

Measuring Long-term Inequality of Opportunity

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Motivation

- A proper understanding of income inequality requires taking longer-term incomes into account (Friedman, 1962)
 - Argument: High annual income inequality might occur side by side with little or no long-term income inequality, if individuals' ranks and/or income shares change over time
- Empirical evidence demonstrate that:
 - Annual incomes fluctuate due to transitory shocks, life-cycle factors and institutional factors (e.g. Mayer and Jencks, 1989; Saez and Chetty, 2003)
 - Individuals save and borrow to smooth consumption, e.g. by housing mortgages, pension schemes, and educational loans (Borsch-Supan, 2003)

EOp versus EO

- The EOp approach assumes that individuals' outcomes arise from two classes of variable: Variables for which they should not be held responsible for (*circumstances*) and variables which belong to the sphere of individuals' responsibility (*effort*)

Two normative principles

- *The Compensation Principle* states that differences in outcomes due to circumstances are ethically unacceptable and should be compensated
- *The Reward Principle* states that differences due to effort are to be considered ethically acceptable and do not need any intervention

Ex ante and ex post approach

- ***Ex ante approach:*** There is equality of opportunity if the set of opportunities is the same for all individuals, regardless of their circumstances. *Inequality of opportunity decreases if inequality between individual opportunity sets decreases* (Van de gaer, 1993) and Peragine, 2004)
- ***The ex-post approach:*** There is equality of opportunity if all those who exert the same effort have the same outcome. *Inequality of opportunity decreases if outcome inequality decreases among individuals who exert the same degree of effort* (Roemer, 1993, 1998)

Long-term (ex ante) Inequality of Opportunity

- Bourguignon, F., F. Ferreira, and M. Wolton, *JoEI*, 2007 introduces the following long-term social welfare function of opportunity

$$W = \min_i \sum_{t=1}^T \phi_t \mu_{it}$$

where μ_{it} is the mean income of type i at time t and ϕ_t is the time preference factor

A two-step aggregation for measuring long-term inequality of opportunity

1. First step: Each individual's income stream is aggregated into an interpersonal comp. measure of permanent income (Aaberge and Mogstad, 2009)
2. Second step: Aggregates permanent incomes across individuals into measures of inequality of opportunity
 - From (1) and (2), introduce a new family of rank-dependent measures of long-term inequality of opportunity

Permanent income measure

Objective:

- Social planner aims at evaluating the income streams of individuals

Assumptions:

- Income streams are exogenous
- Social planner employ the conventional discounted utility model with
 - Perfect foresight, where preferences are intertemporal separable and additive
 - Rate of time preferences that is non-negative and constant over time
- Intertemporal utility function common to all individuals
 - Social planner treats individuals symmetrically after adjusting for relevant non-income factors (see e.g. Hammond, 1991)

- For each individual,
 - Derive the preferred consumption profile and associated utility level
 - The permanent income measure (EAEI) is the minimum annual expenditure required to obtain the maximum utility level
- Analogous to Atkinson's (1970) equally distr. equiv. inc.

Deriving the EAEI under the assumption of a perfect capital market

Step 1) Deriving the optimal consumption profile $(C_1^*, C_2^*, \dots, C_T^*)$ and the associated utility level U

$$(2.1) \quad \max_{C_1, \dots, C_T} \sum_{t=1}^T u(C_t)(1 + \delta)^{1-t}$$

subject to the budget constraint

$$(2.2) \quad \sum_{t=1}^{T-1} C_t \prod_{j=1+t}^T (1 + r_j) + C_T = Y = \sum_{t=1}^{T-1} y_t \prod_{j=1+t}^T (1 + r_j) + y_T$$

$$(2.5) \quad \Rightarrow C_t^* = f_t(Y, \delta, r_2, r_3, \dots, r_T) \text{ for all } t = 1, 2, \dots, T$$

$$(2.6) \quad \hat{U} \equiv \sum_{t=1}^T u(C_t^*)(1 + \delta)^{1-t}$$

Step 2) Deriving the equally-allocated equivalent income Z required to obtain utility level U

$$(2.7) \quad Z = u^{-1}(\Delta^{-1} \hat{U})$$

where $u^{-1}(t) = \inf \{x : u(x) \geq t\}$

is the left inverse of u and Δ is defined by

$$(2.8) \quad \Delta = \sum_{t=1}^T (1 + \delta)^{1-t} = \frac{1 + \delta}{\delta} (1 - (1 + \delta)^{-T})$$

Credit market imperfections

If there are no liquidity constraints, the optimal consumption profile $(C_1^*, C_2^*, \dots, C_T^*)$ is defined as the solution of (2.1) subject to the budget constraints

$$(4.2) \quad \begin{aligned} S_0 &= 0 \\ S_t &= (1 + r\gamma_t)S_{t-1} + y_t - C_t, \quad t = 1, 2, \dots, T-1 \\ S_T &= (1 + r\gamma_T)S_{T-1} + y_T - C_T = 0 \end{aligned}$$

where S_t represents the assets at the end of period t earning an interest rate $r\gamma_{t+1}$, and

$$(4.3) \quad r\gamma_t = \begin{cases} rs_t & \text{if } S_{t-1} \geq 0 \\ rb_t & \text{if } S_{t-1} < 0 \end{cases}, \quad -1 < r\gamma_t < \infty, \quad t = 2, 3, \dots, T.$$

Solving this maximization problem requires comparison of 3^{T-1} conditional consumption profiles.

The optimal consumption profile is determined as the utility maximising choice among the conditional consumption profiles satisfying their respective budget constraints. By inserting the optimal consumption levels into the inter. utility function, the corresponding EAEI is obtained

The presence of liquidity constraints reduces the number of available conditional consumption profiles that have to be compared.

Rank-dependent measures of inequality

Let \mathbf{L} denote the family of Lorenz curves, and let a social planner's ranking of members of \mathbf{L} be represented by a preference ordering \succeq , which will be assumed to satisfy the following basic axioms:

Axiom 1 (Order). \succeq is a transitive and complete ordering on \mathbf{L} .

Axiom 2 (Dominance). Let $L_1, L_2 \in \mathbf{L}$. If $L_1(u) \geq L_2(u)$ for all $u \in [0, 1]$ then $L_1 \succeq L_2$.

Axiom 3 (Continuity). For each $L \in \mathbf{L}$, the sets $\{L^* \in \mathbf{L} : L \succeq L^*\}$ and $\{L^* \in \mathbf{L} : L^* \succeq L\}$ are closed (w.r.t. L_1 -norm).

Given the above continuity and dominance assumptions for the ordering \succeq , Aaberge (2001) demonstrated that the following axiom,

Axiom 4 (Independence). Let L_1, L_2 and L_3 be members of \mathbf{L} and let $\alpha \in [0, 1]$. Then $L_1 \succeq L_2$ implies $\alpha L_1 + (1 - \alpha)L_3 \succeq \alpha L_2 + (1 - \alpha)L_3$,

characterizes the rank-dependent family of inequality measures \tilde{J}_p defined by

$$(3.2) \quad J_p(L) = 1 + \int_0^1 L(v) dp(v) = 1 - \frac{1}{\mu_0} \int_0^1 p(v) F^{-1}(v) dv$$

Circumstances, effort and EOp

Let $F_i^{-1}(s) = \inf \{x : F_i(x) \geq s\}$ represents the income of the person whose rank in the distribution F_i is s , where i represents a subpopulation with equal circumstances, $i= 1, 2, \dots, m$.

Roemer (1993, 1998) suggests to take, as an inter-type comparable measure of effort, the quantile of the effort distribution in the type at which an individual sits; this, given the monotonicity of the income function, will correspond to the quantile in the income distribution of the type. We adopt this solution and hence we say that all individuals at the s quantile of their respective type income distributions have the same effort. Thus, considering types $1, \dots, n$, we define the *tranche* s as the subset of individuals whose incomes are at the s^{th} quantile of their respective type income distributions.

EOp

Ex post approach

Let $\tilde{F}_1^{-1}(s) \leq \tilde{F}_2^{-1}(s) \leq \dots \leq \tilde{F}_m^{-1}(s)$ be the ordering of incomes $F_1^{-1}(s), F_2^{-1}(s), \dots, F_m^{-1}(s)$ across types at trance (quantile) s . Since the type-specific income distributions might intersect note that the type ordering by income might change across quantiles; i.e. $\tilde{F}_k^{-1}(s)$ and $\tilde{F}_k^{-1}(v)$ might represent different types. Accordingly, the proportion of people associated with the lowest income, the second lowest income, etc might change across quantiles. Thus, let $q_i(s)$ be the population share associated with the individual having rank i at trance s . Moreover, let $a_j(s) = \sum_{i=1}^j q_i(s)$ and $b_j(s) = 1 - a_j(s)$.

Illustration

t	$F_1^{-1}(t)$	$F_2^{-1}(t)$	$F_3^{-1}(t)$
0.33	10	15	23
0.67	20	18	28
1.00	30	25	29

t	$\tilde{F}_1^{-1}(t)$	$\tilde{F}_2^{-1}(t)$	$\tilde{F}_3^{-1}(t)$
0.33	10	15	23
0.67	18	20	28
1.00	25	29	30

Quantile-specific measures of inequality

The quantile-specific mean and Lorenz curve are defined by

$$(4.2) \quad \mu(s) = \sum_{i=1}^m q_i(s) \tilde{F}_i^{-1}(s) = \sum_{i=1}^m q_i F_i^{-1}(s)$$

and

$$(4.3) \quad L(a_j(s); s) = \frac{\sum_{i=1}^j q_i(s) \tilde{F}_i^{-1}(s)}{\mu(s)}, \quad j = 1, 2, \dots, m, \quad 0 \leq s \leq 1.$$

Inserting (4.2) and (4.3) in (3.2) yields the following family of *quantile-specific rank-dependent measures of inequality of opportunity*

$$(4.4) \quad J_p(s) = 1 - \frac{1}{h(s) \mu(s)} \sum_{j=1}^m q_j(s) p(a_j(s)) \tilde{F}_j^{-1}(s),$$

where $h(s) = \sum_{j=1}^m q_j(s) p(a_j(s))$, $p(1) = 1$ and $\frac{1}{m-1} \sum_{j=1}^m p(a_j) = 1$.

Overall ex post measures of inequality of opportunity

Principle of Utilitarian Reward introduced by Fleurbaey (2008) states that any “equalizing transfer” between tranches should not change income inequality or social welfare, whatever the effort level of the persons involved in it. That is, an equal weight should be assigned to each tranche-specific inequality measure, whatever degree of effort has been exerted. Accordingly, the average of the tranche-specific inequality of opportunity measures

$$(4.5) \quad J_p = \int_0^1 J_p(s) ds$$

defines a family of rank-dependent measures of *overall inequality of opportunity*.

Maxmin

Note that p_e defined by

$$(4.7) \quad p_e(a_j(s)) = \begin{cases} 1, & j = 1 \\ 0, & j = 2, \dots, m \end{cases}$$

represents the upper limit of inequality aversion exhibited by the family of non-increasing weight functions p . By inserting (4.7) in (4.4), (4.5) and (4.6) we get

$$(4.8) \quad J_e(s) = 1 - \frac{\tilde{F}_1^{-1}(s)}{\mu(s)} = 1 - \frac{\min_{1 \leq i \leq m} F_i^{-1}(s)}{\mu(s)},$$

$$(4.9) \quad J_e = 1 - \int_0^1 \frac{\min_{1 \leq i \leq m} F_i^{-1}(s)}{\mu(s)} ds$$

Ex ante approach

Assume that $\mu_1 \leq \mu_2 \leq \dots \leq \mu_m$ and that q_i is associated with μ_i . Moreover we assume that every individual of a sub-population receives the same income equal to the sub-group mean. Thus, in this case the group-specific distributions are defined by

$$(4.13) \quad F_i^*(x) = \begin{cases} 0, & x < \mu_i \\ 1, & x \geq \mu_i \end{cases}$$

and the associated Lorenz curve by

$$(4.14) \quad L^*(a_j) = \frac{\sum_{i=1}^j q_i \mu_i}{\sum_{i=1}^m q_i \mu_i}, \quad j = 1, 2, \dots, m.$$

The associated rank-dependent family of inequality measures is defined by

$$(4.15) \quad J_p^* = 1 - \frac{\sum_{j=1}^m q_j p(a_j) \mu_j}{\mu \sum_{j=1}^m q_j p(a_j)} .$$

The J_p^* measure captures the inequality between types: this inequality, according to the *Compensation Principle*, can be interpreted as ex ante inequality of opportunity. The welfare function associated with J_p^* is defined by

$$(4.16) \quad W_p^* = \mu(1 - J_p^*) = \frac{\sum_{j=1}^m q_j p(a_j) \mu_j}{\sum_{j=1}^m q_j p(a_j)}$$

Maxmin

$$(4.17) \quad J_e^* = 1 - \frac{\min_{1 \leq i \leq m} \mu_i}{\mu}$$

and

$$(4.18) \quad W_e^* = \min_{1 \leq i \leq m} \mu_i.$$

Note that W_e^* is equal to the EOp welfare function introduced by Roemer (1993, 1998).

Ex post and ex ante Gini measures of inequality of opportunity

Ex post Gini

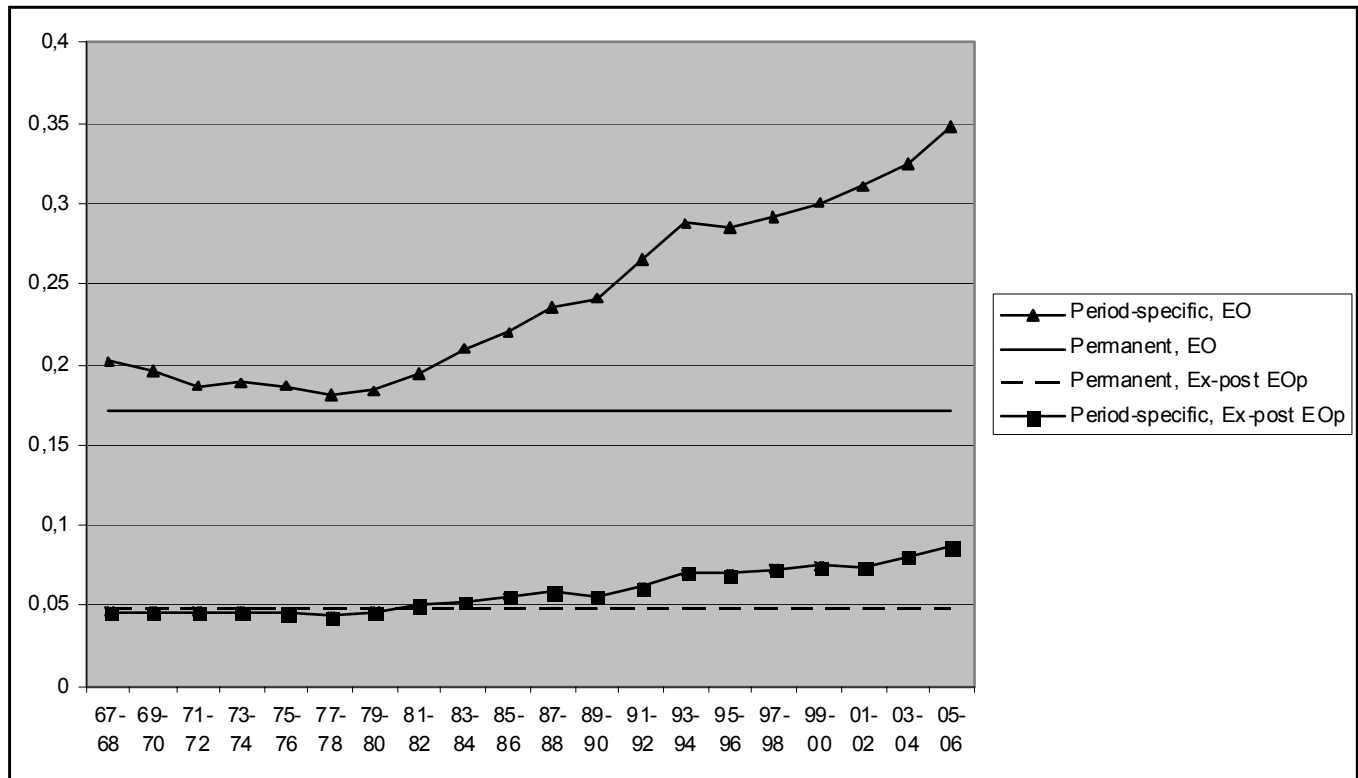
$$G = \int_0^1 G(s) ds = 1 - \frac{\int_0^1 \left[\sum_{j=1}^m q_j(s) b_j(s) \frac{\tilde{F}_i^{-1}(s)}{\mu(s)} \right] ds}{\sum_{j=1}^m q_j b_j}$$

Ex ante Gini

$$G^* = 1 - \frac{\sum_{j=1}^m q_j b_j \mu_j}{\mu \sum_{j=1}^m q_j b_j}$$

Empirical results

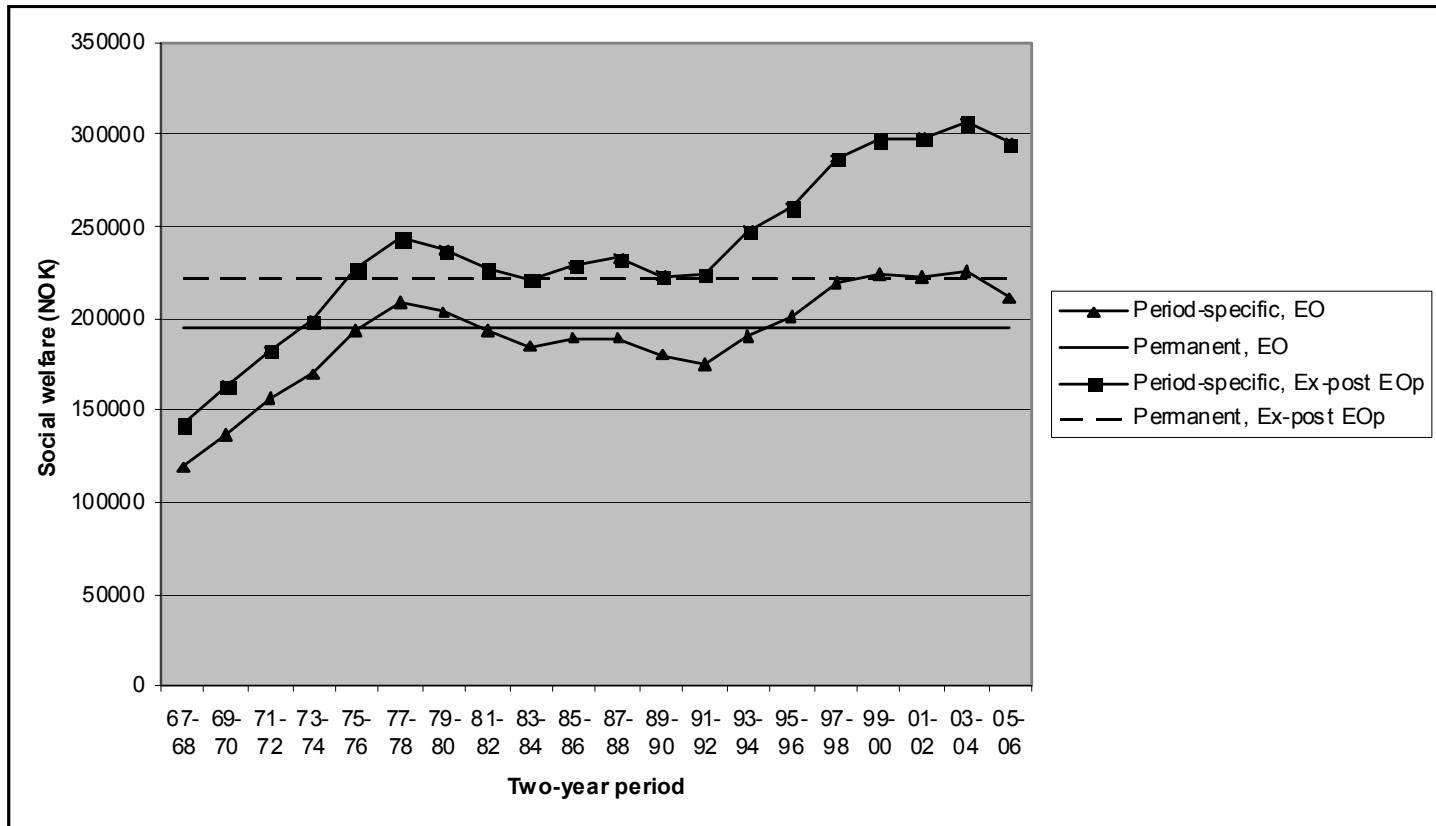
Figure 2. Gini-inequality according to EO and ex-post EO based on period-specific income and permanent income



Notes: Permanent income is defined by (2.1), (2.8) and (2.9) with $\epsilon = 2$ and $\delta = .02$, and annual real interest rates on borrowing and savings given by Table A1 in Appendix A. The ex-post Gini EO measure of inequality is defined by (4.12).

The pattern of social welfare

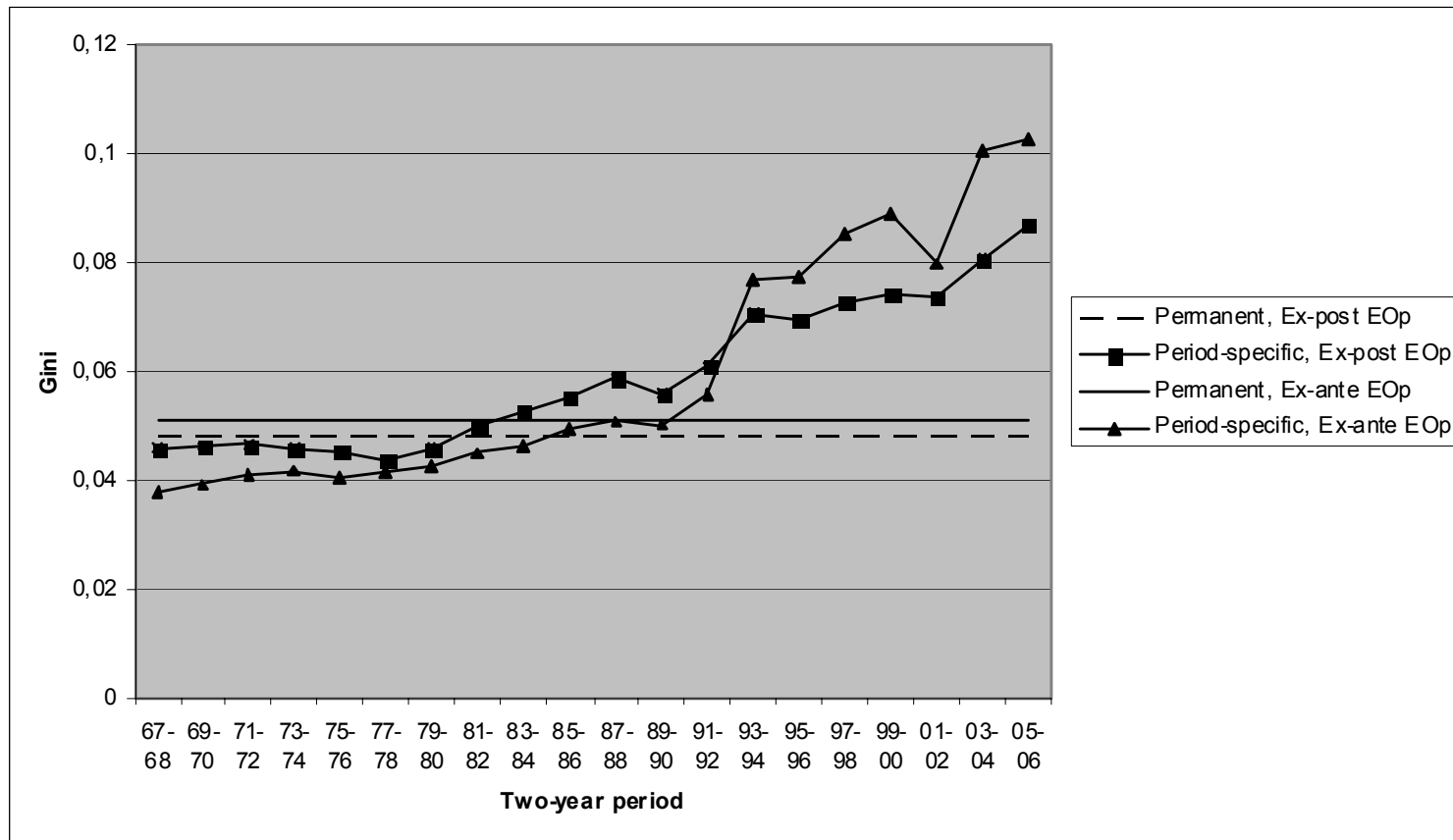
Figure 3. Social welfare according to EO and ex-post EOp based on period-specific income and permanent income



Notes: Permanent income is defined by (2.1), (2.8) and (2.9) with $\epsilon = 2$ and $\delta = .02$, and annual real interest rates on borrowing and savings given by Table A1 in Appendix A. The ex-post EOp social welfare measure is defined by (4.6).

Ex post versus ex ante inequality

Figure 4. Gini-inequality according to ex-ante and ex-post EOp based on period-specific income and permanent income



Notes: Permanent income is defined by (2.1), (2.8) and (2.9) with $\varepsilon = 2$ and $\delta = .02$, and annual real interest rates on borrowing and savings given by Table A1 in Appendix A. The Gini ex-post and ex ante EOp measures of inequality are defined by (4.12) and (4.19).