

# Income mobility and welfare

Markus Jäntti<sup>1</sup>

<sup>1</sup>Swedish Institute for Social Research, Stockholm University

January 13, 2015

# Acknowledgements

This lecture builds on joint work with Stephen P Jenkins

*Markus Jäntti and Stephen Jenkins (2015). “Income mobility”. In: Handbook of Income Distribution. Ed. by Anthony B Atkinson and François Bourguignon. Vol. 2. Elsevier. Chap. 10, pp. 807–935. DOI:*

*doi:10.1016/B978-0-444-59428-0.00011-4.*

*URL: <http://www.sciencedirect.com/science/article/pii/B9780444594280000114>*

and on an unpublished lecture by Tony Atkinson,

*Anthony B Atkinson (2008). “Mobility, Meritocracy and Markets”. Unpublished lecture at Russell Sage Foundation, New York*

# Outline

Introduction

Mobility concepts

Welfare implications of mobility

- Basic setup

- Only inequality aversion

- Inequality and risk aversion

- Inequality and risk aversion and origin independence

- Integrating intra- and inter-generational mobility

Concluding remarks

Tables and figures

# Outline

## Introduction

## Mobility concepts

## Welfare implications of mobility

- Basic setup

- Only inequality aversion

- Inequality and risk aversion

- Inequality and risk aversion and origin independence

- Integrating intra- and inter-generational mobility

## Concluding remarks

## Tables and figures

# Motivation

- ▶ much of mobility measurement concerned with the issue of whether one (at least bivariate) distribution has *more/less* mobility than another
- ▶ we know and understand much less about whether or not more or less mobility is *socially desirable*
- ▶ we will consider both *intra-* and *inter-generational* mobility

# Reading list

- ▶ Markus Jäntti and Stephen Jenkins (2015). “Income mobility”. In: *Handbook of Income Distribution*. Ed. by Anthony B Atkinson and François Bourguignon. Vol. 2. Elsevier. Chap. 10, pp. 807–935. DOI: [doi:10.1016/B978-0-444-59428-0.00011-4](https://doi.org/10.1016/B978-0-444-59428-0.00011-4). URL: <http://www.sciencedirect.com/science/article/pii/B9780444594280000114>, esp. section 2
- ▶ P. Gottschalk and E. Spolaore (2002). “On the Evaluation of Economic Mobility”. In: *Review of Economic Studies* 69, pp. 191–208
- ▶ Anthony B Atkinson (1983). “The measurement of economic mobility”. In: *Social Justice and Public Policy*. Ed. by A. B. Atkinson. Cambridge, MA: MIT Press. Chap. 3, pp. 61–76
- ▶ A. B. Atkinson and F. Bourguignon (1982). “The Comparison of Multi-Dimensioned Distributions of Economic Status”. In: *Review of Economic Studies* 49.2, pp. 183–201
- ▶ A. Markandya (1984). “The Welfare Measurement of Changes in Economic Mobility”. In: *Economica* 51, pp. 457–471
- ▶ Gary S Fields and Efe A Ok (1999). “The Measurement of Income Mobility: An Introduction to the Literature”. In: *Handbook of Income Inequality Measurement*. Ed. by Jacques Silber. Recent Economic Thought. Boston: Kluwer Academic Publishers. Chap. 19, pp. 557–598

# Notation

- ▶ restrict interest to two “periods”, 1 and 2
- ▶ let  $Y_j$ ,  $j = 1, 2$  be the variable of interest, “income”, in the two periods ( $Y_j \geq 0$ )
- ▶ we initially work with the bivariate distribution  $F(Y_1, Y_2)$ ;  $F$  is “well-behaved” (all moments exist and are finite, the marginal distributions have inverse distribution functions and so on)
- ▶ we will be concerned with the problem of *comparing* mobility across two “populations”,  $A$  and  $B$  (each of which has  $F_A$ ,  $F_B$ )
- ▶ let  $M(F)$  be a (statistical) measure of mobility; our concern is *not* to assess if

$$M(F_A) \preceq M(F_B) \quad \text{or} \quad M(F_A) \succeq M(F_B)$$

- ▶ instead, we are concerned with assessing if, given a welfare function(al)  $W$ ,

$$W(F_A) \preceq W(F_B) \quad \text{or} \quad W(F_A) \succeq W(F_B)$$

# Remarks

- ▶ the “periods” could at this stage be different time periods (*intra*-generational mobility), or they could be two generations within the same family or dynasty (*inter*-generational mobility)
- ▶ “income” could be some income variable, or consumption, or wealth) much of the literature is concerned with permanent income); that choice will be taken as given in what follows
- ▶ focussing on only two periods may seem too restrictive, but helps fix ideas
- ▶ we shall mostly look at discrete distributions for analytical tractability



# Outline

Introduction

**Mobility concepts**

Welfare implications of mobility

Basic setup

Only inequality aversion

Inequality and risk aversion

Inequality and risk aversion and origin independence

Integrating intra- and inter-generational mobility

Concluding remarks

Tables and figures

# Mobility concepts

*... the mobility literature does not provide a unified discourse of analysis. This might be because the very notion of income mobility is not well-defined; different studies concentrate on different aspects of this multi-faceted concept. At any rate, it seems safe to say that a considerable degree of confusion confronts a newcomer to the field (Fields and Ok, 1999, p. 557).*

# Mobility concepts

- ▶ focus on the distribution of income in two “periods” (e.g., two years for intra-, generations for inter-generational mobility)
- ▶ income distribution of  $Y = (Y_1, Y_2)'$  with joint density  $f(y_1, y_2)$ ;  $f_1$  and  $f_2$  being the period-specific marginal distributions
- ▶ mobility can be thought of as transformation linking the marginal distribution  $f_1$  with marginal distribution  $f_2$
- ▶ sometimes, study of a single (longitudinal) population can be informative. . .
- ▶ but as a rule, mobility is about *comparing* two populations  $A$  and  $B$  (two countries, two different periods, etc)

# Mobility concepts

- ▶ concepts:
  - ▶ positional change
  - ▶ individual income growth
  - ▶ mobility as inequality reduction
  - ▶ income risk
- ▶ social desirability of mobility
  - ▶ may differ across within/between
  - ▶ may differ across concepts
  - ▶ relationship to equality of opportunity

# Mobility as positional change

- ▶ most easily thought of as defined in terms of not the distribution of income but its inverse (summarize positions not by incomes but by the rank associated with an income)
- ▶ abstract from the shape of (and changes in) the marginal distribution (“exchange” as opposed to “structural” mobility)
- ▶ for every positional change in one direction there must be a corresponding change in the opposite direction
- ▶ “no mobility” occurs when no rank changes take place ( $p_{ij} \equiv 1 \forall i = j, p_{ij} \equiv 0 \forall i \neq j$ )
- ▶ “full” mobility:
  - ▶ origin independence ( $p_{ij} = p_{kl} = 1/n$ ; each row of the transition matrix has identical entries)
  - ▶ rank reversal ( $p_{ij} > 0 \quad i = 1, \dots, n, j = n, \dots, 1$ ; all entries in transition matrix on the anti-diagonal)

# Mobility as individual income growth

- ▶ aggregation of individual income changes (gains or losses)
- ▶ no distinction between exchange and structural mobility:
  - ▶ no standardization of the distributions
  - ▶ all can be upwardly or downwardly mobile
- ▶ immobility:  $y_{i1} = y_{2i} \forall i$
- ▶ mobility: is greater if  $d_i = y_{2i} - y_{1i}$  greater, all else equal
- ▶ measures: directional growth (gains vs. losses) as opposed to non-directional growth

# Mobility as inequality reduction

- ▶ comparison of inequality of marginal with “long-term” distribution; defined in terms of  $\frac{1}{2}(y_{1i} + y_{2i})$
- ▶ no mobility: income of each person in every period is equal to their longer-term income
- ▶ maximum mobility: no inequality in longer-term incomes despite inequality in per-period incomes
- ▶ directional mobility not relevant
- ▶ related to positional change

## Mobility as income risk

- ▶ period-specific income is sum of a 'permanent' component (the longer-term average) and a 'transitory' component (the period-specific deviation from the average)
- ▶ transitory components represent unexpected idiosyncratic shocks to income (long-term income interpreted as "permanent" income)
- ▶ the greater their dispersion across individuals each period, the greater is income risk for this population
- ▶ inequality reduction from longitudinal averaging now re-interpreted as a measure of income risk (and has different normative implications)



# Is income mobility socially desirable?

- ▶ relation to (in)equality of opportunity (but that relationship is complex)
- ▶ differs in the intra- and intergenerational cases
- ▶ positional change: mobility [often] good in inter- but not necessarily in intra-case
- ▶ income growth: gains good, losses bad
- ▶ inequality reduction: good (but for instrumental, not intrinsic, reasons)
- ▶ income risk: mobility bad

# Outline

Introduction

Mobility concepts

Welfare implications of mobility

- Basic setup

- Only inequality aversion

- Inequality and risk aversion

- Inequality and risk aversion and origin independence

- Integrating intra- and inter-generational mobility

Concluding remarks

Tables and figures

## Income mobility and social welfare

- ▶ the social welfare foundations of mobility measurement is small, with contributions including Atkinson (1981), Atkinson and Bourguignon (1982), Markandya (1984), and Gottschalk and Spolaore (2002)
- ▶ social welfare,  $W$ , is the expected value (average) of the utility-of-income functions of individuals.
- ▶ in two-period case, the utility-of-income function is  $U(Y_1, Y_2)$ , and weighted by the joint probability density  $f(y_1, y_2)$ :

$$W = \int_0^{a_2} \int_0^{a_1} U(y_1, y_2) f(y_1, y_2) dy_1 dy_2 \quad (1)$$

where  $U(Y_1, Y_2)$  is differentiable and  $a_1$  and  $a_2$  are the maximum incomes in periods 1 and 2.

- ▶ increases in income in either period assumed desirable (so positive income growth raises utility):  $U_1 \geq 0$  and  $U_2 \geq 0$ .

# Income mobility and social welfare

- ▶ mostly focus is on case where marginal distributions identical (so close to positional mobility analysis)
- ▶ if  $U$  additively separable (so  $U_{12} = 0$ ), mobility is irrelevant and only marginal distributions matter
- ▶ if  $U(Y_1, Y_2)$  is a concave transformation of the sum of the per-period utilities, then  $U_{12} < 0$

# Transition matrices and social welfare

Markandya (1984)

- ▶ focus on a discrete distribution of income with identical marginal distributions in both periods, so
  - ▶  $f_{1i} \equiv f_{2i} \quad i = 1, \dots, n$
  - ▶  $\mathbf{f}'_1 \mathbf{P} = \mathbf{f}_2$
- ▶ consider the problem of choosing the transition matrix  $\mathbf{P}$  that maximizes welfare, given the fixed marginal distribution and a social evaluation function  $U$ :

$$\max_{\mathbf{P}} W = \sum_i \sum_j U(Y_{1i}, Y_{2i}) p_{ij} f_{1i}$$

subject to

$$\sum_i f_{1i} p_{ij} = f_{2j} = f_{1j}, \quad j = 1, \dots, n \quad (2)$$

$$\sum_j p_{ij} = 1, \quad i = 1, \dots, n$$

# Transition matrices and social welfare

Markandya (1984)

- ▶ the crucial “fact” for this problem is the sign of the cross partial derivative,  $U_{12}$ :
  - ▶ for *positive* ( $U_{12} > 0$ ), welfare is maximized by perfect immobility, i.e. choosing the identity matrix  $\mathbf{P} = \mathbf{I}$
  - ▶ for *negative* ( $U_{12} < 0$ ), welfare is maximized by perfect rank reversals (all elements on the anti-diagonal; this solution may not be feasible but transformations that approach it increase welfare)
- ▶ note that “origin independence” plays no role here

## Exchange and structural mobility

- ▶ it is useful to distinguish between changes in mobility that are driven by changes in the marginal distributions (“structural”) and those that are driven by the mapping of  $f_1$  to  $f_2$  (“exchange”)
- ▶ the welfare-based measurement approach allows such a decomposition of mobility
- ▶ consider a two different discrete distributions  $(\mathbf{f}_1^A, \mathbf{P}^A, \mathbf{f}_2^A)$  and  $(\mathbf{f}_1^B, \mathbf{P}^B, \mathbf{f}_2^B)$ ; the move from  $\mathbf{P}^A$  to  $\mathbf{P}^B$  induces both structural and exchange mobility
- ▶ one approach would be purely statistical or mathematical;
  - ▶ subject to a specific distance measure, find  $\tilde{\mathbf{P}}$  that is “closest” to  $\mathbf{P}^A$  subject to  $\tilde{\mathbf{P}}$  being consistent with the marginal distributions
  - ▶ then  $\mathbf{P}^A - \tilde{\mathbf{P}}$  is a measure of exchange mobility;  $\tilde{\mathbf{P}} - \mathbf{P}^B$  a measure of structural mobility
- ▶ an alternative is to rely on the *social evaluation*  $U$  to decompose mobility

## Exchange and structural mobility – welfare-based

- ▶ for each transition matrix  $\mathbf{P}^A$  there is an equilibrium distribution  $\tilde{\mathbf{f}}^A$  such that

$$\tilde{\mathbf{f}}^A' \mathbf{P}^A = \tilde{\mathbf{f}}^A. \quad (3)$$

- ▶ a matrix  $\tilde{\mathbf{P}}$  is “exchange equivalent” to  $\mathbf{P}^A$  if

$$\tilde{\mathbf{f}}^A' \tilde{\mathbf{P}} \mathbf{b} = \tilde{\mathbf{f}}^A. \quad (4)$$

- ▶ the change in *welfare* associated with  $(\tilde{\mathbf{f}}^A, \mathbf{P}^A)$  to that with  $(\tilde{\mathbf{f}}^B, \mathbf{P}^B)$  then considers welfare change induced by  $\mathbf{P}^A \rightarrow \tilde{\mathbf{P}}$  as a measure of exchange mobility;  $\tilde{\mathbf{P}} \rightarrow \mathbf{P}^B$  a measure of structural mobility
- ▶ note that  $\tilde{f}^k, k = A, B$  is a hypothetical steady-state distribution, not the actual



## Exchange and structural mobility – an example

- ▶ to examine this more closely, consider  $n = 2$  and focus on the case of identical marginal distributions in the two time periods:

$$\mathbf{P} = \begin{bmatrix} p_1 & 1 - p_1 \\ 1 - p_2 & p_2 \end{bmatrix} \quad (5)$$

$1 > p_i > 0, i = 1, 2; \quad \mathbf{f} = (f_1, f_2)' = (f_1, 1 - f_1)'$

- ▶ the welfare function (expected/average utility) for this economy is

$$W = U(Y_1, Y_2)p_1f_1 + U(Y_1, Y_2)(1 - p_1)f_1 + U(Y_2, Y_1)(1 - p_2)(1 - f_1) + U(Y_2, Y_2)p_2(1 - f_1) \quad (6)$$

## Exchange and structural mobility – an example

- ▶ this can re-written as

$$W = [\{U(Y_2, Y_2) - U(Y_2, Y_1)\} - \{U(Y_1, Y_2) - U(Y_1, Y_1)\}] p_1 f_1 + C \quad (7)$$

( $C$  does not depend on  $p_1$  or  $p_2$ )

- ▶ to maximize welfare wrt.  $p_1$  we choose a low value when  $\square$  is negative (and high when it is positive); the sign of  $\square$  equals the sign of the cross-partial derivative (as  $Y_1 < Y_2$ )

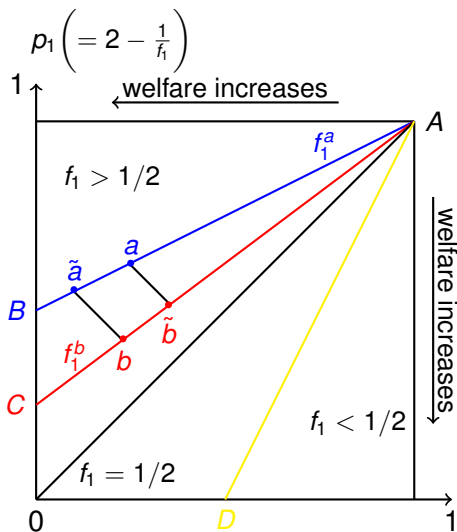
## Exchange and structural mobility – an example

- ▶ the key here is

$$U(Y_2, Y_2) - U(Y_2, Y_1) \stackrel{\leq}{\geq} U(Y_1, Y_2) - U(Y_1, Y_1) \quad (8)$$

- ▶ for a negative cross-partial derivative ( $U_{12} < 0$ ),  $W$  is negative
  - ▶ the decline in utility from going from high income in both periods to low income in the second is less than the increase in utility from going from low in both periods to high in the second
  - ▶ in which case we have a social preference for mobility
  - ▶  $p_1 = p_2 = 0$  has here been ruled out on feasibility grounds so complete rank reversal is not a solution

# Exchange and structural mobility – graphical representation



- ▶ points  $a$  and  $b$  associated with  $\mathbf{P}^a$  and  $\mathbf{P}^b$
- ▶ move along  $f_1^a$  to  $\tilde{a}$  closer to  $b$  is the change in mobility with no structural change
- ▶ move from  $\tilde{a}$  to  $b$  preserves welfare but represents structural mobility  $f_1^a \rightarrow f_1^b$

# Exchange and structural mobility – decomposition

## Decomposition I

$$\text{Total change in welfare} = W^b - W^a$$

$$\text{Exchange mobility} = W^{\tilde{a}} - W^a$$

$$\text{Structural mobility} = W^b - W^{\tilde{a}}$$

## Decomposition II

$$\text{Total change in welfare} = W^b - W^a$$

$$\text{Exchange mobility} = W^b - W^{\tilde{b}}$$

$$\text{Structural mobility} = W^{\tilde{b}} - W^a$$

## Remarks

- ▶ one might also take point A (perfect immobility) as the reference for decomposing, but that would make no use of welfare information.

# Welfare dominance in more general bivariate distributions

Atkinson and Bourguignon (1982)

- ▶ the problem is still to compare two distributions,  $f^A$  and  $f^B$  with

$$\Delta f = f^B - f^A \text{ and } \Delta F = F^B - F^A$$

- ▶ keeping to the two-period case, the difference in welfare is

$$\Delta W = \int_0^{a_2} \int_0^{a_1} U(y_1, y_2) \Delta f(y_1, y_2) dy_1 dy_2 \quad (9)$$

- ▶ we want to know under what conditions  $\Delta W > 0$
- ▶ restrict interest to the case  $U_{12} < 0$

## Welfare dominance in more general bivariate distributions

- ▶ equation 9 can be re-expressed as

$$\begin{aligned} \Delta W = & \underbrace{U(a_1, a_2) \int_0^{a_2} \int_0^{a_1} \Delta f(y_1, y_2) dy_1 dy_2}_{=0} \\ & - \int_0^{a_1} U_1(y_1, a_2) \Delta F_1(y_1) dy_1 - \int_0^{a_2} U_2(a_1, y_2) \Delta F_2(y_2) dy_2 \\ & + \int_0^{a_2} \int_0^{a_1} U_{12}(y_1, y_2) \Delta F(y_1, y_2) dy_1 dy_2 \end{aligned} \tag{10}$$

- ▶ for all  $U$  we are considering, a sufficient condition for  $\Delta W > 0$  is that

$$\Delta F(y_1, y_2) \leq 0$$

- ▶ Atkinson and Bourguignon (1982) consider other classes of  $U$  and derive higher-order dominance conditions

## A closer look at $U$

- ▶ Atkinson and Bourguignon (1982) examine restricted class of utility functions with homothetic preferences
- ▶ consider the following evaluation function

$$U(Y_1, Y_2) = [Y_1^{1-\rho} + Y_2^{1-\rho}]^{(1-\epsilon)/(1-\rho)} \quad (11)$$

- ▶ the two parameters have the following interpretation (Gottschalk and Spolaore, 2002, p. 295):
  - ▶  $\epsilon > 0$  summarizes *aversion to inequality of multi-period utility*,
  - ▶  $\rho > 0$  summarizes the degree of *aversion to inter-temporal fluctuations in income*
- ▶  $U_{12} < 0$  corresponds to  $\epsilon > \rho$ , i.e. multi-period inequality aversion offsets aversion to inter-temporal fluctuations (and reversals are socially valued)
- ▶ when  $\rho = 0$  and perfect substitution of income between periods, one is only interested in the reduction of multi-period inequality



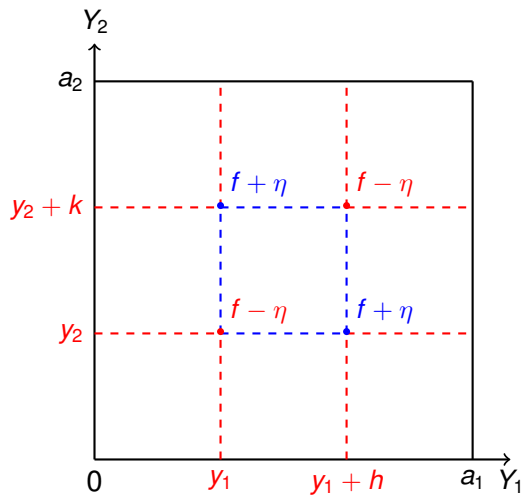
## Mobility dominance

- ▶ an example that would generate a welfare improvement is a 'correlation-reducing transformation' which leaves the marginal distributions unchanged but reduces the correlation between  $Y_1$  and  $Y_2$  (for  $\eta, h, k > 0$ ):

$$\left\{ \begin{array}{cc} y_1 & y_1 + h \\ y_2 & \text{density reduced by } \eta & \text{density increased by } \eta \\ y_2 + k & \text{density increased by } \eta & \text{density reduced by } \eta \end{array} \right\}$$

- ▶ mobility dominance powerful in theory but not used much in practice – results apply to simplified situations (identical margins, homothetic preferences, positional mobility)
- ▶ Dardanoni (1993) provides an alternative approach to dominance (stochastic dominance results for mobility processes summarised by transition matrices with the same steady-state income distribution)

# Mobility dominance – graphical illustration



# Mobility dominance – examples

- ▶ [Go to US transition matrices](#)
- ▶ [Go to IG mobility dominance Germany, the UK, and USA compared](#)

# Welfare dominance with origin independence

Gottschalk and Spolaore (2002)

- ▶ origin independence is an important benchmark in non-welfare-based mobility measurement
- ▶ origin independence has *no* role in the welfare-based approach
- ▶ Gottschalk and Spolaore (2002) introduce origin independence by modifying the evaluation function  $U$
- ▶ in particular, let the certainty equivalent of second-period income be

$$\tilde{Y}_2 = (E_1[Y_2^{1-\gamma}])^{1/(1-\gamma)}. \quad (12)$$

- ▶ the welfare function, using the expectations operator, is then

$$\hat{W} = \{E_0[Y_1^{1-\rho} + (E_1[Y_2^{1-\gamma}])^{1/(1-\gamma)}]^{1-\rho}\}^{1/(1-\epsilon)} \quad (13)$$

# Welfare dominance with origin independence

- ▶ Gottschalk and Spolaore (2002) prove that time independence is value if and only if

$$\epsilon \geq \gamma \text{ and } \rho \geq \gamma$$

i.e., origin independence only matters in the *ex ante* sense that individuals, looking forward, value a sure thing relative to a lottery and that valuation is high enough to dominate aversion to both multiperiod utility ( $\epsilon$ ) and intertemporal variation in income ( $\rho$ )

- ▶ moreover, in the  $2 \times 2$  example, setting  $p_1 = p_2 = p$ , they show that the welfare-maximizing  $p$  depends on the relationship between  $\epsilon$  and  $\rho$

$$p \underset{>}{\overset{\leq}{\equiv}} 1/2 \text{ if } \rho \underset{>}{\overset{\leq}{\equiv}} \epsilon$$

# Measurement of welfare loss

## Welfare measures and extended Atkinson indices

Welfare	Index
No mobility preference: $W_s = \{E_0[Y_1^{1-\rho} + Y_{12}^{1-\rho}]^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)}$	$A_s = 1 - \frac{W_s}{\bar{Y}}$
Reversals improve welfare: $W_r = \{E_0[Y_1^{1-\rho} + Y_2^{1-\rho}]^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)}$	$A_r = 1 - \frac{W_r}{\bar{Y}}$
Origin independence improves welfare: $W_o = \{E_0[Y_1^{1-\rho} + (E_1[Y_2^{1-\gamma}])^{1/(1-\gamma)}]^{1-\rho}\}^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)}$	$A_o = 1 - \frac{W_o}{\bar{Y}}$

Note:  $Y_{12}$  is income in period 2 under the assumption of no mobility, i.e.,  
 $Y_{12} = F_2^{-1}[F_1(Y_1)]$ .

# Measurement of welfare loss – empirical illustration

## Decomposition of welfare gains from mobility

	$\underbrace{A_o - A_s}_{\text{Overall diff}}$	=	$\underbrace{A_o - A_r}_{\text{diff from origin independence}}$	+	$\underbrace{A_r - A_o}_{\text{diff from reversals}}$
Germany	-.096		-.041		-.055
US	-.090		-.044		-.046

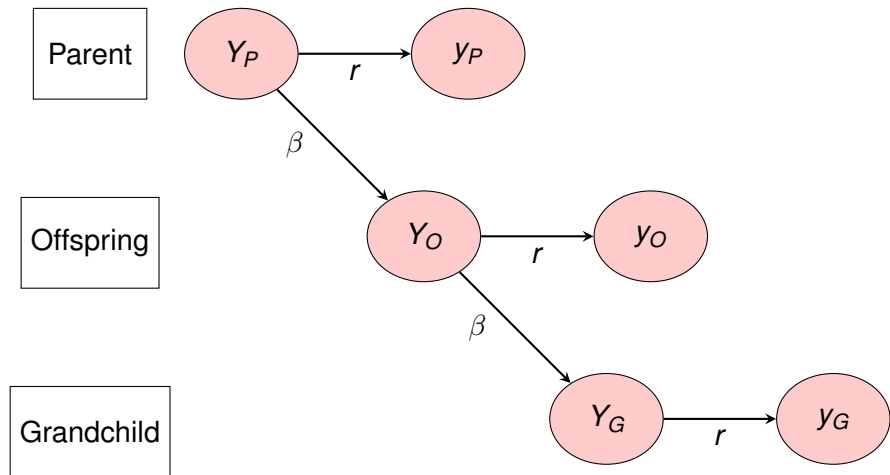
Source: Gottschalk and Spolaore (2002), Table 1, p 202

# Intra- or inter-generational mobility

- ▶ hitherto, analysis thought to be applicable to both intra- and inter-generational mobility
- ▶ IMHO, the Gottschalk and Spolaore (2002) framework (that introduces origin independence!) harder to justify in intergenerational case
- ▶ the “plasticity” of the framework hides the fact that in intergenerational analysis, individuals experience (welfare-reducing) income fluctuations within generations
- ▶ next, we'll look at a simple way of integrating intra- and inter-generational mobility based on Atkinson (2008)



# Intra- and inter-generational mobility



## Inter- and intragenerational mobility

- ▶ focus for now on the 2-generation case, but allow each generation to have annual income that fluctuates around the long-run average such that

$$Y_j = \prod_{t_1}^T \tilde{y}_{jt}^{1/T} \text{ and } \ln Y_j = \frac{1}{T} \sum_{t=1}^T y_{jt} \quad j = F, S \quad (14)$$

- ▶ a parent's utility (or the ex ante evaluation) is

$$U(Y_P, Y_O) = [\ln Y_P + \delta \ln Y_O] / \Delta, \quad \Delta = 1 + \delta \quad (15)$$

- ▶ we'll measure social welfare by  $-\text{Var}[\cdot]$ , so we need

$$\text{Var}[U(Y_P, Y_O)] = \text{Var}[\ln Y_P] + \delta^2 \text{Var}[\ln Y_O] + \delta 2\beta \text{Var}[\ln Y_P]^{1/2} \text{Var}[\ln Y_O]^{1/2} \quad (16)$$

( $\beta$  is the intergenerational income *correlation*;  $\delta$  is the discount rate)

## Inter- and intragenerational mobility

- ▶ assuming a within-person correlation  $r_j$  and stationary transitory error variance  $\sigma_{V_j}^2$ , the welfare function is

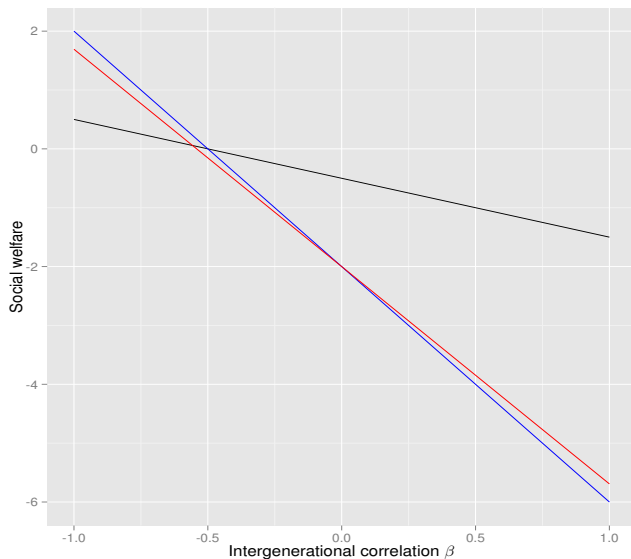
$$\begin{aligned} W = -\text{Var}[U(Y_P, Y_O)] = & - \left\{ \sigma_P^2 \left( \frac{1}{T} + \frac{T-1}{T} r_P \right) + \frac{\sigma_{V_P}^2}{T} + \right. \\ & \delta^2 \left[ \sigma_O^2 \left( \frac{1}{T} + \frac{T-1}{T} r_O \right) + \frac{\sigma_{V_O}^2}{T} \right] + \\ & \delta 2\beta \sqrt{\sigma_P^2 \left( \frac{1}{T} + \frac{T-1}{T} r_P \right) + \frac{\sigma_{V_P}^2}{T}} \times \\ & \left. \sqrt{\sigma_O^2 \left( \frac{1}{T} + \frac{T-1}{T} r_O \right) + \frac{\sigma_{V_O}^2}{T}} \right\} / \Delta^2 \end{aligned} \quad (17)$$

# Inter- and intragenerational mobility

- ▶ assume  $T$  large and impose stationarity  
( $\sigma_P = \sigma_O = \sigma$ ;  $r_P = r_O = r$ ):

$$W = -\text{Var}[U(Y_P, Y_O)] = -\sigma^2[r(1 + \delta^2) + \delta 2\beta]/\Delta^2 \quad (18)$$

# Welfare and intergenerational correlation (2-gen)

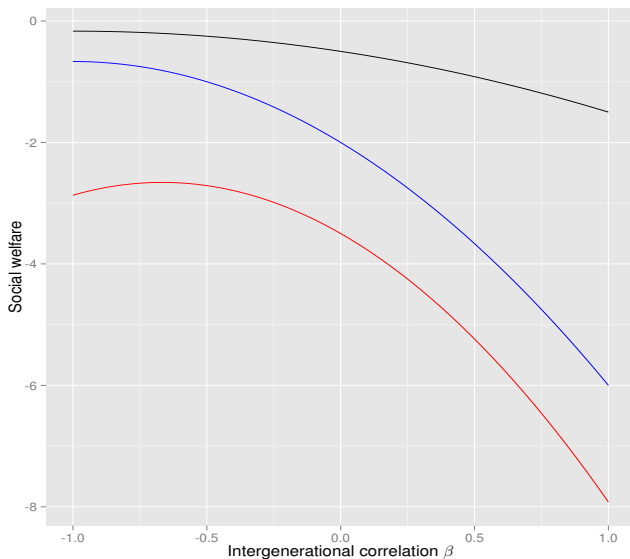


(black= $\sigma^2 = 1; \delta = 1$ ; blue= $\sigma^2 = 2; \delta = 1$ ; red= $\sigma^2 = 2; \delta = 1.5$ )

# Welfare and intergenerational correlation (3-gen)

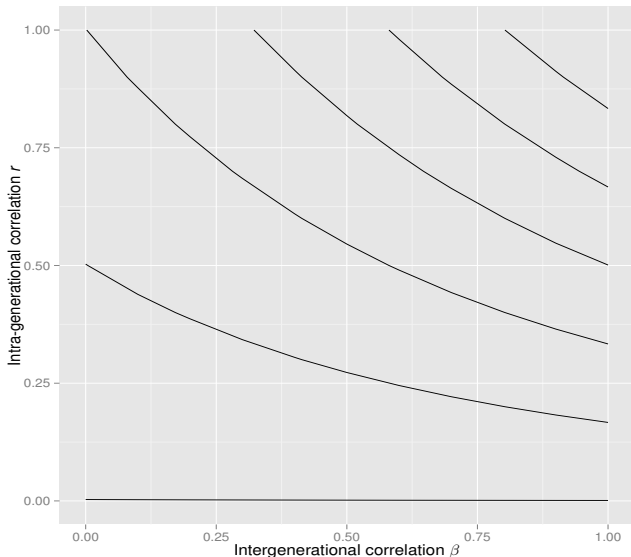
- ▶ taking a 3-generation perspective changes this only a little
- ▶ welfare is now non-linear (in fact, quadratic) in the intergenerational correlation so it is more sensitive to generational variance and discount factor

# Welfare and intergenerational correlation (3-gen)



(black= $\sigma^2 = 1; \delta = 1$ ; blue= $\sigma^2 = 2; \delta = 1$ ; red= $\sigma^2 = 2; \delta = 1.5$ )

# Intra- and intergenerational correlation – trade-off





# Outline

Introduction

Mobility concepts

Welfare implications of mobility

- Basic setup

- Only inequality aversion

- Inequality and risk aversion

- Inequality and risk aversion and origin independence

- Integrating intra- and inter-generational mobility

Concluding remarks

Tables and figures

## Concluding comments

- ▶ focus on exchange mobility (incomes could be and often are replaced by ranks)
- ▶ welfare implications demanding but can (and should) be studied
- ▶ the role of the (in period 1) uncertain lottery in generating value for time dependence underlines a difference between welfare analysis of intra- vs. intergenerational mobility:
  - ▶ it is not clear why *society* should value a sure thing for the offspring generation (“period 2”) relative to the uncertain lottery
  - ▶ it is more clear that such valuations make sense within the same individual
- ▶ integration of intra- within intergenerational analysis promising, but more complex processes likely useful
  - ▶ Homoscedastic transitory variances? (Bingley and Cappellari, 2012)

# Outline

Introduction

Mobility concepts

Welfare implications of mobility

- Basic setup

- Only inequality aversion

- Inequality and risk aversion

- Inequality and risk aversion and origin independence

- Integrating intra- and inter-generational mobility

Concluding remarks

Tables and figures

# Decile transition matrices: USA, (a) 1979–1988

Note: Income refers to equivalized real annual family disposable income, distributed among all individuals (adults and children). The decile groups are ordered from poorest (1) to richest (10). Source: Hungerford (2011, Tables 2 and 3), based on PSID data.

[▶ Go back](#)

Origin	Destination										
	1	2	3	4	5	6	7	8	9	10	
<b>1979</b>					<b>1988</b>						
1	44.3	18.3	12.4	9.2	7.1	3.0	1.8	2.0	0.7	1.3	
2	18.1	25.3	21.0	11.7	7.5	5.4	4.7	3.2	1.9	1.1	
3	10.6	18.2	15.3	16.8	11.6	9.0	8.8	4.9	3.1	1.7	
4	7.2	8.9	14.0	14.0	14.7	15.7	12.0	5.6	6.0	2.1	
5	6.1	9.2	10.9	12.8	13.3	16.9	12.3	7.5	7.7	3.4	
6	4.1	5.2	8.8	10.3	11.8	10.0	14.2	16.9	12.6	6.2	
7	3.5	6.5	6.9	8.6	10.4	13.4	13.3	16.8	13.4	7.2	
8	3.1	4.6	3.2	7.7	12.3	9.5	12.6	15.7	17.7	13.6	
9	1.2	2.2	4.8	6.3	6.9	10.2	12.2	14.7	18.0	23.5	
10	2.1	1.5	2.8	2.5	4.2	7.0	8.5	12.8	18.6	40.0	

# Decile transition matrices: USA, (b) 1989–1998

Note: Income refers to equivalized real annual family disposable income, distributed among all individuals (adults and children). The decile groups are ordered from poorest (1) to richest (10). Source: Hungerford (2011, Tables 2 and 3), based on PSID data.

▶ Go back

Origin	Destination									
	1	2	3	4	5	6	7	8	9	10
<b>1989</b>					<b>1998</b>					
1	41.9	21.6	13.7	7.0	4.6	3.7	2.7	2.2	1.9	0.7
2	20.4	22.5	15.4	11.6	11.0	8.1	4.0	4.0	1.7	1.2
3	12.5	20.8	17.1	16.4	10.9	10.3	5.2	3.2	1.7	1.9
4	6.9	11.6	15.5	16.9	14.5	11.4	10.1	7.7	2.3	3.1
5	4.8	6.2	12.2	13.8	16.0	14.2	12.4	7.1	7.5	5.8
6	3.2	3.7	9.1	11.6	16.0	14.4	15.7	11.7	7.7	6.9
7	3.2	4.5	7.6	9.3	8.7	12.2	16.3	15.6	16.8	5.8
8	3.0	4.7	5.2	5.4	7.9	12.1	17.2	17.0	19.3	8.3
9	2.5	3.1	4.0	4.9	7.5	7.1	10.7	18.2	21.8	20.3
10	1.7	1.0	0.4	3.2	3.0	6.3	6.0	13.1	19.3	46.1

# Differences in cumulative density: USA, 1979–1988 versus 1989–1998

Source: Authors' calculations from (Hungerford, 2011, Tables 2 and 3), based on PSID data.

[▶ Go back](#)

Origin group	Destination group								
	1	2	3	4	5	6	7	8	9
1	0.2	-0.1	-0.2	0.0	0.3	0.2	0.1	0.1	-0.1
2	0.0	0.0	0.4	0.6	0.5	0.2	0.2	0.1	0.0
3	-0.2	-0.5	-0.2	0.0	0.0	-0.5	-0.1	-0.1	0.0
4	-0.2	-0.7	-0.6	-0.6	-0.7	-0.7	-0.2	-0.3	0.1
5	0.0	-0.3	-0.3	-0.5	-0.7	-0.5	0.0	-0.1	0.4
6	0.1	-0.1	-0.1	-0.4	-1.1	-1.3	-0.9	-0.5	0.4
7	0.1	0.2	0.0	-0.3	-0.8	-0.9	-0.8	-0.3	0.3
8	0.1	0.2	-0.2	-0.2	-0.3	-0.7	-1.1	-0.7	-0.3
9	0.0	-0.1	-0.3	-0.2	-0.4	-0.4	-0.7	-0.6	-0.6
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

# Intergenerational transition matrices in disposable income among all persons for Germany, the UK and the USA

Source: Authors' calculations from Eberharter (2013, Table 3). [▶ Go back](#)

		A. Germany				
		<b>Offspring</b>				
		1	2	3	4	5
<b>Father</b>						
1		34	29	14	17	7
2		15	23	32	15	16
3		12	16	22	26	24
4		9	11	18	29	33
5		7	11	19	25	39

		B. UK				
		<b>Offspring</b>				
		1	2	3	4	5
<b>Father</b>						
1		48	22	14	12	5
2		22	26	21	22	10
3		11	18	25	25	21
4		6	16	25	26	25
5		4	16	16	27	36

		C. USA				
		<b>Offspring</b>				
		1	2	3	4	5
<b>Father</b>						
1		37	31	13	13	5
2		21	23	24	17	15
3		12	23	18	24	24
4		9	11	21	33	26
5		2	10	15	26	46

# Cumulated differences in intergenerational transition matrices in disposable income among all persons for Germany, the UK and the USA

Source: Authors' calculations from Eberharter (2013, Table 3). [▶ Go back](#)

A. USA – Germany

		Offspring				
		1	2	3	4	5
Father	1	3	5	5	1	0
	2	9	11	4	2	0
	3	9	18	6	2	0
	4	9	18	9	9	0
	5	4	13	1	2	0

B. USA – UK

		Offspring				
		1	2	3	4	5
Father	1	-10	-1	-1	0	0
	2	-11	-5	-2	-6	0
	3	-11	1	-4	-9	0
	4	-8	-3	-12	-10	-1
	5	-10	-11	-21	-20	-1

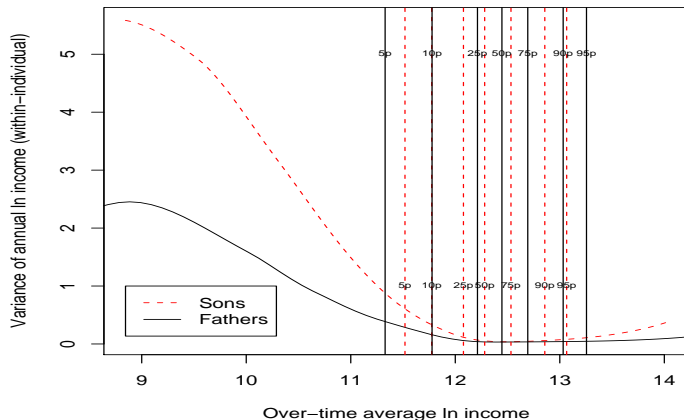
C. UK – Germany

		Offspring				
		1	2	3	4	5
Father	1	14	6	7	2	0
	2	20	16	6	8	0
	3	20	18	11	11	0
	4	17	20	21	19	1
	5	15	24	22	23	1








# Transitory errors and long-run income

The variation of annual ln income across over-time mean of ln income – Swedish fathers and sons



-  Atkinson, A. B. and F. Bourguignon (1982). “The Comparison of Multi-Dimensioned Distributions of Economic Status”. In: *Review of Economic Studies* 49.2, pp. 183–201.
-  Atkinson, Anthony B (1981). “The Measurement of Economic Mobility”. In: *Essays in Honor of Jan Pen*. Ed. by P. J. Eigjelshoven and L. J. van Gemerden. Utrecht: Het Spectrum.
-  – (1983). “The measurement of economic mobility”. In: *Social Justice and Public Policy*. Ed. by A. B. Atkinson. Cambridge, MA: MIT Press. Chap. 3, pp. 61–76.
-  – (2008). “Mobility, Meritocracy and Markets”. Unpublished lecture at Russell Sage Foundation, New York.
-  Bingley, Paul and Lorenzo Cappellari (2012). *Alike in many ways: Intergenerational and sibling correlations of brothers' earnings*. CESIFO Working Paper 3994.  
[http://www.cesifo-](http://www.cesifo-group.de/DocDL/cesifo1_wp3994.pdf)  
[group.de/DocDL/cesifo1\\_wp3994.pdf](http://www.cesifo-group.de/DocDL/cesifo1_wp3994.pdf). Munich:  
CESifo.

-  Dardanoni, V. (1993). “Measuring Social Mobility”. In: *Journal of Economic Theory* 61, pp. 372–394.
-  Eberharther, Veronika V (2013). “The Intergenerational Dynamics of Social Inequality – Empirical Evidence from Europe and the United States”. Paper presented at the ECINEQ Conference, Bari, July 2013, [http://www.ecineq.org/ecineq\\_bari13/FILESxBari13/CR2/p118.pdf](http://www.ecineq.org/ecineq_bari13/FILESxBari13/CR2/p118.pdf).
-  Fields, Gary S and Efe A Ok (1999). “The Measurement of Income Mobility: An Introduction to the Literature”. In: *Handbook of Income Inequality Measurement*. Ed. by Jacques Silber. Recent Economic Thought. Boston: Kluwer Academic Publishers. Chap. 19, pp. 557–598.
-  Gottschalk, P. and E. Spolaore (2002). “On the Evaluation of Economic Mobility”. In: *Review of Economic Studies* 69, pp. 191–208.
-  Hungerford, T. L. (2011). “How Income Mobility Affects Income Inequality: US Evidence in the 1980s and 1990s”. In: *Journal of Income Distribution* 20.1, pp. 83–103.



Jäntti, Markus and Stephen Jenkins (2015). “Income mobility”.

In: *Handbook of Income Distribution*. Ed. by

Anthony B Atkinson and François Bourguignon. Vol. 2.

Elsevier. Chap. 10, pp. 807–935. DOI:

[doi:10.1016/B978-0-444-59428-0.00011-4](https://doi.org/10.1016/B978-0-444-59428-0.00011-4). URL:

<http://www.sciencedirect.com/science/article/pii/B9780444594280000114>.



Markandya, A. (1984). “The Welfare Measurement of Changes in Economic Mobility”. In: *Economica* 51, pp. 457–471.