_{i j}^{88} * w p_{j}^{s}+\left(I_{i j}^{88} * S_{i j}^{88}\right) s p_{j}^{s}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 16.7 | 0.699 | 0.774 | 0.60 |
| $\exp \left(X_{i j}^{88} \beta^{88}+I_{i j}^{s} * w p_{j}^{s}+\left(I_{i j}^{s} * S_{i j}^{88}\right) s p_{j}^{s}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 14.6 | 0.653 | 0.73 | 0.59 |
| $=\exp \left(X_{i j}^{88} \beta^{s}+I_{i j}^{s} * w p_{j}^{95}+\left(I_{i j}^{s} * S_{i j}^{88}\right) s p_{j}^{95}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 12.9 | 0.600 | 0.66 | 0.57 |
| $=\exp \left(X_{i j}^{88} \beta^{95}+I_{i j}^{s} * w p_{j}^{95}+\left(I_{i j}^{s} * S_{i j}^{88}\right) s p_{j}^{95}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 12.3 | 0.581 | 0.65 | 0.56 |
| $=\exp \left(X_{i j}^{88} \beta^{95}+I_{i j}^{s} * w p_{j}^{95}+\left(I_{i j}^{s} * S_{i j}^{88}\right) s p_{j}^{95}+F_{95}^{-1}\left(\theta_{i 88}\right)\right)$ | 12.0 | 0.587 | 0.691 | 0.57 |
| $=\exp \left(X_{i j}^{95} \beta^{95}+I_{i j}^{95} * w p_{j}^{95}+\left(I_{i j}^{95} * S_{i j}^{95}\right) s p_{j}^{95}+F_{95}^{-1}\left(\theta_{i 95}\right)\right)$ | 12.4 | 0.617 | 0.715 | 0.582 |

Source: Author's Calculation from PNADs.

## 3. Results (Wage Decomposition)

Figure 6: Observed and counterfactual wage growth incidence curves, 1995-1988: industry wage premia.


## 3. Results (Wage Decomposition)

Figure 7: Observed and counterfactual wage growth incidence curves, 1995-1988: industry and skill wage premia.


## 3. Results (Wage Decomposition)

Figure 8: Observed and counterfactual wage growth incidence curves, 1995-1988: all trade-mandated changes from 2nd stage.


## 3. Results (Wage Decomposition)

Table 8: Actual and Counterfactual Hourly Wage Distributions

|  | $\mathrm{P}_{90} / \mathrm{P}_{10}$ | GE (0) | GE (1) | Gini |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{3}=\exp \left(X_{i j}^{88} \beta^{88}+I_{i j}^{88} * w p_{j}^{88}+\left(I_{i j}^{88} * S_{i j}^{88}\right) s p_{j}^{88}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 16.9 | 0.703 | 0.780 | 0.611 |
| $=\exp \left(X_{i j}^{88} \beta^{88}+I_{i j}^{88} * w p_{j}^{s}+\left(I_{i j}^{88} * S_{i j}^{88}\right) s p_{j}^{88}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 16.9 | 0.705 | 0.784 | 0.611 |
| $=\exp \left(X_{i j}^{88} \beta^{88}+I_{i j}^{88} * w p_{j}^{s}+\left(I_{i j}^{88} * S_{i j}^{88}\right) s p_{j}^{s}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 16.7 | 0.699 | 0.774 | 0.609 |
| $=\exp \left(X_{i j}^{88} \beta^{88}+I_{i j}^{s} * w p_{j}^{s}+\left(I_{i j}^{s} * S_{i j}^{88}\right) s p_{j}^{s}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 14.6 | 0.653 | 0.731 | 0.593 |
| $=\exp \left(X_{i j}^{88} \beta^{s}+I_{i j}^{s} * w p_{j}^{95}+\left(I_{i j}^{s} * S_{i j}^{88}\right) s p_{j}^{95}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 12.9 | 0.600 | 0.66 | 0.57 |
| $=\exp \left(X_{i j}^{88} \beta^{95}+I_{i j}^{s} * w p_{j}^{95}+\left(I_{i j}^{s} * S_{i j}^{88}\right) s p_{j}^{95}+F_{88}^{-1}\left(\theta_{i 88}\right)\right)$ | 12.3 | 0.581 | 0.65 | 0.56 |
| $=\exp \left(X_{i j}^{88} \beta^{95}+I_{i j}^{s} * w p_{j}^{95}+\left(I_{i j}^{s} * S_{i j}^{88}\right) s p_{j}^{95}+F_{95}^{-1}\left(\theta_{i 88}\right)\right)$ | 12.0 | 0.587 | 0.691 | 0.571 |
| ${ }^{;}=\exp \left(X_{i j}^{95} \beta^{95}+I_{i j}^{95} * w p_{j}^{95}+\left(I_{i j}^{95} * S_{i j}^{95}\right) s p_{j}^{95}+F_{95}^{-1}\left(\theta_{i 95}\right)\right)$ | 12.4 | 0.617 | 0.715 | 0.582 |

Source: Author's Calculation from PNADs.

## 3. Results (Wage Decomposition)

Figure 9: Observed and counterfactual wage growth incidence curves,
1995-1988: upper-bound on trade effects.


## 3. Results (Wage Decomposition)

Figure 10: Observed and counterfactual wage growth incidence curves,
1995-1988: trade effects + other price changes.


## 3. Results (Wage Decomposition)

Figure 11: Observed and counterfactual wage growth incidence curves, 1995-1988: trade effects, price changes + changes in residuals.


### 3.2. General Equilibrium Approaches (Economy-wide policies)

- This paper adopted a "reduced-form" approach to the general equilibrium processes through which price changes (in tradable goods markets) affected other prices, wages, output and employment levels in the economy. An alternative is to model these general equilibrium processes explicitly.
- There are two basic approaches to generating GE-compatible counterfactual income distributions (and thus counterfactual GICs):
- Fully disaggregated CGE models, where each household is individually linked to the production and consumption modules. E.g. Chen and Ravallion, 2003, for China.
- "Leaner" macroeconomic models linked to microsimulation modules on a household survey dataset. E.g. Bourguignon, Robilliard and Robinson, 2005, for Indonesia.


### 3.2. General Equilibrium Approaches (Economy-wide policies)

Distributional Impact of China's accession to the WTO. (Chen \& Ravallion, 2003) GE-compatible counterfactual GICs corresponding to a specific policy.

Figure 3b: Mean percentage gain by income percentile


### 3.2. General Equilibrium Approaches (Economy-wide policies)

- In the Macro-Micro approach, some key counterfactual linkage variables are generated in a "leaner" macro model, whose parameters may have been calibrated or estimated from a timeseries, and then fed into sector-specific equations estimated in the household survey, to generate a counterfactual GIC.


Household income micro-simulation model

## An example: LAV structure (Wages: One for Urban; one for Rural)

## Sectors

## HH <br> Groups

|  | Formal <br> Tradable | Formal <br> Non-Tradable | Informal |  |
| :--- | :--- | :--- | :--- | :--- |
| Low Skill | W | W | W |  |
| Int. Skill | W | W | W |  |
| High Skill | W | W | W |  |

Note: In rural areas, intermediate and high skill groups were pooled.
Occupations: Urban x Rural; household heads, spouses, others.

Results: Changes in Unemployment Levels by Skill

| Changes in <br> Unemployment | Simulated | Actual | Error |
| :--- | :---: | :---: | :---: |
| Low Skill (U) | $7.22 \%$ | $8.49 \%$ | $-1.27 \%$ |
|  | $6.25 \%$ | $3.75 \%$ | $2.50 \%$ |
| Intermediate | $10.14 \%$ | $9.49 \%$ | $0.65 \%$ |
| Skill | $12.05 \%$ | $9.45 \%$ | $2.61 \%$ |
| High Skill (U) | $11.72 \%$ | $14.56 \%$ | $-2.84 \%$ |

Note: In rural areas, intermediate and high skill groups were pooled.

## Results: Household Incomes

Figure 7 - Comparison between
Actual Observed Changes \&
Experiment 1 - using Representative Households Groups (RHG)


## Conclusions

1. Growth, changes in poverty and changes in inequality are all summary measures of changes in the disaggregated distribution of incomes.
2. Understanding these changes requires understanding the determinants of changes in the growth incidence curve.
3. Counterfactual simulations that isolate the individual impacts of changes in prices, in occupational structure, in the distribution of household endowments, or in transfers, are a useful first step.
4. Counterfactual GICs that are consistent with (partial or general) economic equilibria are more difficult to estimate, as they involve modeling behavior. But starts have been made.

- Beware of the Lucas critique and the 'black-box' critique.

