Statistical Methods for Distributional Analysis

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Outline

Introduction

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  - volume 2
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  - survey of theory, methods underlying good practice
  - guide to the tools available to the practitioner in this field
- Not just standard inference
  - how to model distributions
  - how to handle data problems

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- *Distribution* F. Set of all distribution functions will be denoted $\mathbb{F}$.
- *Indicator function* $\iota(\cdot)$. For logical condition D:

$$\iota(D) = \begin{cases} 1 & \text{if } D \text{ is true} \\ 0 & \text{if } D \text{ is not true} \end{cases}$$

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  - may exclude some sections of the population.

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  - *Stratification*: oversampling certain categories to ensure that adequate representation of certain types

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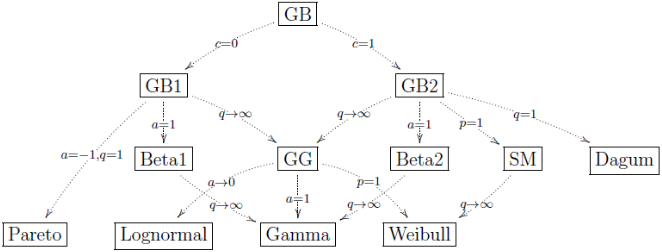
- C: Extension of estimation problem with grouped data
- D: Trimming

Outline

Parametric density estimation

Most of the standard parametric income distributions are special cases of the Generalized Beta distribution:



Goodness of Fit

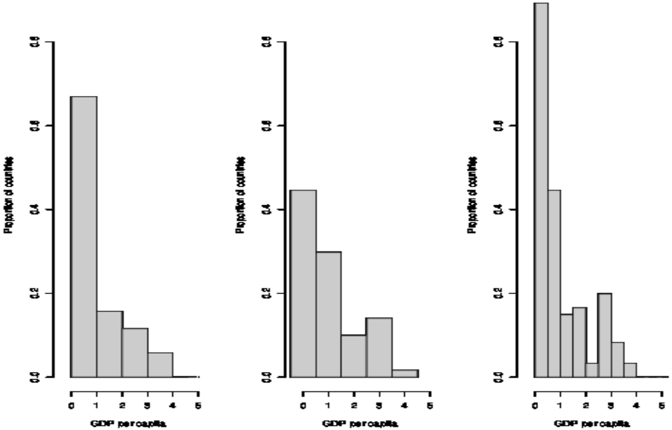
- Density based statistic: Pearson chi-squared (χ^2)
- EDF based statistic: $\begin{cases} \text{Kolmogorov-Smirnov} \\ \text{Anderson-Darling} \\ \text{Cramér-von-Mises} \end{cases}$
- Cowell et al. (2011) developed a GoF test based on axiomatic discussion, with social-welfare foundations:

$$G_\xi = \frac{1}{\xi^2 - \xi} \sum_{i=1}^{n} \left[\left[\frac{u_i}{\mu_u} \right]^{\xi} \left[\frac{2i}{n+1} \right]^{1-\xi} - 1 \right],$$

where $u_i = F(y_{(i)}; \hat{\theta})$ and $y_{(i)}$ is the ith smallest observat.
- The pearson χ^2 statistic has poor finite sample properties

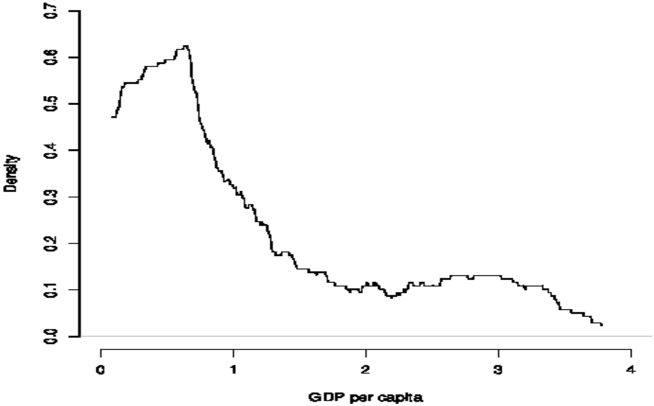
Outline

Histograms



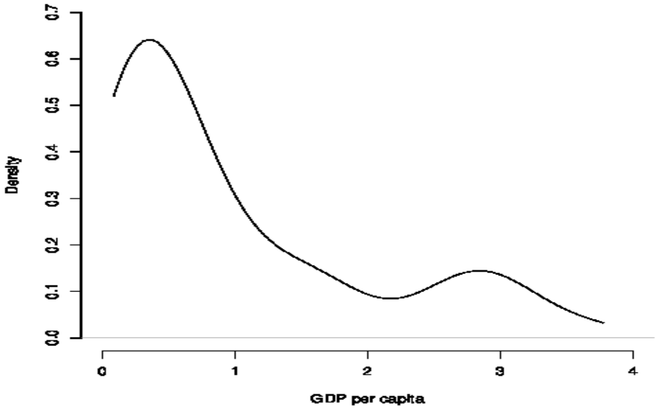
Histogram's sensitivity to the position and the number of bins

Naive estimator



Naive estimator of GDP per capita

Kernel estimator



Kernel estimator of GDP per capita

Bandwidth selection

The kernel density estimator

$$\hat{f}(y) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{y - y_i}{h}\right)$$

is not really affected by the choice of the kernel $K()$, but it is sensitive to the choice of the bandwidth h

Bandwidth selection:

- Silverman's rule-of-thumb, $\hat{h}_{opt} = 0.9 \min\left(\hat{\sigma}\,; \frac{\hat{q}_3 - \hat{q}_1}{1.349}\right) n^{-\frac{1}{5}}$.
- Plug-in method
- Cross-validation

Adaptive kernel

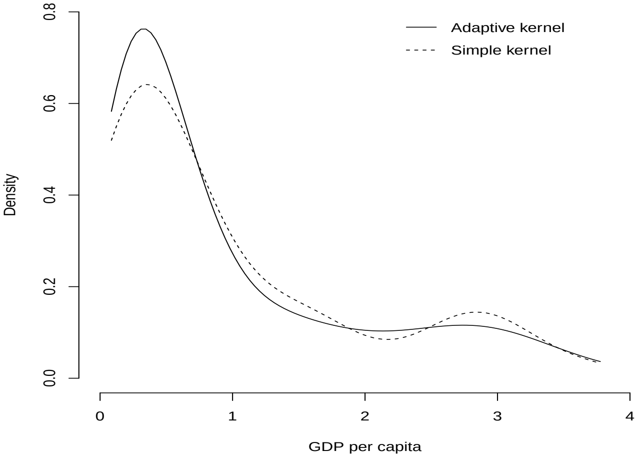
- When the concentration of the data is markedly heterogeneous, a fixed bandwidth may be quite restrictive.

- The adaptive kernel estimator is defined as follows:

$$\hat{f}(y) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h\lambda_i}K\left(\frac{y-y_i}{h\lambda_i}\right),$$

where λ_i is a parameter that varies with the local concentration of the data, $\lambda_i = [g/\tilde{f}(y_i)]^{\alpha}$.

Adaptive kernel



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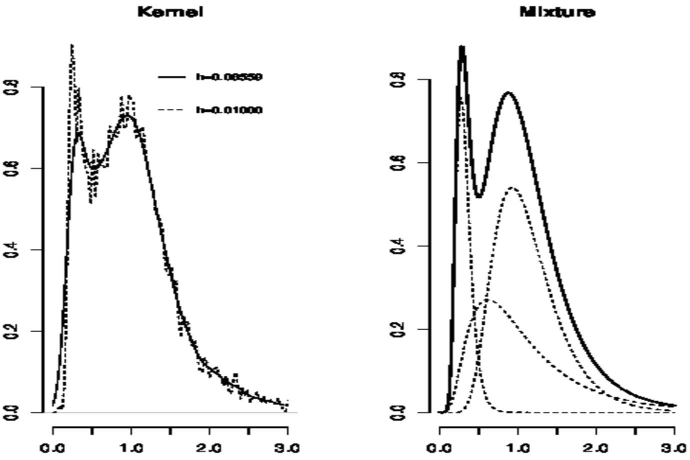
Finite-mixture models

- Under regularity conditions, any distribution can be consistently estimated by a mixture of Normal distributions
- Estimate any income distrib. with a mixture of lognormals :

$$f(\log y; \Theta) = \sum_{k=1}^{K} \pi_k \Phi(y_k; \mu_k, \sigma_k)$$

- Interpretation: a (heterogeneous) population can be decomposed into several distinct (homogeneous) groups
- Brings out the link between parametric and nonparametric estimator ($K = 1$ and $K = n$, $\pi_k = 1/n$)

Finite-mixture models



Income distribution in the United Kingdom in 1973

Finite-mixture models with covariates

- Covariates can be introduced in probabilities to characterized group profiles

$$f(\log y|z;\Theta) = \sum_{k=1}^{K} \pi_k(z_k;\alpha_k)\Phi(y_k;\mu_k,\sigma_k)$$

- Covariates can be introduced into the modeling of the densities in each of the groups, leading us to consider mixture of regression models

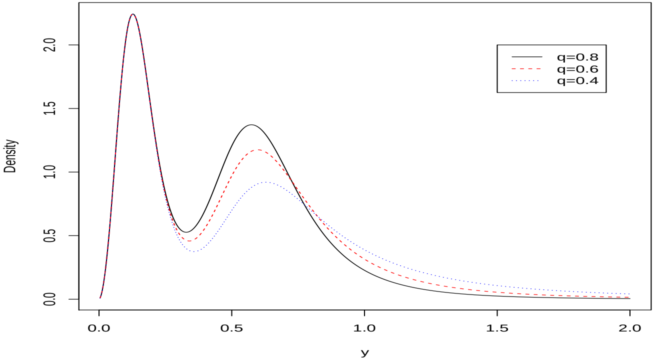
$$f(\log y|x;\Theta) = \sum_{k=1}^{K} \pi_k\Phi(y_k|x_k;\mu_k,\sigma_k)$$

- Covariates can be introduced in both components

Outline

Finite sample properties

- Quality of the fit: MIAE $= E\left(\int_0^\infty \left|\hat{f}(y) - f(y)\right| \mathrm{d}y\right)$.
- Data are drawn from lognormals, Singh-Maddala and mixtures of two SM distributions.



Finite sample properties

	Standard kernel			Adaptive kernel			Mixture
	Silv.	CV	Plug-in	Silv.	CV	Plug-in	lognormal
Lognormal							
$\sigma = 0.5$	0.1044	0.1094	0.1033	0.0982	0.1098	0.1028	0.0407
$\sigma = 0.75$	0.1326	0.1326	0.1252	0.1098	0.1283	0.1179	0.0407
$\sigma = 1$	0.1643	0.1716	0.1522	0.1262	0.1609	0.1362	0.0407
Singh-Maddala							
$q = 1.7$	0.0942	0.1009	0.0951	0.0915	0.0994	0.0934	0.0840
$q = 1.2$	0.1039	0.1100	0.1048	0.0947	0.1050	0.0994	0.0920
$q = 0.7$	0.1346	0.1482	0.1326	0.1049	0.1349	0.1175	0.0873
Mixture of two Singh-Maddala							
$q = 0.8$	0.2080	0.1390	0.1328	0.1577	0.1356	0.1224	0.1367
$q = 0.6$	0.2458	0.1528	0.1463	0.1896	0.1457	0.1293	0.1464
$q = 0.4$	0.2885	0.1953	0.1733	0.2234	0.1812	0.1450	0.1366

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  - Examples:
    - $c_1 = C(F;1) = \mu(F)$, mean of distribution F
    - $\frac{c_q}{c_1} = \frac{C(F;q)}{C(F;1)}$, income share of bottom $100q$ percent

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- How "important" is point mass at z?

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- Provides key to large class of indices used in economics

Outline

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Outline

Stat methods

FAC-EF

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  - $\bar{Z} = n^{-1} \sum_{i=1}^{n} Z_i$

Outline

Finite sample properties

	Theil		Gini		SST	
	asym	boot	asym	boot	asym	boot
Lognormal						
$\sigma = 0.5$	0.927	0.936	0.942	0.943	0.926	0.952
$\sigma = 1.0$	0.871	0.913	0.922	0.936	0.945	0.940
$\sigma = 1.5$	0.746	0.854	0.876	0.920	0.964	0.937
Singh-Maddala						
$q = 1.7$	0.915	0.931	0.945	0.944	0.945	0.950
$q = 1.2$	0.856	0.905	0.925	0.934	0.945	0.951
$q = 0.7$	0.647	0.802	0.847	0.906	0.939	0.946

Table: Coverage of asymptotic and bootstrap confidence intervals at the 95% level for the Theil, Gini and SST indices, $n = 500$.

Inequality indices: unreliable CI with heavy-tailed distributions!

Inference with heavy-tailed distributions

	asym	boot	varstab	semip	mixture
Lognormal					
$\sigma = 0.5$	0.927	0.936	0.939	0.937	0.942
$\sigma = 1.0$	0.871	0.913	0.907	0.921	0.946
$\sigma = 1.5$	0.746	0.854	0.850	0.915	0.944
Singh-Maddala					
$q = 1.7$	0.915	0.931	0.933	0.926	0.928
$q = 1.2$	0.856	0.905	0.899	0.905	0.912
$q = 0.7$	0.647	0.802	0.796	0.871	0.789

Table: Coverage of asymptotic and bootstrap confidence intervals at the 95% level for the Theil index, for several bootstrap approaches, $n = 500$.

Testing equality of inequality measures

- Null hypothesis:

$$H_0 : W_x = W_y$$

- Independent samples: $X = \{x_1, \ldots, x_n\}, Y = \{y_1, \ldots, y_m\}$
- Test statistic:

$$\tau = (\hat{W}_x - \hat{W}_y)/[\widehat{\text{var}}(\hat{W}_x) + \widehat{\text{var}}(\hat{W}_y)]^{1/2}$$

- Monte Carlo permutation tests:
  - $F_x = F_y$: exact inference!![1]
  - $F_x \neq F_y$: not valid
- Dufour et al. (2013) propose a new bootstrap method:
  - with exact inference when $F_x = F_y$
  - valid when $F_x \neq F_y$

[1] even with very heavy-tailed distr. and very small samples

Standard bootstrap

- With independent samples, we test $H_0 : W_x = W_y$ with

$$\tau = (\hat{W}_x - \hat{W}_y)/[\widehat{\text{var}}(\hat{W}_x) + \widehat{\text{var}}(\hat{W}_y)]^{1/2}$$

- Bootstrap samples:
  - $X^\star$: resample with replacement n observations from X.
  - $Y^\star$: resample with replacement m observations from Y.
- Bootstrap test:

$$\tau_b^\star = [\hat{W}_{x_b^\star} - \hat{W}_{y_b^\star} - (\hat{W}_x - \hat{W}_y)]/[\widehat{\text{var}}(\hat{W}_{x_b^\star}) + \widehat{\text{var}}(\hat{W}_{y_b^\star})]^{1/2}$$

- Bootstrap distribution:

  EDF of the B bootstrap statistics, $\tau_b^\star$ for $b = 1, \ldots, B$

New bootstrap method

- Dufour et al (2013) propose generating bootstrap samples
  - $X^{\star\star}$: resample with replacement n observations from Z.
  - $Y^{\star\star}$: resample with replacement m observations from Z.

$$Z = \left\{ \frac{x_1}{\bar{x}}, \ldots, \frac{x_n}{\bar{x}}, \frac{y_1}{\bar{y}}, \ldots, \frac{y_m}{\bar{y}} \right\}$$

where $\bar{x}$ and $\bar{y}$ are sample means. The bootstrap test is

$$\tau_b^{\star\star} = [\hat{W}_{x_b^{\star\star}} - \hat{W}_{y_b^{\star\star}}]/[\widehat{\text{var}}(\hat{W}_{x_b^{\star\star}}) + \widehat{\text{var}}(\hat{W}_{y_b^{\star\star}})]^{1/2}$$

- This bootstrap procedure is
  - closely related to permutation test when $F_x = F_y$
  - still valid when $F_x \neq F_y$
  - respects the null hypothesis (Golden Rule)

Finite sample properties

$\alpha_x - \alpha_y$	asym	boot	bootH_0
0.00	0.0770	0.0614	0.0500
0.58	0.0796	0.0612	0.0496
0.96	0.0825	0.0639	0.0510
1.23	0.0865	0.0668	0.0539
1.57	0.0956	0.0719	0.0586
1.97	0.1138	0.0824	0.0705
2.57	0.1289	0.0911	0.0805

Table: Rejection frequencies for the Gini index, $H_0 : I_{\text{Gini}}(F_x) = I_{\text{Gini}}(F_y)$, as F_x moves away from F_y (as $\alpha_x - \alpha_y$ increases), at nominal level 0.05, $n = 50$.

Outline

First-order Dominance

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- "F first-order dominates G"

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  - $\forall q \in \mathbb{Q} : Q(F,q) \geq Q(G,q)$
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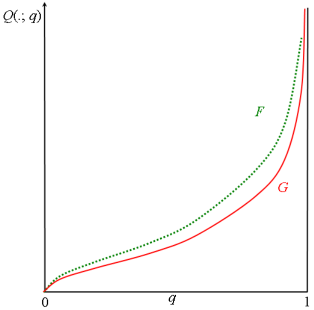

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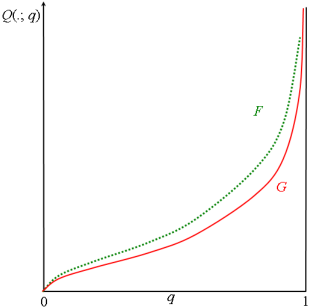
  - 

- "$W(F) \geq W(G)$, for any $W \in \mathbb{W}_1$"
  - $\mathbb{W}_1 := \{W | W(F) = \int \phi(y) \, \mathrm{d}F(y), \phi'(y) > 0\}$

Second-order Dominance

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Second-order Dominance

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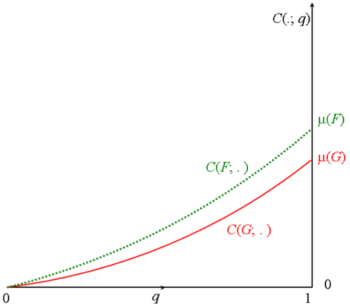

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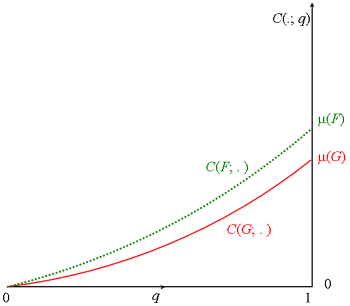
  - 

- "$W(F) \geq W(G)$, for any $W \in \mathbb{W}_2$"
  - $\mathbb{W}_2 := \{W | W(F) = \int \phi(y)\, dF(y), \phi'(y) > 0, \phi''(y) \leq 0\}$

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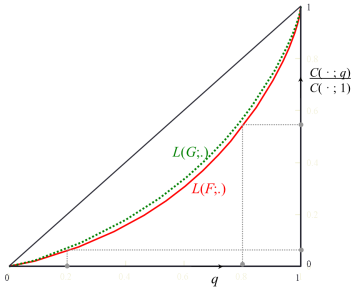

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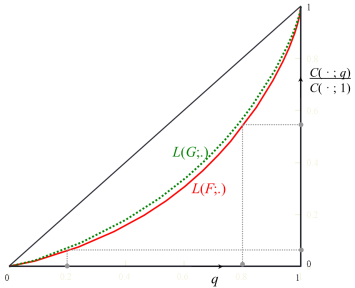

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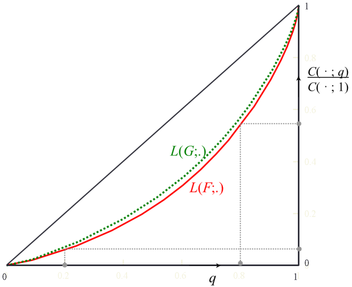
  - 

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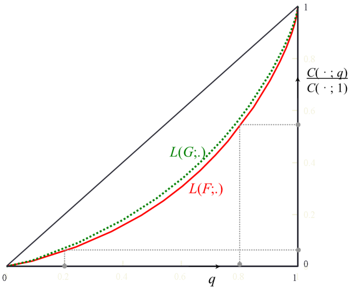
  - 

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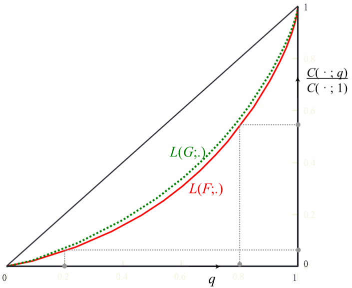

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  - So, given that $\iota(x_{q'} \ge z) = 1$ whenever $\iota(x_q \ge z) = 1$:
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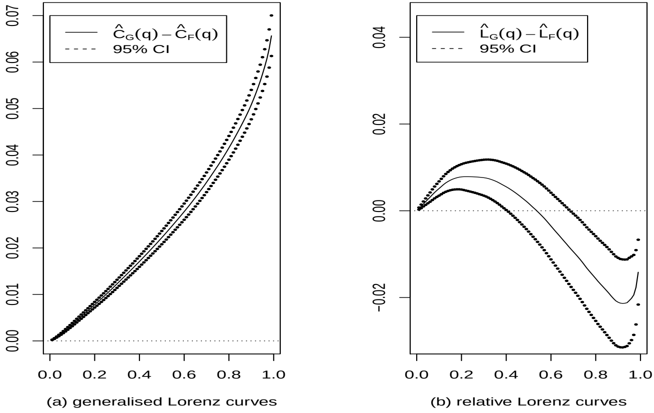
Outline

Dominance: an intuitive application



Difference between two empirical Lorenz curves, $n = 5\,000$

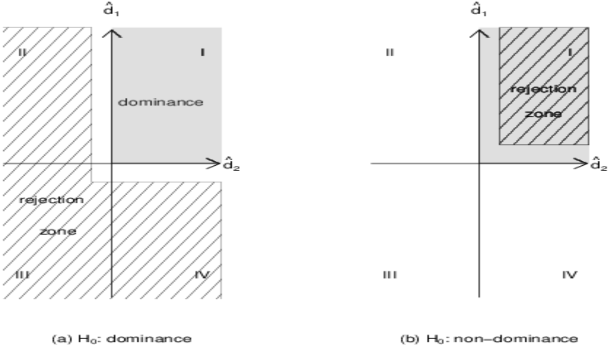
Dominance: an intuitive application

	Distribution F		Distribution G	
	Index	$CI_{95\%}$	Index	$CI_{95\%}$
Poverty measures*				
P^0_{FGT}	0.1134	*[0.1046;0.1222]*	0.0260	*[0.0216;0.0304]*
P^1_{FGT}	0.0299	*[0.0270;0.0329]*	0.0053	*[0.0042;0.0065]*
P_{Sen}	0.0426	*[0.0385;0.0466]*	0.0077	*[0.0061;0.0093]*
P_{SST}	0.0579	*[0.0523;0.0635]*	0.0106	*[0.0083;0.0129]*
Generalised Entropy measures				
I^{-1}_{GE}	0.1803	*[0.1694;0.1913]*	0.1568	*[0.1468;0.1667]*
I^{0}_{GE}	0.1416	*[0.1351;0.1481]*	0.1420	*[0.1324;0.1516]*
I^{1}_{GE}	0.1360	*[0.1289;0.1430]*	0.1570	*[0.1411;0.1729]*
I^{2}_{GE}	0.1548	*[0.1431;0.1665]*	0.2266	*[0.1798;0.2734]*
I_{Gini}	0.2849	*[0.2785;0.2913]*	0.2909	*[0.2816;0.3001]*

* The poverty line is half the median of the sample drawn from distribution F: $\zeta_0 = 0.07565776$.

Outline

The null hypothesis: dominance or non-dominance



Tests of dominance and non-dominance. The first quadrant, I, corresponds to dominance of G by F in the sample (grey area). The quadrants II, III and IV correspond to non-dominance.

Outline

Hypothesis testing

- Under the null of dominance (F dominates G), we have

$$\begin{aligned} H_0: &\quad D_F^s(y) \leq D_G^s(y), \quad \text{for all } y \in \mathbb{Y}, \\ H_1: &\quad D_F^s(y) > D_G^s(y), \quad \text{for some } y \in \mathbb{Y}. \end{aligned}$$

  Test based on the supremum of individual differences:

$$\tau = \sup_{y \in \mathbb{Y}} \left(\hat{D}_F^s(y) - \hat{D}_G^s(y) \right).$$

- Under the null of non-dominance (F does not dominate G):

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  Test based on the infinum of individual differences:

$$\tau' = \inf_{y \in \mathbb{Y}^r} \left(\hat{D}_G^s(y) - \hat{D}_F^s(y) \right).$$

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   - $IF(z;Q(\cdot,0.5),F) = \frac{q-\iota(q\geq F(z))}{f(Q(F,0.5))} = \frac{q-\iota(y_{0.5}\geq z)}{f(y_{0.5})}$

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  - weights: $W_c(x;\theta) = \min\left\{ 1 \; ; \frac{c}{\|A(\theta)[s(x;\theta)-a(\theta)]\|} \right\}$
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- $p \times p$ matrix $A(\theta)$ and $a(\theta) \in \mathbb{R}^p$:
  - $E\left[\psi(x;\theta)\psi(x;\theta)^T\right] = \left[A(\theta)^T A(\theta)\right]^{-1}; E[\psi(x;\theta)] = 0$
  - c: regulator between efficiency (high) and robustness (low)

Outline

Censoring and Truncation

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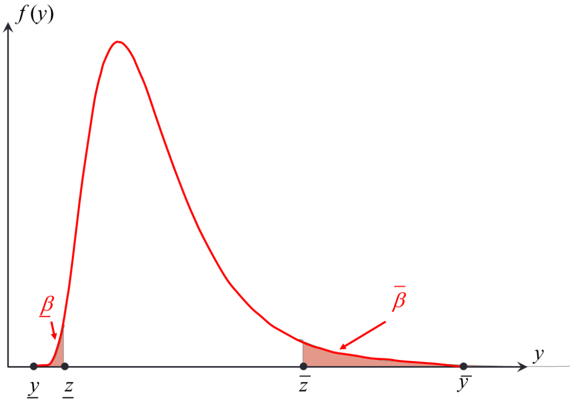

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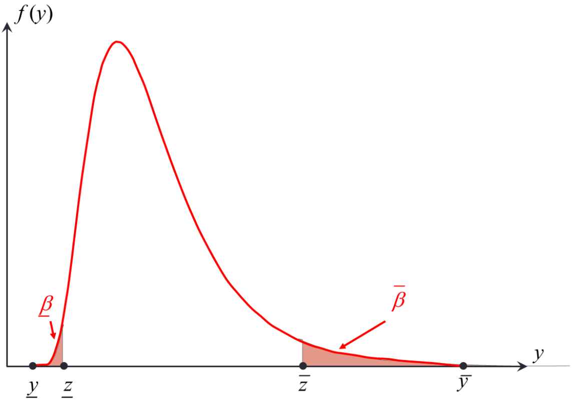

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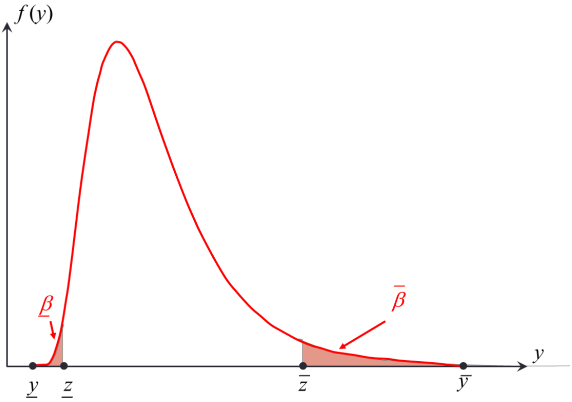

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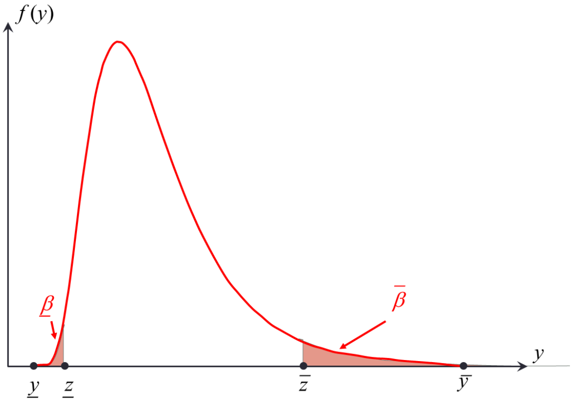

- Empirical distribution is random
- Fixed boundaries $(\underline{z}, \bar{z})$ on excluded portion
- Therefore size of excluded portions $(\underline{\beta}, 1 - \bar{\beta})$ is random

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  - carry out inference on Lorenz-curve ordinates and some welfare indices

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- Asymptotic covariance:
  - $\widehat{\omega}_{qq'} := \widehat{s}_q + \left[q\widehat{y}_q - \widehat{c}_q\right]\left[\widehat{y}_{q'} - q'\widehat{y}_{q'} + \widehat{c}_{q'}\right] - \hat{y}_q\widehat{c}_q$
  - $\widehat{c}_q := \widehat{c}_{\text{low}} + \frac{1}{n}\sum_{i=\kappa(n,\underline{\beta})+1}^{\kappa(n,q)} y_{(i)}$
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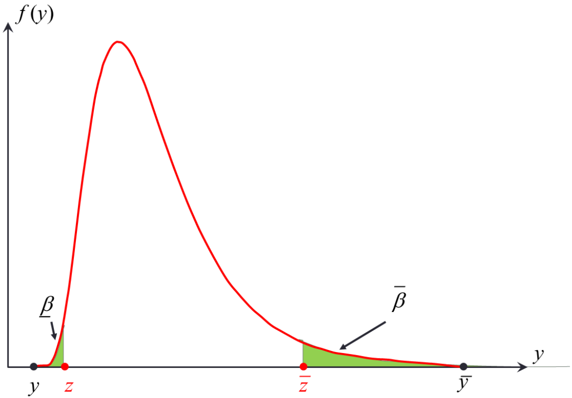

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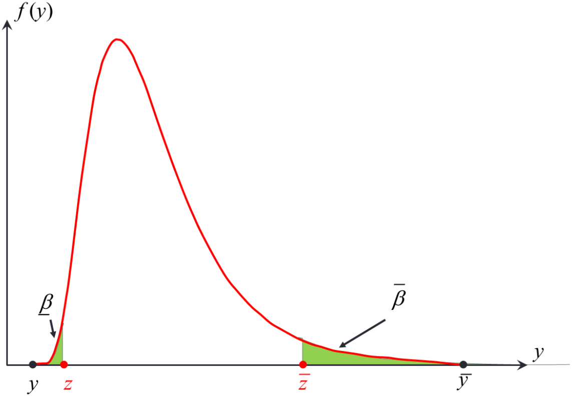

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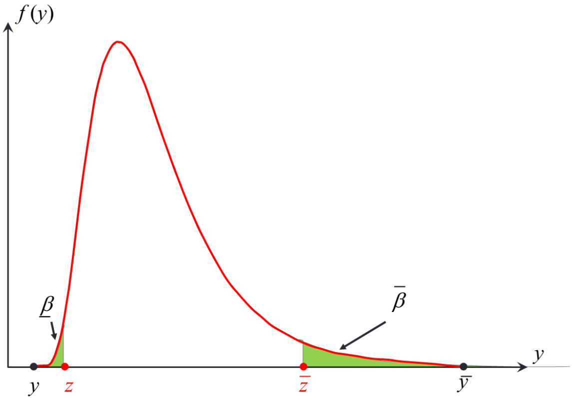

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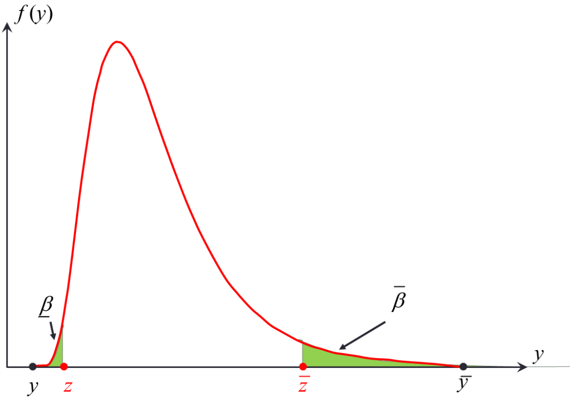

- Fixed proportion of the sample discarded
  - remove outliers for robustness reasons?
  - proportions $(\underline{\beta}, 1 - \bar{\beta})$ removed from the (bottom, top)
  - $y_{\underline{\beta}}$ and $y_{\bar{\beta}}$ are *random*

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  - $b := 1/\left[\bar{\beta} - \underline{\beta}\right]$
- Key statistics:
  - income cumulations $c_{\beta,q} := C(\tilde{F}_\beta; q) = b\int_{y_{\underline{\beta}}}^{y_q} y\,\mathrm{d}F(y)$
  - mean $\mu_\beta := \mu(\tilde{F}_\beta)$
  - $s_{\beta,q} := S(\tilde{F}_\beta; q) := b\int_{y_{\underline{\beta}}}^{y_q} y^2\,\mathrm{d}F(y)$

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- $IF(z; C(\cdot;q), \tilde{F}_\beta) =$
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  - $= -c_{\beta,q} + b\left[q y_q - \underline{\beta} y_{\underline{\beta}} - \iota(y_q \geq z) y_q + \iota(y_{\underline{\beta}} \geq z) y_{\underline{\beta}}\right] +$
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    $b\left[\iota(y_q \geq z) - \iota(y_{\underline{\beta}} \geq z)\right]$
- So the asymptotic covariance of $\sqrt{n}\hat{c}_{\beta,q}$, $\sqrt{n}\hat{c}_{\beta,q'}$ ($q \leq q'$) is
  - $\varpi_{qq'} = b^2\left[\omega_{qq'} + \omega_{\underline{\beta}\underline{\beta}} - \omega_{\underline{\beta}q} - \omega_{\underline{\beta}q'}\right]$

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- Covariance of $\sqrt{n}\hat{c}_{\beta,q}$, $\sqrt{n}\hat{c}_{\beta,q'}$ $(q \leq q'$) estimated by
  - $\widehat{\varpi}_{q_iq_j} = \left[q_iy_{(i)} - \underline{\beta}y_{(1)} - \sum_{k=1}^i \frac{y_{(k)}}{bn_\beta}\right] \times \left[[1-q_j]\,y_{(j)} - \left[1-\underline{\beta}\right]y_{(1)} + \sum_{k=1}^j \frac{y_{(k)}}{bn_\beta}\right] - \sum_{k=1}^i \frac{y_{(i)}y_{(k)} - y_{(k)}^2}{bn_\beta} + y_{(1)}\left[q_iy_{(i)} - \underline{\beta}y_{(i)} - \sum_{k=1}^i \frac{y_{(i)}}{bn_\beta}\right]$

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  - $IF(z; W_{\mathrm{QAD}}, \tilde{F}_\beta) = b\varphi\left(\max\left(y_{\underline{\beta}}, \min(z, y_{\overline{\beta}})\right), \mu(\tilde{F}_\beta)\right) - W_{\mathrm{QAD}}(\tilde{F}_\beta) + bIF(z, C(\cdot;\overline{\beta}), \tilde{F}_\beta) \int_{Q(F,\underline{\beta})}^{Q(F,\overline{\beta})} \varphi_\mu\left(x, \mu(\tilde{F}_\beta)\right) \mathrm{d}F(x)$

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    - $\hat{w}_{\text{QAD},\beta} := W_{\text{QAD}}(\tilde{F}_\beta^{(n)}) := \frac{b}{n} \sum_{i=1}^{n} \varphi\left(y_{(i)}, \hat{\mu}_\beta\right) \iota(\kappa(n, \underline{\beta}) + 1 < i < \kappa(n, \overline{\beta}))$
    - $IF(z; W_{\text{QAD}}, \tilde{F}_\beta) = b\varphi\left(\max\left(y_{\underline{\beta}}, \min(z, y_{\overline{\beta}})\right), \mu(\tilde{F}_\beta)\right) - W_{\text{QAD}}(\tilde{F}_\beta) + bIF(z, C(\cdot; \overline{\beta}), \tilde{F}_\beta) \int_{Q(F,\underline{\beta})}^{Q(F,\overline{\beta})} \varphi_\mu\left(x, \mu(\tilde{F}_\beta)\right) \mathrm{d}F(x)$
- Estimate of AV of $\sqrt{n} W_{\text{QAD}}(\tilde{F}_\beta^{(n)})$ found by computing the mean of squares of $IF(z; W_{\text{QAD}}, \tilde{F}_\beta)$, $z = y_i$, $i = 1, \ldots, n$.

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  - The asymptotic variance of $\sqrt{n} W_{\text{QAD}}(\tilde{F}_\beta^{(n)})$
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    $2b^3 \text{cov}\left(x, \varphi\left(x, \mu(\tilde{F}_\beta)\right); F_\beta^*\right) \int_{Q(F,\underline{\beta})}^{Q(F,\overline{\beta})} \varphi_\mu\left(x, \mu(\tilde{F}_\beta)\right) \mathrm{d}F(x) +$
    $b^4 \text{var}(x; F_\beta^*) \left[\int_{Q(F,\underline{\beta})}^{Q(F,\overline{\beta})} \varphi_\mu\left(x, \mu(\tilde{F}_\beta)\right) \mathrm{d}F(x)\right]^2$

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  - $\vartheta_\beta = \mu_\beta^2 \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\beta}}^{q} \varpi_{q'q}\,\mathrm{d}q'\,\mathrm{d}q + \mu_\beta^2 \int_{\underline{\beta}}^{\overline{\beta}} \int_{q}^{\overline{\beta}} \varpi_{qq'}\,\mathrm{d}q\,\mathrm{d}q + \varpi_{\overline{\beta}\overline{\beta}} \left[\int_{\underline{\beta}}^{\overline{\beta}} c_{\beta,q}\,\mathrm{d}q\right]^2 - 2\mu_\beta \int_{\underline{\beta}}^{\overline{\beta}} c_{\beta,q}\,\mathrm{d}q \int_{\underline{\beta}}^{\overline{\beta}} \varpi_{q\overline{\beta}}\,\mathrm{d}q$

Outline

Semi-parametric modelling

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- Main issues
  1. What parametric model should be used for the tail?
  2. How should the model be estimated?
  3. How should the proportion β be chosen?
  4. What implications for welfare indices, dominance criteria?

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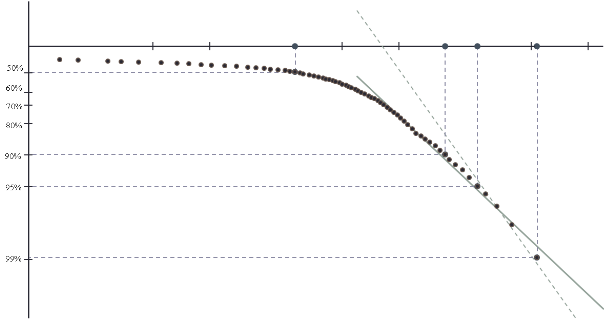

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Semi-parametric: UK Lorenz curves

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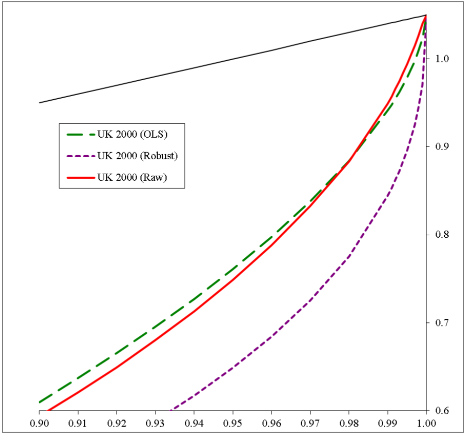

Semi-parametric: Sweden Lorenz curves

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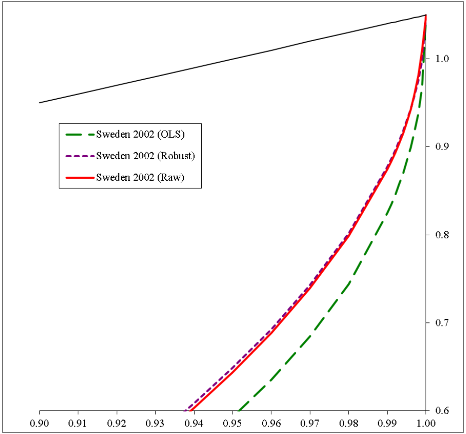

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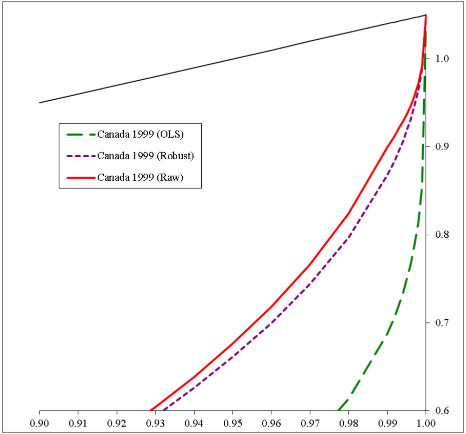

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- More appropriate method
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- Necessary to analyse the tails
  - plot of Hill estimators
  - use appropriate methods with heavy-tailed distributions

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- Special attention to the way the parameters of the model are to be estimated