Upward Structural Mobility Measurement: Subgroup Consistency versus Rank-dependency

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1 Introduction

Fields (2007): mobility as a multifaced concept. Here we deal with 2 aspects of mobility:

Exchange mobility: increases if the correlation between incomes in two successive periods decreases while keeping the marginal income distributions constant.

Structural mobility compares situations that have different marginal distributions in the two periods. It amounts to factual mobility "caused by differential change in the stratum distribution" (Yasuda (1964), p.16) or -in an intergenerational context- by "the amount of mobility generated by the fact that the distribution among social strata experienced by the sons differs from the corresponding experience of their fathers" (Boudon (1973), p.17).

The formal literature has focused on exchange mobility, whereas the concept of upward structural mobility has received only little attention.

This talk: focus on (upward) structural mobility, using an axiomatic approach.

Based on joint work with Christian Schluter [RISC, RIW forthcoming] and Thomas Demuynck [RD, working paper].

We'll be confronted with several issues already mentioned during the past few days:

- how to aggregate individual mobilities (Frank Cowell).

- measurement of mobility: similar to the measurement of multidimensional inequality?

Empirical Illustration: comparison of mobility US/ Germany. (PSID, GSOEP)

- Conventional wisdom: US more mobile than Germany.

- Standard mobility measures: Germany more mobile than US (Burkhauser et al (1997), Maasoumi and Trede (2001), Gottschalk and spolaore (2002), Schluter and Trede (2003))

- Subgroup consistent measure: US more mobile than Germany.

- rank dependent measure: Germany more mobile than US.

2 RISC mobility

2.1 Notation and Core Axioms:

$y_1 \in \mathbb{R}^n_{++} \to y_2 \in \mathbb{R}^n_{++}$ and $(y_1, y_2) \in D = \cup_{n=1}^{\infty} \mathbb{R}^{2n}_{++}$.

Consider only two groups of individuals with $g \in \{1, 2\}$. Let $P = \left\{N^1, N^2\right\}$ be a partition of the set $N = \{1, \ldots, n\}$ in two non-overlapping subsets and let $\mathcal{P}$ denote the set of all such possible partitions of N. For each group g of size N^g, the income vectors are partitioned correspondingly into $\left(y_1^g, y_2^g\right)$.

A replication invariant subgroup consistent (RISC) mobility index is a non-constant function $M : D \to \mathbb{R}$, continuous in its arguments, whose value indicates the amount of mobility in moving from a distribution y_1 to y_2, and which satisfies the following axioms:

RISC.1 [Anonymity] $M(y_1, y_2)$ is symmetric: $M(y_1, y_2) = M\left(y_1', y_2'\right)$ whenever the vectors y_1' and y_2' are obtained after applying the same permutation on y_1 and y_2.

RISC.2 [Replication Invariance] $M(y_1, y_2) = M\left(y_1', y_2'\right)$, whenever the vectors y_1' and y_2' are obtained after applying the same replication on y_1 and y_2.

RISC.3 [Subgroup Consistency] For all $P \in \mathcal{P}$:

$M\left(y_1^1, y_1^2, y_2^1, y_2^2\right) > M\left(y_1^{1\prime}, y_1^{2\prime}, y_2^{1\prime}, y_2^{2\prime}\right)$ whenever $M\left(y_1^1, y_2^1\right) > M\left(y_1^{1\prime}, y_2^{1\prime}\right)$ and $M\left(y_1^2, y_2^2\right) = M\left(y_1^{2\prime}, y_2^{2\prime}\right)$.

We now formalize our mobility properties.

(1) Exchange Mobility.

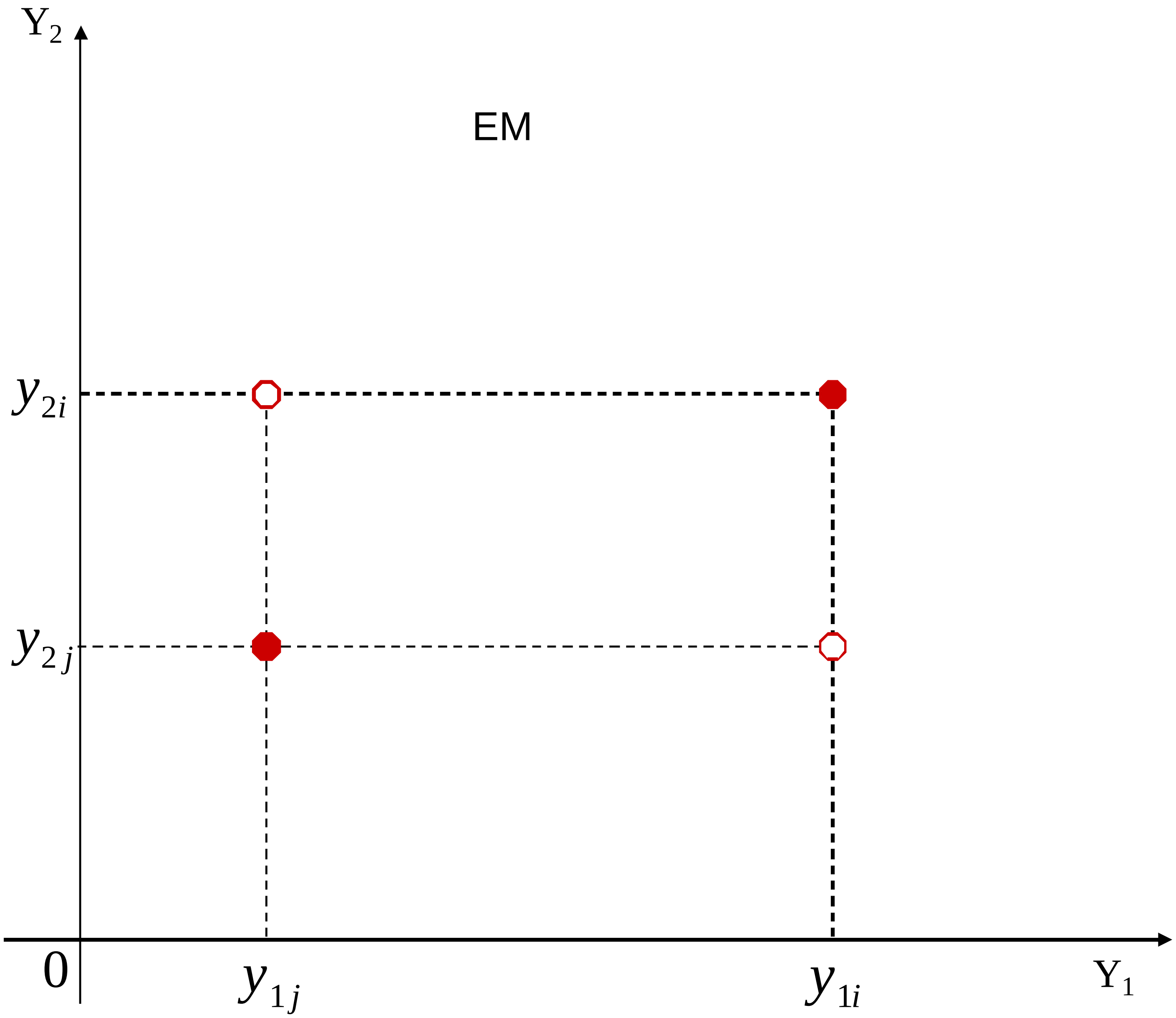
Given a vector $x = \left(x_1, \ldots, x_i, \ldots, x_j, \ldots x_n\right) \in \mathbb{R}^n_{++}$,

define $x\left(\sigma_{ij}\right) = \left(x_1, \ldots, x_j, \ldots, x_i, \ldots x_n\right) \in \mathbb{R}^n_{++}$.

EM For all $i, j \in N$ and all $y_1, y_2, y_1\left(\sigma_{ij}\right), y_2\left(\sigma_{ij}\right) \in \mathbb{R}^n_{++}$:

$\left(y_{1i} - y_{1j}\right)\left(y_{2i} - y_{2j}\right) > 0$

$\Rightarrow M\left(y_1, y_2\left(\sigma_{ij}\right)\right) > M\left(y_1, y_2\right)$ and $M\left(y_1\left(\sigma_{ij}\right), y_2\right) > M\left(y_1, y_2\right)$.



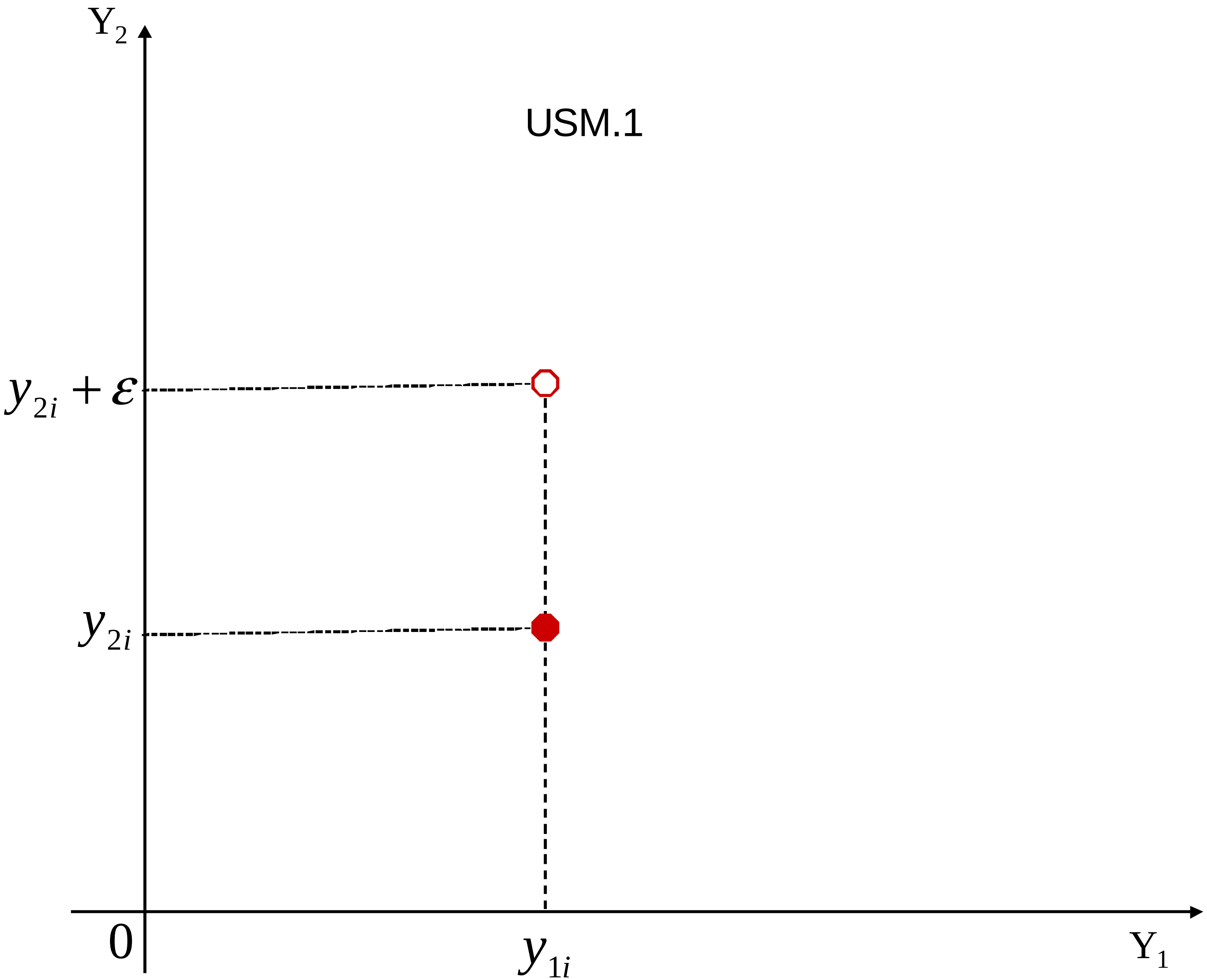
Y_2
EM
y_{2i}
y_{2j}
0
y_{1j}
y_{1i}
Y_1

(2) Upward Structural Mobility.

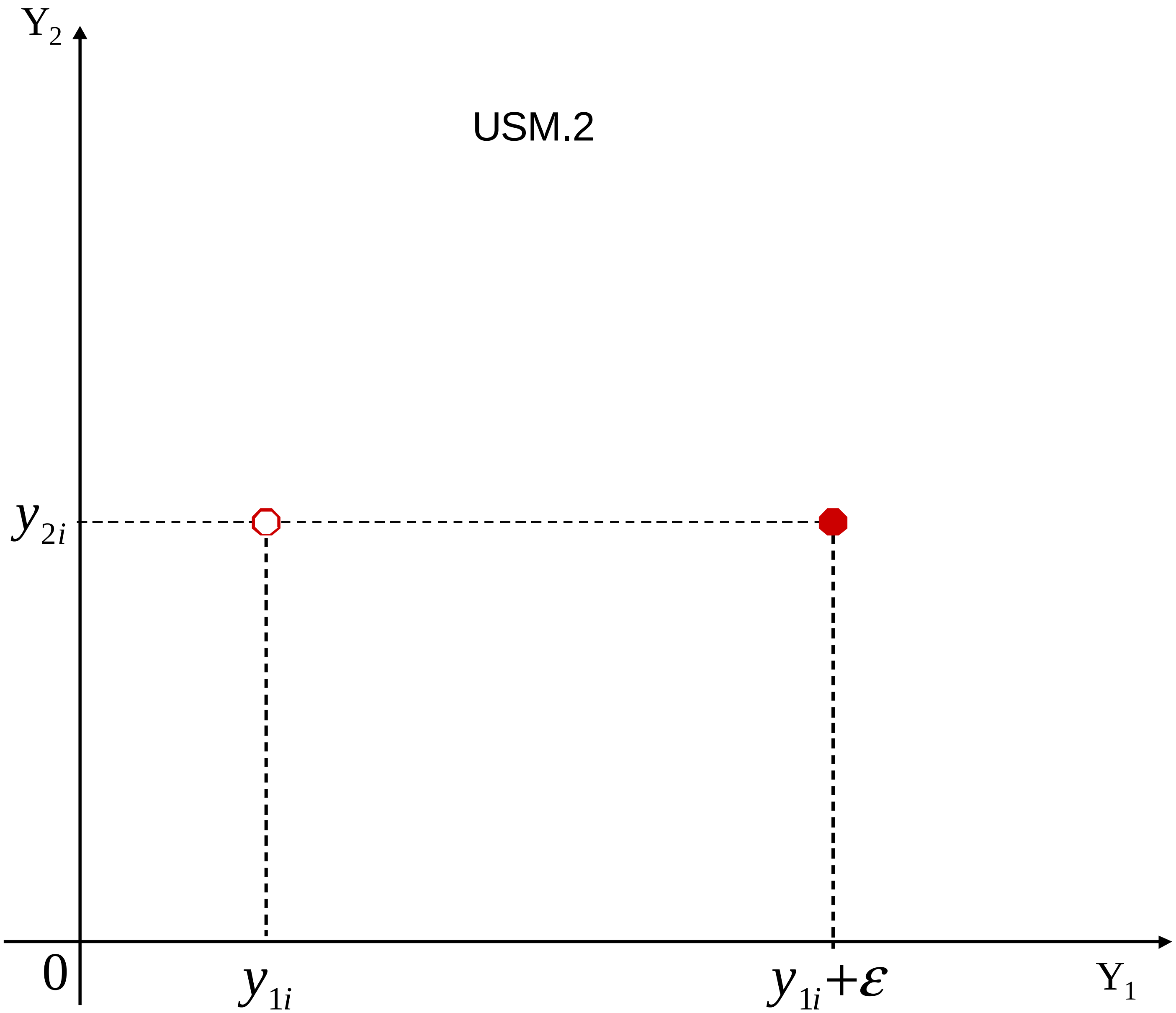
Given a vector $x = (x_1, \ldots, x_i, \ldots x_n) \in \mathbb{R}^n_{++}$, consider any $\varepsilon \in \mathbb{R}_{++}$ such that $x^{+\varepsilon(i)} = (x_1, \ldots, x_i + \varepsilon, \ldots x_n) \in \mathbb{R}^n_{++}$.

USM.1 For all $i \in N$ and all $y_1, y_2 \in \mathbb{R}^n_{++} : M\left(y_1, y_2^{+\varepsilon(i)}\right) > M(y_1, y_2)$.

USM.2 For all $i \in N$ and all $y_1, y_2 \in \mathbb{R}^n_{++} : M(y_1, y_2) > M\left(y_1^{+\varepsilon(i)}, y_2\right)$.



Y_2
USM.1
$y_{2i} + \varepsilon$
y_{2i}
0
y_{1i}
Y_1


USM.2
Y_2
y_{2i}
0
y_{1i}
$y_{1i}+\varepsilon$
Y_1

(3) Further structural mobility axioms.

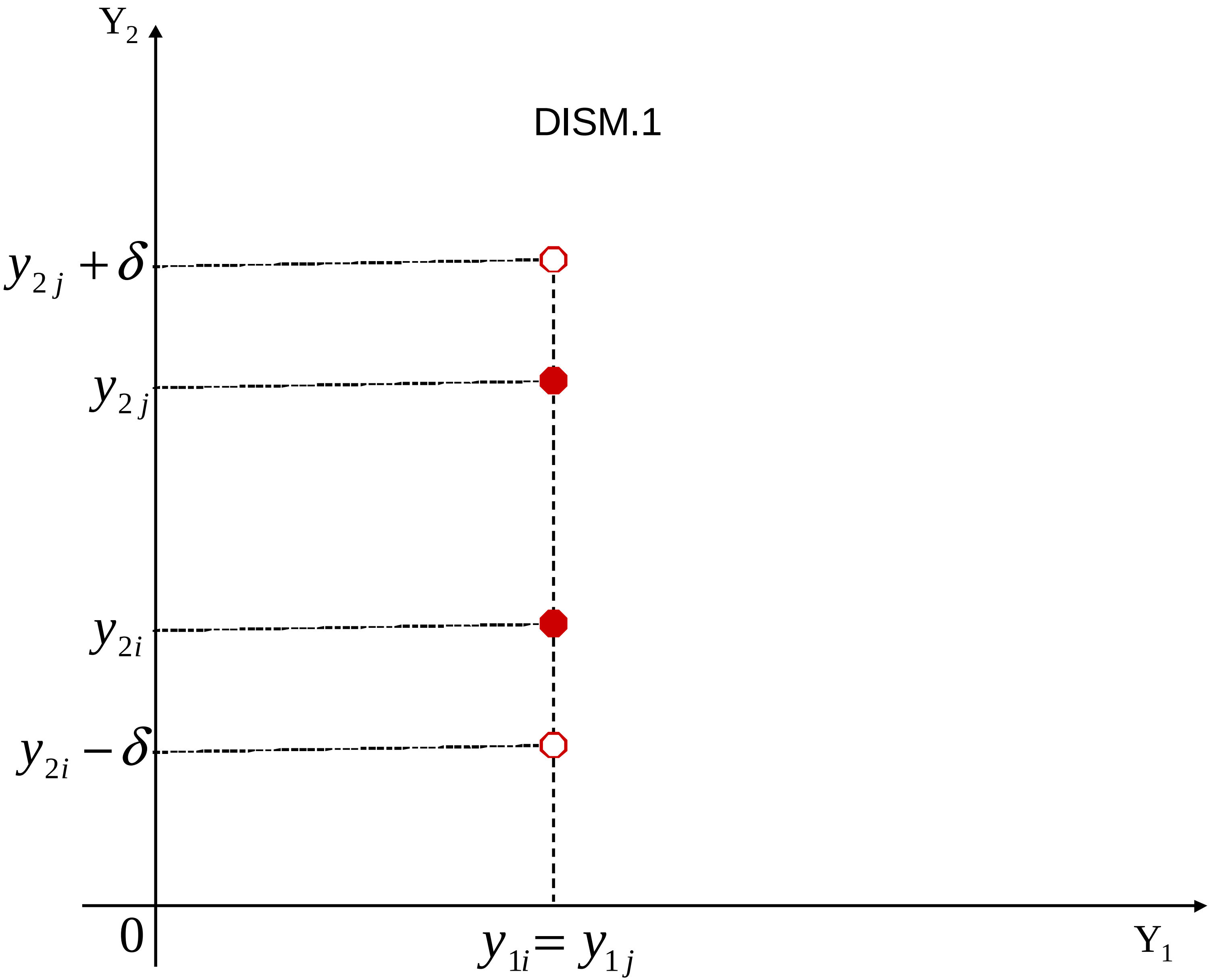
Given a vector $x = \left(x_1, \ldots, x_i, \ldots, x_j, \ldots x_n\right) \in \mathbb{R}^n_{++}$, consider $\delta \in \mathbb{R}_{++}$ such that $x^{\delta(ij)} = \left(x_1, \ldots, x_i - \delta, \ldots, x_j + \delta, \ldots x_n\right) \in \mathbb{R}^n_{++}$.

Given $i, j \in N, R^n_{++}(ij) = \left\{x \in \mathbb{R}^n_{++} : x_i = x_j\right\}$.

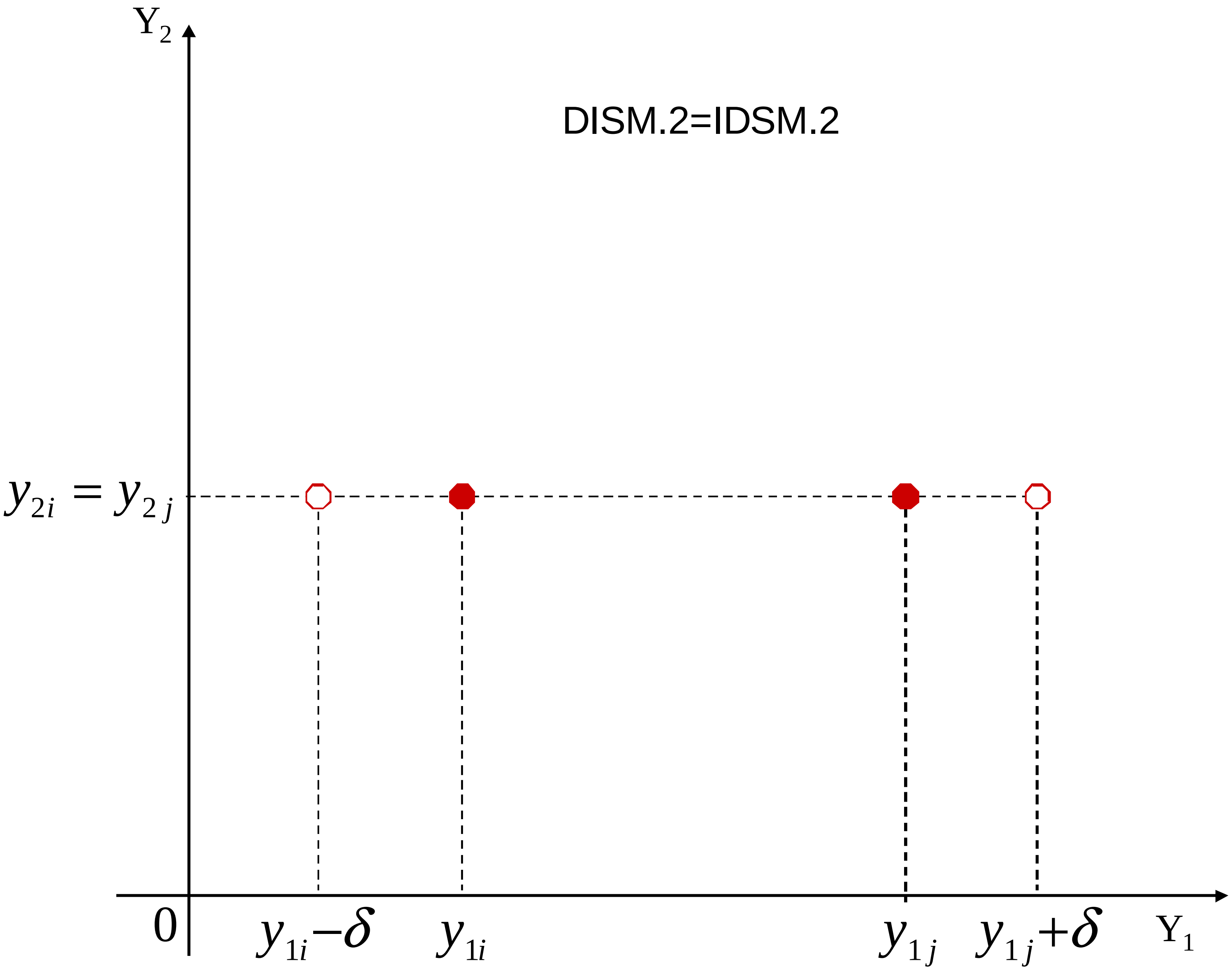
Distance Increasing Structural Mobility.

DISM.1 For all $i, j \in N$ with $y_{2j} \geq y_{2i} > \delta \in \mathbb{R}_{++}$, all $y_1 \in R^n_{++}(ij)$ and all $y_2 \in \mathbb{R}^n_{++} : M\left(y_1, y_2^{\delta(ij)}\right) > M\left(y_1, y_2\right)$.

DISM.2 For all $i, j \in N$ with $y_{1j} \geq y_{1i} > \delta \in \mathbb{R}_{++}$, all $y_2 \in R^n_{++}(ij)$ and all $y_1 \in \mathbb{R}^n_{++} : M\left(y_1^{\delta(ij)}, y_2\right) > M\left(y_1, y_2\right)$.



Y_2
DISM.1
$y_{2j} + \delta$
y_{2j}
y_{2i}
$y_{2i} - \delta$
0
$y_{1i} = y_{1j}$
Y_1

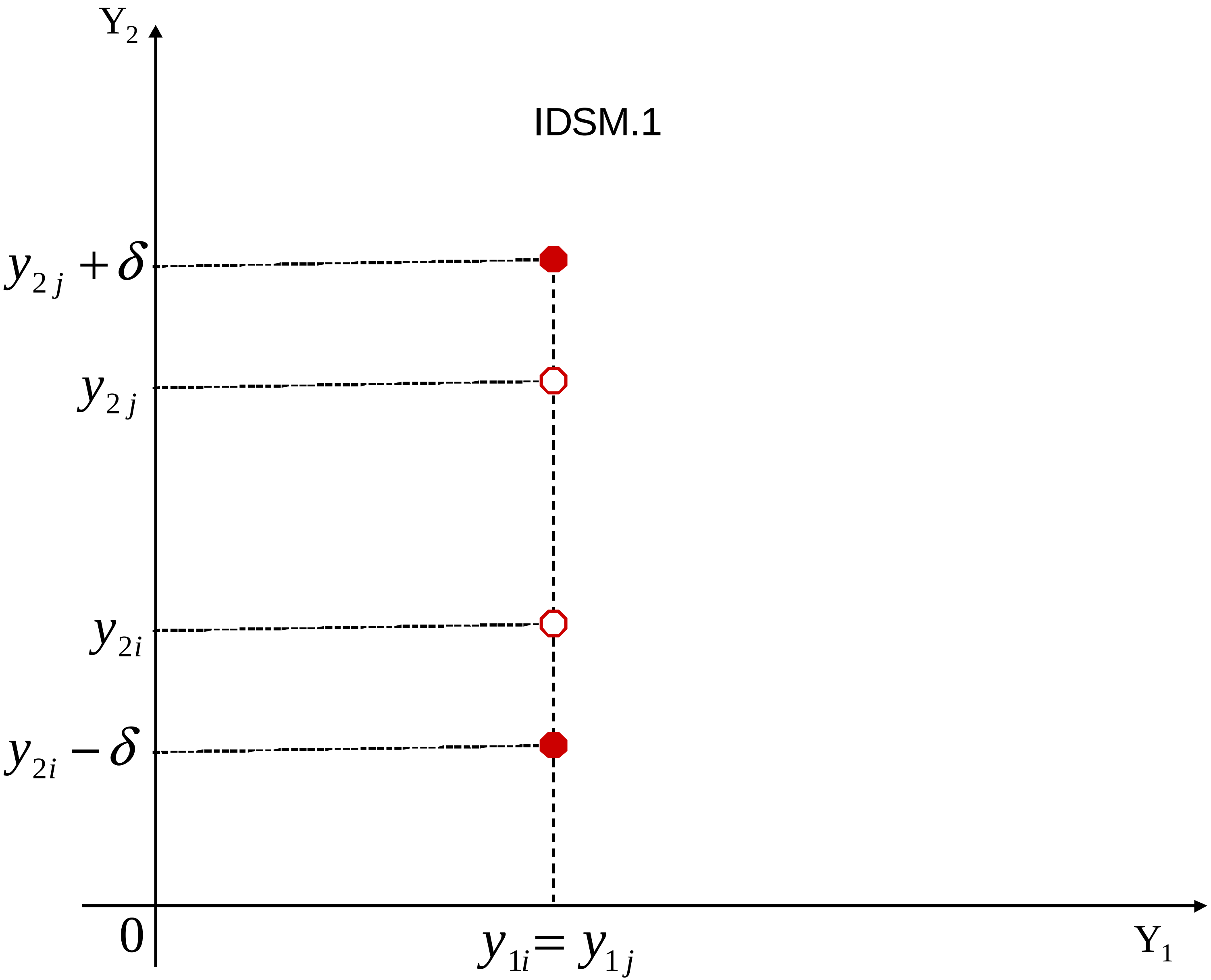


Y_2
DISM.2=IDSM.2
$y_{2i} = y_{2j}$
0
$y_{1i} - \delta$
y_{1i}
y_{1j}
$y_{1j} + \delta$
Y_1

Inequality Decreasing Structural Mobility.

IDSM.1 For all $i, j \in N$ with $y_{2j} \geq y_{2i} > \delta \in \mathbb{R}_{++}$, all $y_1 \in R^n_{++}(ij)$ and all $y_2 \in \mathbb{R}^n_{++}$: $M(y_1, y_2) > M\left(y_1, y_2^{\delta(ij)}\right)$.

IDSM.2 For all $i, j \in N$ with $y_{1j} \geq y_{1i} > \delta \in \mathbb{R}_{++}$, all $y_2 \in R^n_{++}(ij)$ and all $y_1 \in \mathbb{R}^n_{++}$: $M\left(y_1^{\delta(ij)}, y_2\right) > M(y_1, y_2)$.



IDSM.1
Y_2
$y_{2j}+\delta$
y_{2j}
y_{2i}
$y_{2i}-\delta$
0
$y_{1i}=y_{1j}$
Y_1

2.2 Results

Lemma 2.1 *:* Replication Invariant Subgroup Consistent (RISC) Mobility Measures*.*

For each $n \geq 1$ *and every* $(y_1, y_2) \in \mathbb{R}^{2n}_{++}$ *a replication invariant subgroup consistent mobility measure can be written as*

$$F\left(\frac{1}{n}\sum_{i=1}^{n}\phi\left(y_{1i}, y_{2i}\right)\right), \tag{1}$$

where $F : \phi\left(\mathbb{R}^2_{++}\right) \rightarrow \mathbb{R}$ *is continuous and increasing and* $\phi : \mathbb{R}^2_{++} \rightarrow \mathbb{R}$ *is continuous.*

Particular RISC mobility measures have been proposed by Fields and Ok (1996, 1999), namely

$$\begin{aligned} M_{FO_1} &= \frac{1}{n}\sum_{i=1}^{n}\left|y_{2i}-y_{1i}\right|^{\alpha}, \quad M_{F0_2}=\frac{1}{n}\sum_{i=1}^{n}\left|\log\left(\frac{y_{2i}}{y_{1i}}\right)\right|^{\alpha} \\ M_{FO_3} &= \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{y_{2i}}{y_{1i}}\right), \end{aligned}$$

where $\alpha \in \mathbb{R}_{++}$,

$$\begin{aligned} M_{FO_4} &= \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(y_{2i})^{1-\sigma}}{1-\sigma}-\frac{(y_{1i})^{1-\sigma}}{1-\sigma}\right| \text{ for } 0 \leq \sigma \neq 1 \\ \text{and } M_{FO_4} &= \frac{1}{n}\sum_{i=1}^{n}\left|\log\left(\frac{y_{2i}}{y_{1i}}\right)\right| \text{ for } \sigma = 1, \end{aligned}$$

where $\sigma \in \mathbb{R}_{++}$ and by D'Agostino and Dardanoni (2007),

$$M_{D_1}=\frac{1}{n}\sum_{i=1}^{n}\left(y_{1i}-y_{2i}\right)^2 \text{ and } M_{D_2}=\frac{1}{n}\sum_{i=1}^{n}\left(g\left(y_{1i}\right)-g\left(y_{2i}\right)\right)^2,$$

where $g\left(.\right)$ is a continuous and increasing function.

Corollary 2.2 $M_{FO_1}, M_{F0_2}, M_{FO_3}, M_{FO_4}, M_{D_1}$ *and* M_{D_2} *are RISC measures of mobility.*

Table 1: existing RISC measures and their properties

measure satisfies axiom ?	M_{FO_1}	M_{F0_2}	M_{FO_3}	M_{FO_4}	M_{D_1}	M_{D_2}
EM	No	No	No	No	Yes	Yes
USM	No	No	Yes	No	No	No
DISM	No	No	No	No	Yes	No
IDSM	No	No	Yes	No	No	No

Relative Mobility Measurement:

A specific functional form for a mobility measure satisfying EM and USM can be obtained by imposing two additional standard axioms:

RSI [Ratio-scale Invariance] For all $y_1, y_2, x_1, x_2 \in \mathbb{R}^n_{++}$ and for all $\lambda_1, \lambda_2 \in \mathbb{R}_{++}$: $M(y_1, y_2) = M(x_1, x_2) \Leftrightarrow M(\lambda_1 y_1, \lambda_2 y_2) = M(\lambda_1 x_1, \lambda_2 x_2)$.

SI [Scale Invariance] For all $y_1, y_2 \in \mathbb{R}^n_{++}$ and for all $\lambda \in \mathbb{R}_{++}$: $M(\lambda y_1, \lambda y_2) = M(y_1, y_2)$.

Theorem 2.3 : A new index of relative mobility.

A replication invariant, subgroup consistent mobility index satisfies EM, USM, RSI and SI if and only if for each $n \geq 1$ *and every* $(y_1, y_2) \in \mathbb{R}^{2n}_{++}$ *it can be written as*

$$F\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_{2i}}{y_{1i}}\right)^{r}\right), \tag{2}$$

where $F : \phi\left(\mathbb{R}^{2}_{++}\right) \rightarrow \mathbb{R}$ *is continuous and increasing and* $r \in \mathbb{R}_{++}$*. In addition, the measure satisfies DISM if and only if* $r > 1$ *and IDSM if and only if* $r < 1$*.*

Remark: replace EM by not EM and we have a characterization of

$$F\left(\frac{1}{n}\sum_{i=1}^{n}\left(\log\left(y_{2i}\right)-\log\left(y_{1i}\right)\right)\right),$$

where $F : \phi\left(\mathbb{R}^2_{++}\right) \rightarrow \mathbb{R}$ is continuous and increasing. The measure always satisfies IDSM.

-$>$ an alternative characterization of M_{FO_3}, as the only "pure" measure of upward structural mobility satisfying the other axioms.

Absolute Mobility Measurement

Replace $\mathbb{R}^n_{++}$ by $\mathbb{R}^n$ in all domain definitions, define ι as an n-dimensional vector of ones and replace RSI and SI by, respectively,

TSI [Translation-scale Invariance] For all $y_1, y_2, x_1, x_2 \in \mathbb{R}^n$ and for all $\kappa_1, \kappa_2 \in \mathbb{R}$: $M(y_1, y_2) = M(x_1, x_2) \Leftrightarrow M(y_1 + \kappa_1 \imath, y_2 + \kappa_2 \imath) = M(x_1 + \kappa_1 \imath, x_2 + \kappa_2 \imath)$.

AI [Addition Invariance] For all $y_1, y_2 \in \mathbb{R}^n$ and for all $\kappa \in \mathbb{R}_{++}$:

$$M(y_1 + \kappa\iota, y_2 + \kappa\iota) = M(y_1, y_2).$$

Theorem 2.4 : A new index of absolute mobility.

A replication invariant, subgroup consistent mobility index satisfies EM, USM, TSI and AI if and only if for each $n \geq 1$ *and every* $(y_1, y_2) \in \mathbb{R}^{2n}$ *it can be written as*

$$F\left(\frac{1}{n}\sum_{i=1}^{n} \exp\left[c\left(y_{2i} - y_{1i}\right)\right]\right), \tag{3}$$

where $F : \phi\left(\mathbb{R}^2_{++}\right) \to \mathbb{R}$ *is continuous and increasing and* $c \in \mathbb{R}_{++}$*. The measure always satisfies DISM.*

2.3 Income Mobility in the USA and Germany Revisited

Received wisdom: Germany exhibits both lower income inequality and lower income mobility.

Burkhauser et al. (1997a, 1997b), using Shorrocks (1978) indices: Germany is typically ranked more mobile than the US.

Gottschalk and Spolaore (2002): mobility leads to a larger decline in inequality in the US than in Germany, but its effect on intertemporal fluctuations and aversion to second period risk makes the impact of mobility in total more similar.

Schluter and Trede (2003): the difference between the distribution of time averaged income and the average of the marginal distributions is larger in Germany than in the US for low incomes, and the Shorrocks indices (with G.E. and Gini) give a larger weight to what happens in this part of the distribution, resulting in more mobility in Germany than in the US.

Data: “Equivalent Data Files” of the US Panel Study of Income Dynamics (PSID) and the German Socio-Economic Panel (GSOEP).

The unit of analysis is the person, the income concept is net (i.e. post-tax post-benefit) income equivalised using the OECD scale (equal to the square root of the household size) in 1996 prices. We follow the literature and trim each sample at the 1% and 99% quantile. The resulting samples are in excess of 10,000 persons.

period	$M(.2)$	$M(.4)$	$M(.7)$	$M(1)$	$M(1.5)$	$M(2)$
			PSID			
1984/85	1.009	1.027	1.072	1.151	1.460	2.703
	(0.0007)	(0.0015)	(0.0035)	(0.0088)	(0.0570)	(0.4268)
1985/86	1.008	1.024	1.065	1.132	1.351	1.889
	(0.0007)	(0.0014)	(0.0030)	(0.0058)	(0.0194)	(0.0758)
1986/87	1.008	1.023	1.062	1.126	1.330	1.811
	(0.0006)	(0.0014)	(0.0029)	(0.0055)	(0.0173)	(0.0611)
1987/88	1.014	1.035	1.083	1.157	1.400	2.202
	(0.0006)	(0.0013)	(0.0030)	(0.0070)	(0.0440)	(0.3444)
1988/89	1.010	1.028	1.069	1.136	1.339	1.804
	(0.0006)	(0.0013)	(0.0028)	(0.0054)	(0.0167)	(0.0595)
1989/90	1.002	1.012	1.040	1.090	1.250	1.628
	(0.0006)	(0.0013)	(0.0026)	(0.0049)	(0.0175)	(0.0859)
1990/91	1.003	1.014	1.049	1.116	1.389	2.355
	(0.0007)	(0.0014)	(0.0034)	(0.0078)	(0.0386)	(0.2217)
1991/92	1.002	1.013	1.045	1.104	1.311	1.861
	(0.0007)	(0.0014)	(0.0030)	(0.0060)	(0.0213)	(0.0886)

period	$M\,(.2)$	$M\,(.4)$	$M\,(.7)$	$M\,(1)$	$M\,(1.5)$	$M\,(2)$
GSOEP						
1984/85	1.003	1.01	1.031	1.069	1.205	1.585
	0.0006	0.0013	0.0029	0.0059	0.0222	0.0945
1985/86	1.005	1.014	1.034	1.068	1.177	1.485
	(0.0006)	(0.0012)	(0.0026)	(0.0054)	(0.0237)	(0.1232)
1986/87	1.015	1.033	1.066	1.109	1.211	1.374
	(0.0005)	(0.0011)	(0.0021)	(0.0035)	(0.0081)	(0.0194)
1987/88	1.007	1.017	1.037	1.063	1.127	1.224
	(0.0005)	(0.0010)	(0.0018)	(0.0029)	(0.0058)	(0.0118)
1988/89	1.007	1.017	1.038	1.066	1.139	1.269
	(0.0005)	(0.0010)	(0.0020)	(0.0035)	(0.0090)	(0.0260)
1989/90	1.004	1.011	1.025	1.047	1.101	1.192
	(0.0005)	(0.0010)	(0.0019)	(0.0031)	(0.0065)	(0.0147)
1990/91	1.01	1.022	1.047	1.078	1.151	1.262
	(0.0005)	(0.0010)	(0.0020)	(0.0031)	(0.0066)	(0.0149)
1991/92	1.005	1.013	1.031	1.057	1.122	1.231
	(0.0005)	(0.0011)	(0.0021)	(0.0034)	(0.0071)	(0.0159)

period	$M(.2)$	$M(.4)$	$M(.7)$	$M(1)$	$M(1.5)$	$M(2)$
			$M_{PSID}(r) > M_{GSOEP}(r)$			
1984/85	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
1985/86	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
1986/87	FALSE	FALSE	FALSE^{ns}	TRUE	TRUE	TRUE
1987/88	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
1988/89	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
1989/90	FALSE	TRUE^{ns}	TRUE	TRUE	TRUE	TRUE
1990/91	FALSE	FALSE	TRUE^{ns}	TRUE	TRUE	TRUE
1991/92	FALSE	FALSE^{ns}	TRUE	TRUE	TRUE	TRUE

We conclude that the US exhibits more (joint) structural and exchange mobility than Germany.

[possible exception: low values for r, later years in the sample.]

Decompositions by population subgroups.

$$M^g(r) = \frac{1}{n_g} \sum_{i \in I_g} \left(\frac{y_{2i}}{y_{1i}} \right)^r$$

where I_g denotes the set of individuals in group g which is of size n_g. The overall index is thus

$$M(r) = \sum_g (n_g/n) \, M^g(r) \, .$$

3 Rank dependent mobility

Consider the RISC Mobility measures again. They can be written as

$$F\left(\frac{1}{n}\sum_{i=1}^{n}\phi\left(y_{1i},y_{2i}\right)\right), \tag{4}$$

where $F : \phi\left(\mathbb{R}^2_{++}\right) \longrightarrow \mathbb{R}$ is continuous and increasing and $\phi : \mathbb{R}^2_{++} \longrightarrow \mathbb{R}$ is continuous.

Natural interpretation: ordinally equivalent to the average of the individual mobility functions $\phi\left(y_{1i},y_{2i}\right)$.

Idea of the paper: why use an unweighted average of the individual mobilities?

Example: $\mathbf{x} = (10, 10, 10)$, $\mathbf{y} = (10, 20, 30)$ and $\mathbf{z} = (20, 20, 20)$. Individual mobility: income growth:

$\mathbf{x} \to \mathbf{y} : (1, 2, 3)$ and $\mathbf{x} \to \mathbf{z} : (2, 2, 2)$

unweighted mobility: both transitions are equivalent.

We propose an axiomatisation of a specific aggregation procedure, allowing the researcher to weight individual mobilities on the basis of their rank order.

Empirical Illustration: comparison of mobility US/ Germany. (PSID, GSOEP)

3.1 Notation and Core Axioms

$y_1 \in \mathbb{R}^n_{++} \rightarrow y_2 \in \mathbb{R}^n_{++}$.

$z_i = (y_{1i}, y_{2i})\,, z = (z_1, z_2, ..., z_n)\,, Z^n = \mathbb{R}^{2n}_{++}$.

A mobility index is a non-constant function $M^n(z) : Z^n \rightarrow \mathbb{R}$, whose value indicates the amount of mobility in the population.

Measuring Individual Mobility

M [Monotonicity]: $M^1(y_{1i}, y_{2i})$ increasing in y_{2i}, decreasing in y_{1i}.

SI [Scale Invariance]: $M^1(y_{1i}, y_{2i}) = M^1(\lambda y_{1i}, \lambda y_{2i})$, $\lambda \in \mathbb{R}_+$.

MPI [Multiplicative Path Independence]:

$y_{1i} \rightarrow y_{2i} \rightarrow y_{3i}$: $M^1(y_{1i}, y_{3i}) = M^1(y_{1i}, y_{2i}) \cdot M^1(y_{2i}, y_{3i})$.

API [Additive Path Independence]:

$y_{1i} \rightarrow y_{2i} \rightarrow y_{3i}$: $M^1(y_{1i}, y_{3i}) = M^1(y_{1i}, y_{2i}) + M^1(y_{2i}, y_{3i})$.

LEMMA:

- $M^1(y_{1i}, y_{2i})$ satisfies M, SI and MPI iff $M^1(y_{1i}, y_{2i}) = (y_{2i}/y_{1i})^r$ for some $r \in \mathbb{R}_+$;

- $M^1(y_{1i}, y_{2i})$ satisfies M, SI and API iff $M^1(y_{1i}, y_{2i}) = c \ln(y_{2i}/y_{1i})$ for some $c \in \mathbb{R}_+$.

For convenience, write $m(z_1) \equiv M^1(z_1)$.

Let $\widehat{z}$ be a permutation of z such that $m(\widehat{z}_1) \geq m(\widehat{z}_2) \geq \ldots \geq m(\widehat{z}_n)$.

[Framework: similar to Bossert (1990, JET), characterization of the single-series Gini and use of Donaldson and Weymark (1980, JET)]

Axioms:

(1) Two of the RISC axioms are maintained:

AN [Anonymity, was RISC.1] $M^n(z)$ is symmetric: $M^n(z) = M^n(z')$ whenever z' is obtained after applying a permutation on z,

PI [Population Invariance, was RISC.2] $M^n(z) = M^{kn}(z')$, whenever z' is obtained as a $k-$fold replication ($k \in \mathbb{N}_{++}$) of z.

(2) RISC.3 [Subgroup consistency] is replaced by two axioms that, together, are logically weaker.

WD [Weak Decomposabilty] Total mobility is a weakly increasing function of individual mobilities.

D-HM [Decomposability w.r.t. Highest Mobilities] Total mobility depends on the mobility in the population consisting of the subpopulation of the $n-1$ highest individual mobilities and the value of $\widehat{z}_n$.

(3) Two transformation axioms of the individual mobilities are added.

TI [Translation Invariance] Adding a positive constant to all individual mobilities in two equally mobile situations does not change the equality of their mobility indices.

RI [Relative Invariance] Multiplying all individual mobilities with a positive constant in two equally mobile situations does not change the equality of their indices.

(4) distributional Axiom:

PLM [Priority to lower mobilities]: increasing a lower individual mobility by a given amount has a larger effect than increasing a higher individual mobility by the same amount.

3.2 Results

Theorem 3.1 *For all $n \in \mathbb{N}$ if M^n satisfies M, SI, MPI, RI, TI, D-HM, PI and PLM, then there exist a $\delta \geq 1$ and a number $r > 0$, such that:*

$$M^n(\mathbf{y}_1, \mathbf{y}_2) = \frac{1}{n^\delta} \sum_{i=1}^{n} (i^\delta - (i-1)^\delta) \ \tilde{m}_i.$$

with,

$$m_i = M^1(y_{1i}, y_{2i}) = (y_{2i}/y_{1i})^r.$$

If API is satisfied instead of MPI, then,

$$m_i = M^1(y_{1i}, y_{2i}) = r \ln(y_{2i}/y_{1i}).$$

$\delta \geq 0$ is a sensitivity parameter: as δ increases, more weight is given to lower levels of mobility: $\delta = 0$: only $m(\widehat{z}_1)$ matters, $\delta = 1$: all individual mobilities get equal weight, $\delta = 2$ traditional Gini weights, $\delta = \infty$: only $m(\widehat{z}_n)$ matters.

Remember: the RISC mobility measures mentioned in the previous paper belong to this class with $\delta = 1$!

Income Mobility in the USA and Germany Revisited Again

Data: the same as before (but includes the more recent period).

Individual mobilities:

(1) $m(z_i) = \log(y_{2i}) - \log(y_{1i})$

(2) $m(z_i) = \left(\frac{y_{2i}}{y_{1i}}\right)^r, r \in \mathbb{R}_{++}$

Inference: based on bootstrap procedure.

1984/85	log	$M(.2)$	$M(.4)$	$M(.7)$	$M(1)$	$M(1.5)$	$M(2)$
δ	$M_{PSID}(r) > M_{GSOEP}(r)$						
1	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
2	F^{ns}	F^{ns}	T^{ns}	T^{ns}	TRUE	TRUE	TRUE
4	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	F^{ns}
6	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
8	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

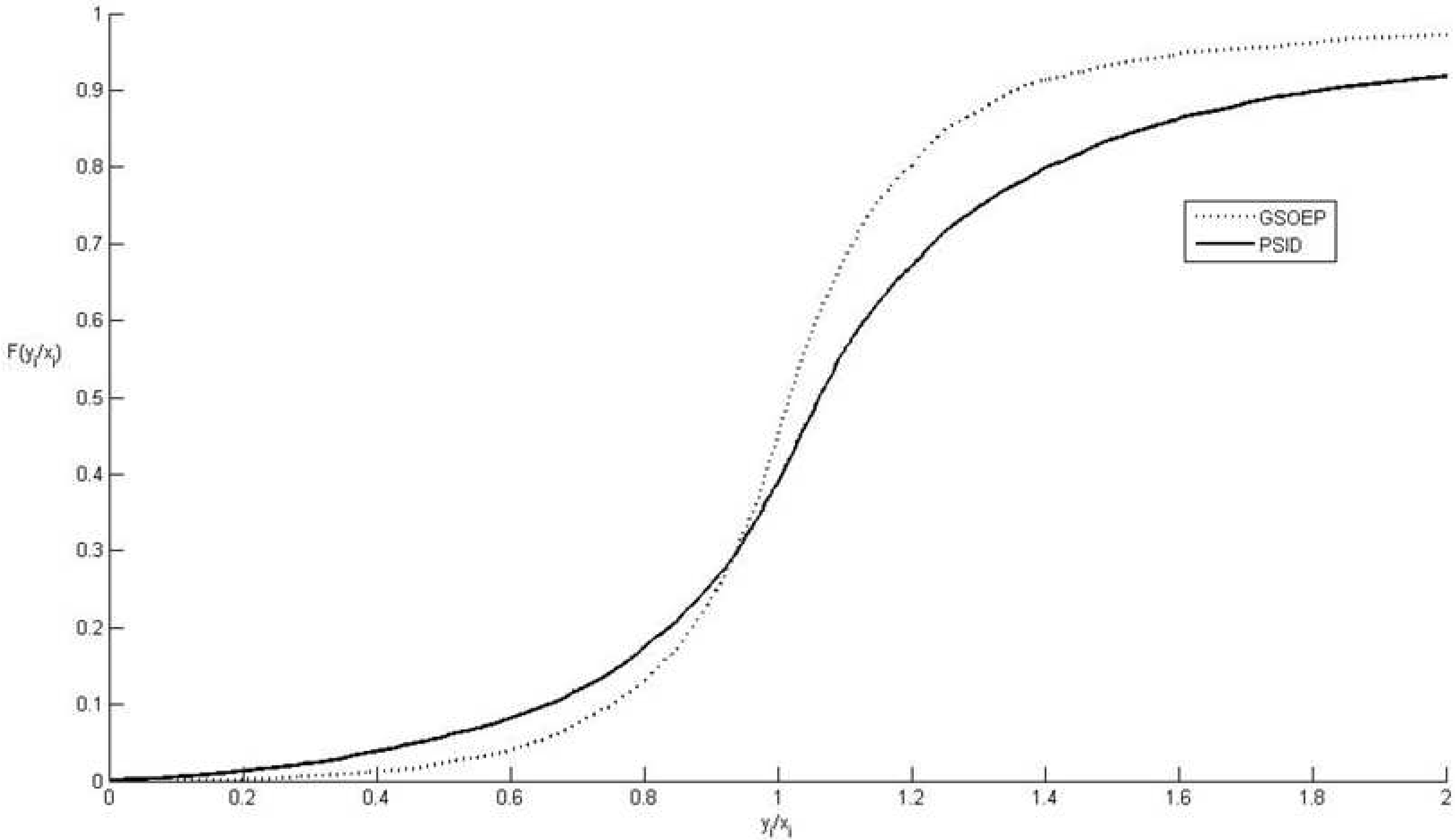
1996/97	log	$M(.2)$	$M(.4)$	$M(.7)$	$M(1)$	$M(1.5)$	$M(2)$
δ	$M_{PSID}(r) > M_{GSOEP}(r)$						
1	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
2	FALSE	FALSE	FALSE	FALSE	F^{ns}	TRUE	TRUE
4	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
6	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
8	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

Comparison US-Germany:

- $\delta = 1$, we get the result of the previous section;

- as δ increases, the picture changes: Germany becomes more mobile than the US. Reason: higher $\delta \Rightarrow$ larger weight for lower ranked individual mobilities. Hence, those with lower ranked indivudual mobilities in Germany have a higher value for $m(z_i)$ than their equally lowly ranked counterparts in the US. Meaning: those that experience a *drop* in individual incomes in Germany have a smaller percentage decrease in their incomes than their equally lowly ranked counterparts in the US.

- as r increases a higher value for δ is necessary before the ranking changes. Reason: increases in r do not change the ranking of individual mobilities but increase the differences between low and high mobilities such that the weight given to low mobilities has to increase before the ranking switches.



F(y_i/x_i)
y_i/x_i
GSOEP
PSID
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

4 Conclusion

(1) A priori, it is not self-evident that individual mobilities should aggregated in an unweighted way (what is done by many mobility measures), especially when mobility measures something ”good”, like income growth.

(2) Empirical illustration shows that the way individual mobilities are weighted on the basis of their rank order can matter in the comparison of mobility.

General message: when using a mobility measure, be careful. A researcher faces several choices:

- "welfare evaluation" versus "upward structural mobility": increasing in first period income or decreasing?

- distance increasing structural mobility versus inequality decreasing structural mobility.

- subgroup consistency versus rank dependency: weighted versus unweighted individual mobilities? Observe that choices have consequences, e.g., the weighted mobility measures respond non trivially to EM type transformations.

- (not dealt with here): other decomposition axioms exist, e.g., let the population be partitioned in k subgroups of sizes n_i $(i = 1, ..., k)$ and require that mobility in the total population is a function of mobility in each subgroup, the mean first and second period incomes in each subgroup and each of the n_i. This is the decomposition axiom used in Cowell (1985) for the characterization of distributional change.