Fairness and Well-Being

F. Maniquet[1]

Canazei Winter School, January 2015

[1] CORE (UCL)

Introduction

Based on:

Fleurbaey, M. and F. Maniquet 2014, "Fairness and Well-Being Measurement," in progress.

Fleurbaey, M. and F. Maniquet 2011, "A Theory of Fairness and Social Welfare," CUP.

Decancq, K. M. Fleurbaey and F. Maniquet 2014, "Multidimensional poverty measurement with individual preferences," mimeo.

Fleurbaey, M. and Blanchet 2014, "Beyond GDP,"

Motivation

Comparing **gains and losses in well-being** is necessary for policy evaluation (social indexes, fair allocation, optimal taxation, etc).

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- Here: **only** well-being measurement.
- Two solutions to the price normalization problem: **money-metric utility** (Samuelson, 1974) and **ray utility** (Samuelson, 1977). Axiomatic foundations?

Well-Being Measurement:

- consumption set: X
- set of admissible preferences: $\mathscr{R}$
- Well-Being Measure: $W : X \times \mathscr{R} \to \mathbb{R}$ such that

$$W(x,R) \geq W(x',R) \Leftrightarrow xRx'.$$

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Remark III: Can all that be **applied**? Ask Koen...

Main ideas

1. Desirable, divisible and cardinal goods: ray utility and money-metric utility: **focal** ethical well-being measures.
2. Building comparability **more intuitive** than building cardinalization (with consequences on aggregation).
3. Multiple ways to combine RU and MMU with well-being measures in the presence of **bounded**, **discrete**, **non-desirable**, and/or **non-cardinal** commodities.

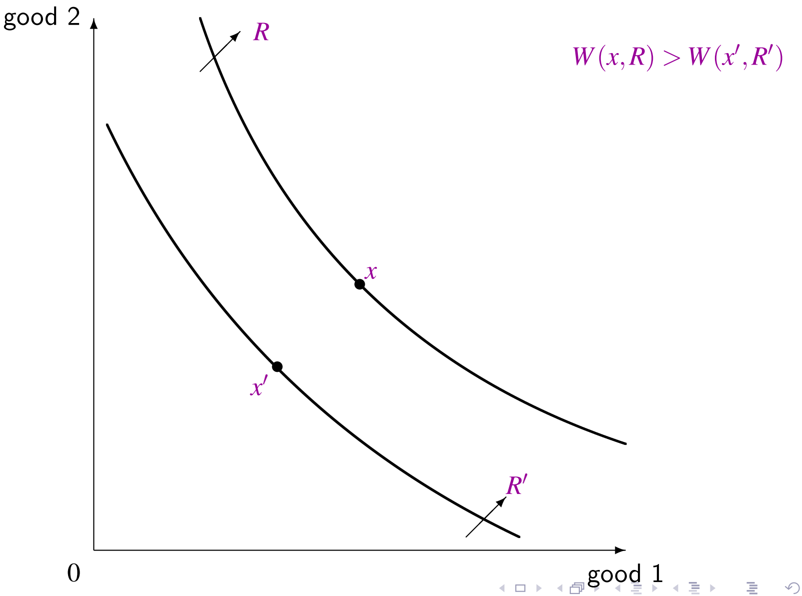
Step 1: desirable, divisible and cardinal goods

$X \subseteq \mathbb{R}_+^K$

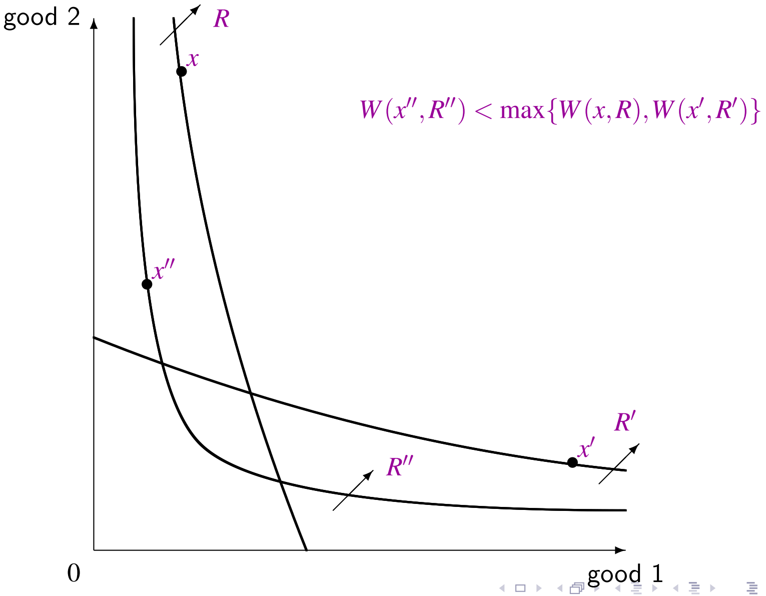
$R \in \mathscr{R}$: monotonic, convex, continuous.

W: continuous in x.

Nested Contour



Lower Contour Inclusion



Worst preferences

Axiom

WORST PREFERENCES

There exists $R^w \in \mathscr{R}$ *such that for all* $x \in X$, $R \in \mathscr{R}$, $W(x,R^w) \leq W(x,R)$.

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Theorem

Let X *be a convex and compact set. A well-being measure* W *over* X *satisfies Lower Contour Inclusion if and only if it satisfies Nested Contour and Worst Preferences. Moreover, for worst preferences* $R^w \in \mathscr{R}$, *the well-being measure is defined by: for all* $x \in X$ *and* $R \in \mathscr{R}$:

$$W(x, R) = \max_{x' \in L(x,R)} W(x', R^w).$$

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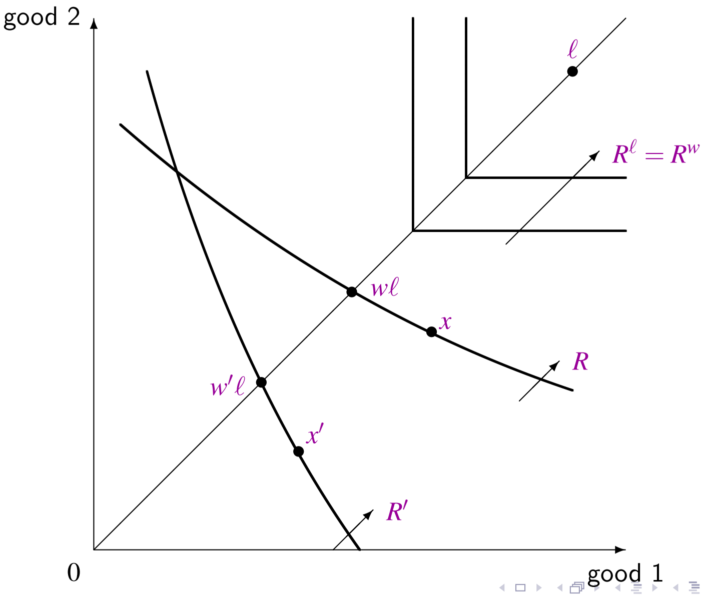
$$W(x, R) = \max_{x' \in L(x,R)} W(x', R^w).$$

- **No restriction** on the choice of the worst preferences.
- Holds on any **preference domain** that is closed under...
- An illustration with **Leontieff** preferences.

W^ℓ (ray utility)



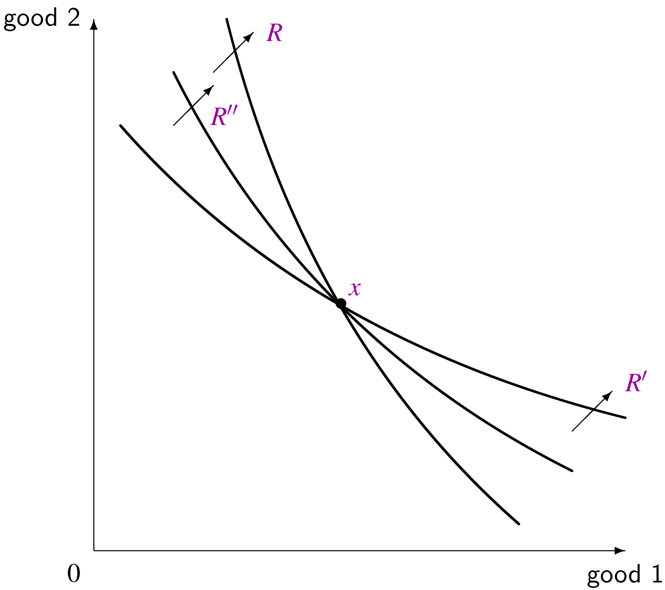


Figure : *Intermediary Preferences I*: $W(x, R'') \in [W(x, R), W(x, R')]$.

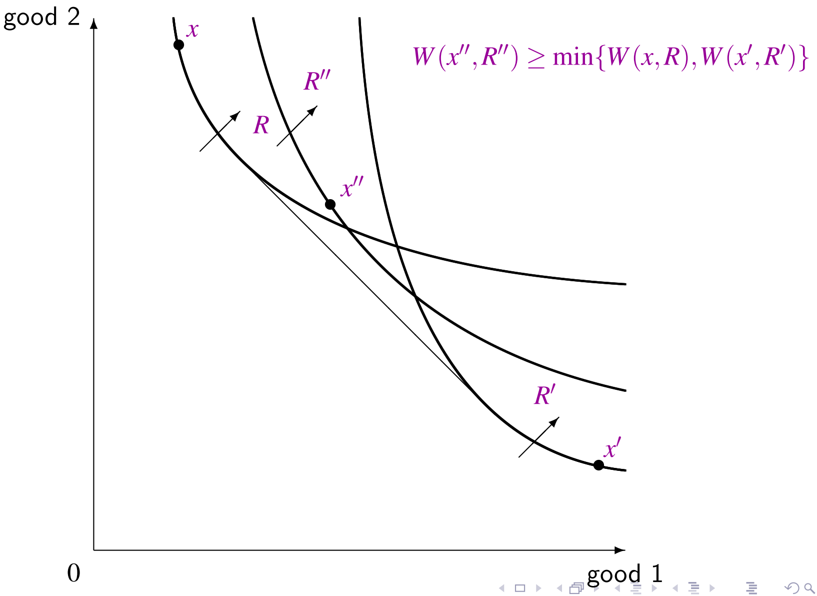
Convex Hull Inclusion

Intuition: if the consumption is intermediary, then the well-being is intermediary.

$$x'' = \lambda x + (1-\lambda)x' \Rightarrow W(x'', R'') \in [W(x,R), W(x',R')]$$

and the same is true for any y'' indifferent to x'', y indifferent to x and y' indifferent to x'.

Convex Hull Inclusion



Best preferences

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There exists $R^b \in \mathscr{R}$ *such that for all* $x \in X$, $R \in \mathscr{R}$, $W(x, R^w) \geq W(x, R)$.

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Theorem

Let X *be a convex and compact set. A well-being measure* W *over* X *satisfies* *Convex Hull Inclusion* *if and only if it satisfies* *Nested Contour* *and* *Best Preferences*. *Moreover, for best preferences* $R^b \in \mathscr{R}$, *the well-being measure is defined by: for all* $x \in X$ *and* $R \in \mathscr{R}$:

$$W(x,R) = \min_{x' \in U(x,R)} W(x', R^b).$$

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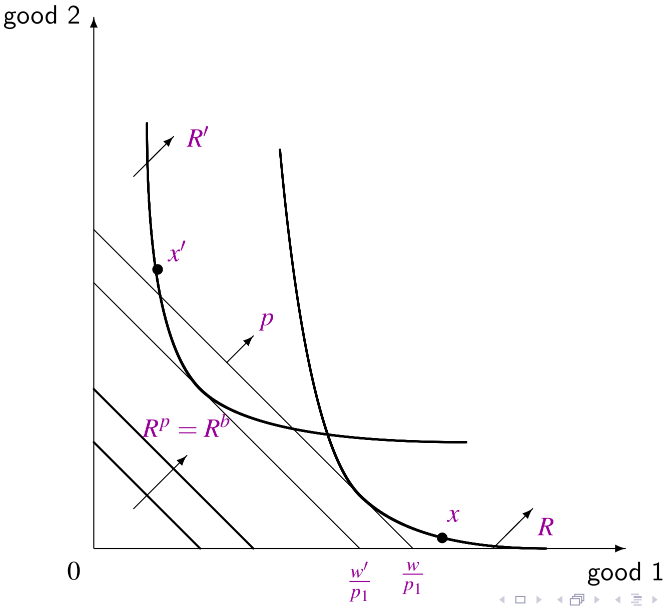
$$W(x, R) = \min_{x' \in U(x,R)} W(x', R^b).$$

- **No restriction** on the choice of the best preferences.
- Holds on any **preference domain** that is closed under...
- An illustration with **linear** preferences.

W^p (money-metric utility)



Intermediary Preferences II

Axiom

INTERMEDIARY PREFERENCES II

For all $x, x', x'' \in X$, $R, R', R'' \in \mathscr{R}$, *if*

$$U(x'', R'') = \frac{U(x, R) + U(x', R')}{2},$$

then $W(x, R'') \in [W(x, R), W(x, R')]$.

Intermediary Preferences II

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HOMOTHETICITY
For all $x, x' \in X$, $R, R' \in \mathscr{R}^H$, $\lambda \in \mathbb{R}$, *if* $W(x,R) = W(x',R')$ *then* $W(\lambda x, R) = W(\lambda x', R')$.

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Theorem

Let X *be a convex and compact set.*

- *A well-being measure* W *over* X *satisfies Lower Contour Inclusion, Intermediary Preferences I and Homotheticity if and only if it is ordinary equivalent to the Ray Utility Measure.*
- *A well-being measure* W *over* X *satisfies Convex Hull Inclusion, Intermediary Preferences II and Homotheticity if and only if it is ordinary equivalent to the Money-Metric Utility Measure.*

Desirable and cardinal commodities: summary

Nested Contour

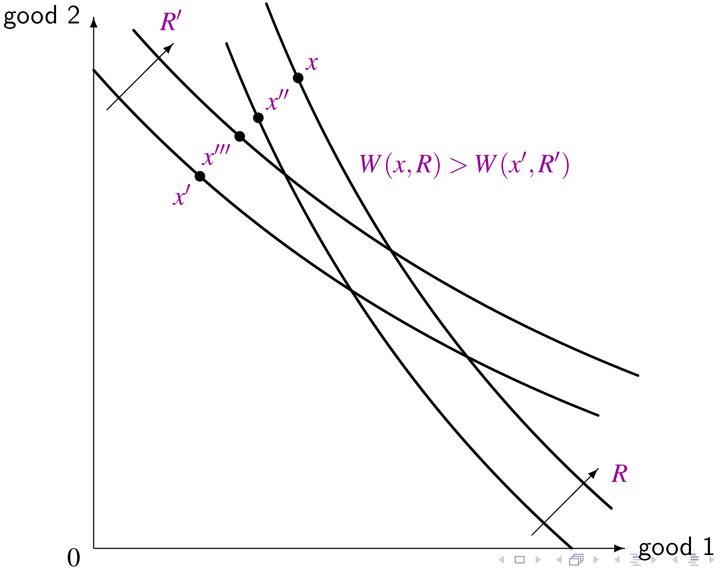
→ Lower Convex Inclusion + Intermediary Pref. I + Homotheticity

⇔ Worst Preferences R^w — R^w has  IC's — ray utility

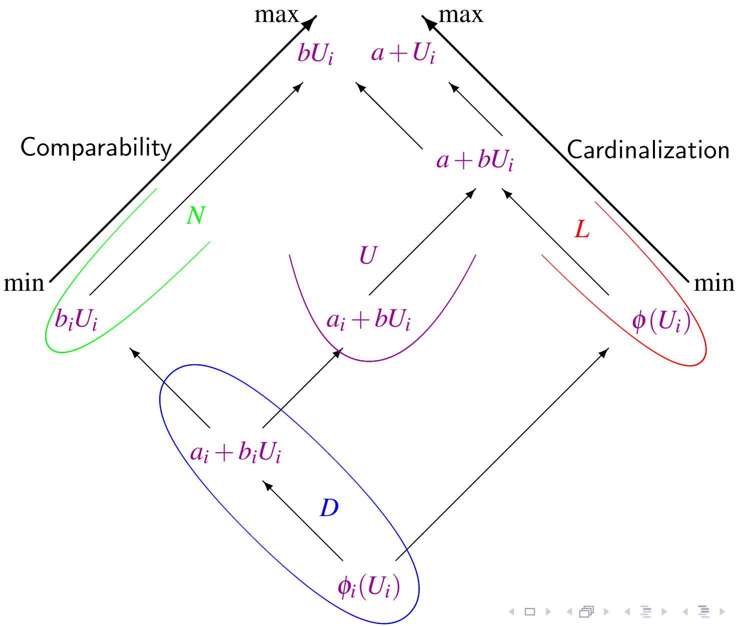
→ Convex Hull Inclusion + Intermediary Pref. II + Homotheticity

⇔ Best Preferences R^b — R^b has  IC's — money metric utility

(Pigou-Dalton) Transfer



Combining WB measures with welfarist aggregators



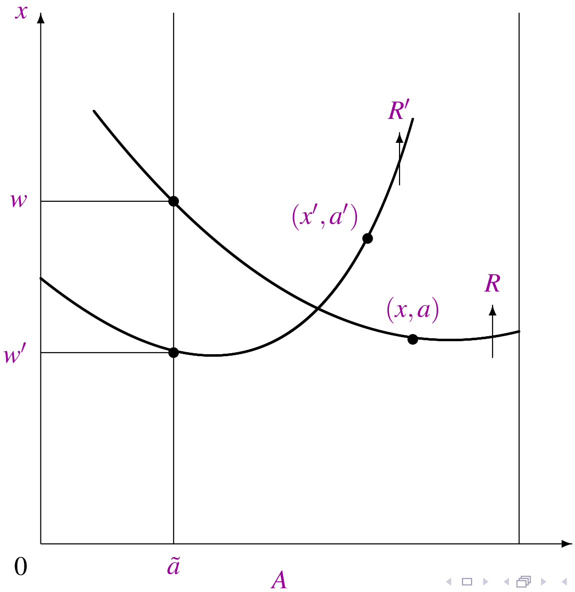
Step 2: satiation, ordinal goods, discrete goods

$X = \mathbb{R}_+ \times A$

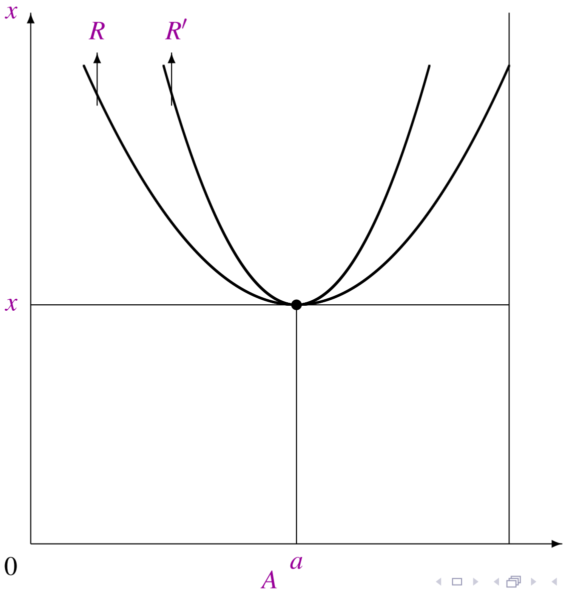
Lower Contour Inclusion does not imply Worst Preferences (and Leontieff preferences not well defined).

Convex Hull Inclusion not well defined.

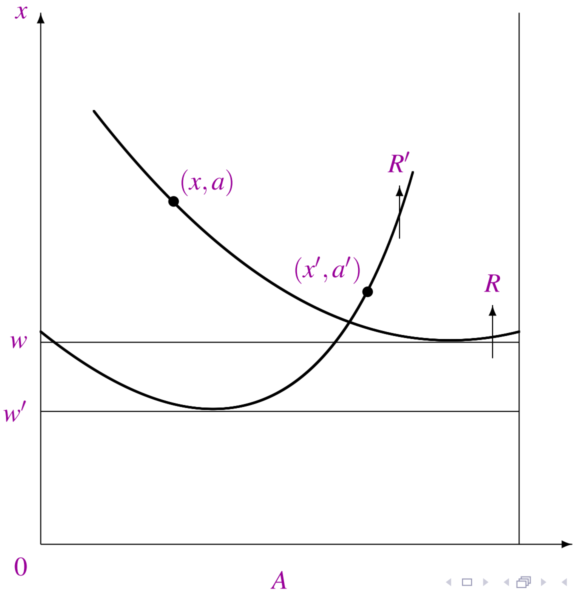
$W^{\tilde{a}}$



Equal Well-Being at Preferred Attribute



$W^{a_{\max}}$



Four different well-being measures

1. Combining W^{ℓ} and $W^{\tilde{a}}$, we can define $W^{\ell\tilde{a}}$ as follows: for all $(x,a) \in X \times A$, all $R \in \mathscr{R}$,

$$W^{\ell\tilde{a}}(x,a) = w \Leftrightarrow (x,a)\, I\, (w\ell, \tilde{a}).$$

2. Combining W^{ℓ} and $W^{a_{\max}}$, we can define $W^{\ell a_{\max}}$ as follows: for all $(x,a) \in X \times A$, all $R \in \mathscr{R}$,

$$W^{\ell a_{\max}}(x,a) = w \Leftrightarrow (x,a)\, I\, (w\ell, a_{\max}(w\ell, R)).$$

3. Combining W^{p} and $W^{\tilde{a}}$, we can define $W^{p\tilde{a}}$ as follows: for all $(x,a) \in X \times A$, all $R \in \mathscr{R}$,

$$W^{p\tilde{a}}(x,a) = w \Leftrightarrow (x,a)\, I \max\left(R, \{(x', \tilde{a}) \in X \times A | px' \leq w\}\right).$$

4. Combining W^{p} and $W^{a_{\max}}$, we can define $W^{pa_{\max}}$ as follows: for all $(x,a) \in X \times A$, all $R \in \mathscr{R}$,

$$W^{pa_{\max}}(x,a) = w \Leftrightarrow (x,a)\, I \max\left(R, \{(x', a') \in X \times A | px' \leq w, a' \in A\}\right).$$

Conclusion

- It is possible to build ethical well-being measures based on fairness views.
- The nature of the goods matters.
- Classical goods: Two families of well-being measures; dual characterization: Worst vs Best Preferences. Fairness $\Rightarrow$ well-being is closely related to the ability to trade-off between goods.
- Axiomatic foundation to money-metric + ray utility.
- Sheds light on the dichotomy between money-metric and ray utility.
- Other goods: other measures + combination with money-metric + ray utility.
- Open new possibilities to the FSO literature:
  - fairness requirements lead to constructing comparabilities rather than cardinalization,
  - but possibilities exist: escape maximin.