Inequality at the Top of the Distribution

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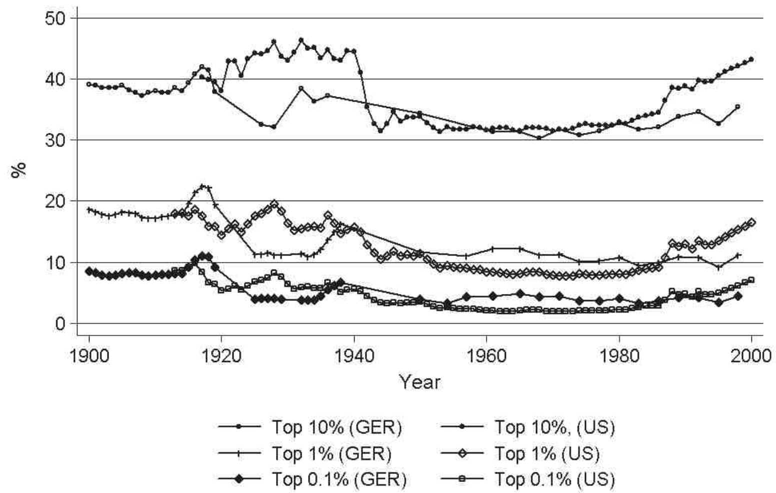

Figure: Alvaredo et al. (2011): "The World Top Incomes Database"

- Increasing inequality (and awareness of it) around the world
- Growing interest in top of income distribution:
  Piketty (2001/3/5); Piketty/Saez (2006); Atkinson/Piketty (2007,2010); **Atkinson/Piketty/Saez (2011)**; Aaberge/Atkinson (2010); Roine/Waldenström (2008); Jäntti et al. (2010); **Peichl/Schaefer/Scheicher (2010)**

Why care about the top? (Atkinson 2007; Waldenström 2009):

- heterogeneity among “the rich”
- command over resources (taxable capacity) and people (power)
- global significance
- impact on growth:
  - ΔY US(75-06): 32.2% [without T1%: 17.9%]
  - ΔY FR(75-06): 27.1% [without T1%: 26.4%]
- source of inequality
  - Δ Gini US(76-06): $+7.2$ p.p. [without T1%: -1.2 p.p.]
- design of public policies

2. Measuring Richness

- Outcome distribution $x = (x_1, x_2, \ldots, x_n) \in R_+^n$,
  π : poverty line (eg. 60% of median income),
  $p = \#\{i | x_i < \pi,\ i = 1, 2, \ldots, n\}$ number of poor people
- Headcount index (fraction poor people):

$$\varphi_{HC}(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{x_i < \pi} = \frac{p}{n},$$

- Foster-Greer-Thorbecke (1984):

$$\varphi_{FGT}(x) = \frac{1}{n} \sum_{i=1}^{n} \left(\left(\frac{\pi - x_i}{\pi} \right)_+ \right)^{\alpha},$$

  ($\alpha > 0$ und $y_+ := \max\{y, 0\}$.)

- ρ richness line, $r = \#\{i | x_i > \rho,\ i = 1, 2, \ldots, n\}$ number rich people.
- Headcount ratio (HCR):

$$R_{HC}(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{x_i > \rho} = \frac{r}{n}.$$

- Income shares of the top $p\%$ (TIS) of the income distribution (Atkinson/Piketty/Saez):

$$IS_p(\mathbf{x}) = \frac{\sum_{i=1}^{n} x_i \mathbf{1}_{x_i > q_{1-p}}}{\sum_{i=1}^{n} x_i}$$

with q_p being the $(1-p)\%$ quantile.

- Advantage: simple descriptive stats, no normative choices
- Problems:
  - HCR only concerned with number of individuals above fixed cutoff level without taking income variation into account
  - TIS do not account for changes in the composition of the population nor changes in the distribution of income among the top
- Solution 1: compute HCR using different richness lines and different TIS to capture some information about distribution

- Solution 2: simultaneously account for composition and distribution with same measure (cf. poverty measurement, e.g.: FGT).
- Medeiros (2006) defines (absolute) affluence gap by

$$R^{Med}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \rho)_+ = \frac{1}{n}\sum_{i=1}^{n}\max\{x_i - \rho, 0\}\,. \tag{1}$$

  - Advantage: increasing in income.
  - But: absolute measure that is proportional to income, i.e. transfer between two rich individuals will not change index.

- Peichl, Schaefer & Scheicher (2006,2010): class of richness measures that take into account the number of rich people as well as the intensity (distribution and amount) of richness:

$$R(\mathbf{x},\rho) = \frac{1}{n}\sum_{i=1}^{n} f\left(\frac{x_i}{\rho}\right) ,$$

  where f is continuous, strictly increasing function measuring the individual contribution to overall richness
- This weighting function shall have some desirable properties which are derived following the literature on axioms for poverty indices
- Transfer axiom: concave or convex

3. Examples

	1	2	3	4	5	6	7	8	9	10
w	1	1	1	1	1	1	1	1	1	1
y1	5	5	5	5	5	5	5	5	5	55
x1	4	4	4	4	4	4	4	4	4	64
y2	5	5	5	5	5	5	5	5	11	49
y3	5	5	5	5	5	5	5	5	30	30

	1	2	3	4	5	6	7	8	9	10
w	1	1	1	1	1	1	1	1	1	1
y1	5	5	5	5	5	5	5	5	5	55
x1	4	4	4	4	4	4	4	4	4	64
y2	5	5	5	5	5	5	5	5	11	49
y3	5	5	5	5	5	5	5	5	30	30

	RL	HCR	Concave	Convex	Absolute	T10
y1	10	0.100	0.082	2.025	4.500	0.550
x1	10	0.100	0.084	2.916	5.400	0.640
y2	10	0.200	0.089	1.522	4.000	0.490
y3	10	0.200	0.133	0.800	4.000	0.300

	1	2	3	4	5	6	7	8	9	10	11
w	9.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
y1	5.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
x1	4.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0
x2	4.0	55.0	57.0	59.0	61.0	63.0	65.0	67.0	69.0	71.0	73.0
y4	5.0	46.0	48.0	50.0	52.0	54.0	56.0	58.0	60.0	62.0	64.0

	1	2	3	4	5	6	7	8	9	10	11
w	9.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
y1	5.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
x1	4.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0
x2	4.0	55.0	57.0	59.0	61.0	63.0	65.0	67.0	69.0	71.0	73.0
y4	5.0	46.0	48.0	50.0	52.0	54.0	56.0	58.0	60.0	62.0	64.0

	RL	HCR	Concave	Convex	Absolute	T10	T01
y1	10	0.100	0.082	2.025	4.500	0.550	0.055
x1	10	0.100	0.084	2.916	5.400	0.640	0.064
x2	10	0.100	0.084	2.949	5.400	0.640	0.073
y4	10	0.100	0.082	2.058	4.500	0.550	0.064

	1	2	3	4	5	6	7	8	9	10	11
w	9.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
y1	5.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
x1	4.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0
x2	4.0	55.0	57.0	59.0	61.0	63.0	65.0	67.0	69.0	71.0	73.0
y4	5.0	46.0	48.0	50.0	52.0	54.0	56.0	58.0	60.0	62.0	64.0

	1	2	3	4	5	6	7	8	9	10	11
w	9.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
y1	5.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
x1	4.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0
x2	4.0	55.0	57.0	59.0	61.0	63.0	65.0	67.0	69.0	71.0	73.0
y4	5.0	46.0	48.0	50.0	52.0	54.0	56.0	58.0	60.0	62.0	64.0

	RL	HCR	Concave	Convex	Absolute	T10	T01
y1	50	0.100	0.009	0.001	0.500	0.550	0.055
x1	50	0.100	0.022	0.008	1.400	0.640	0.064
x2	50	0.100	0.021	0.009	1.400	0.640	0.073
y4	50	0.070	0.009	0.002	0.560	0.550	0.064

4. Empirical Application

	Tax return data	Survey data
Samples	large	small
Representativeness	taxpayers	whole population (less for top 1%)
Income	taxable Y	gross & net
Socio-demographics	little	detailed
Problems	avoidance & evasion	measurement error
	varying definitions	survey / sampling methods
	(income, tax unit)	

- Pareto distribution for income y:
  density: $f(y) = \alpha \frac{k^{\alpha}}{y^{(1+\alpha)}}$, $(k > 0, \alpha > 1)$
  α: Pareto parameter; k scale parameter
  $\beta = \frac{\alpha}{(\alpha-1)}$: inverted Pareto parameter;
  lower α (higher β): more inequality [fatter upper tail]

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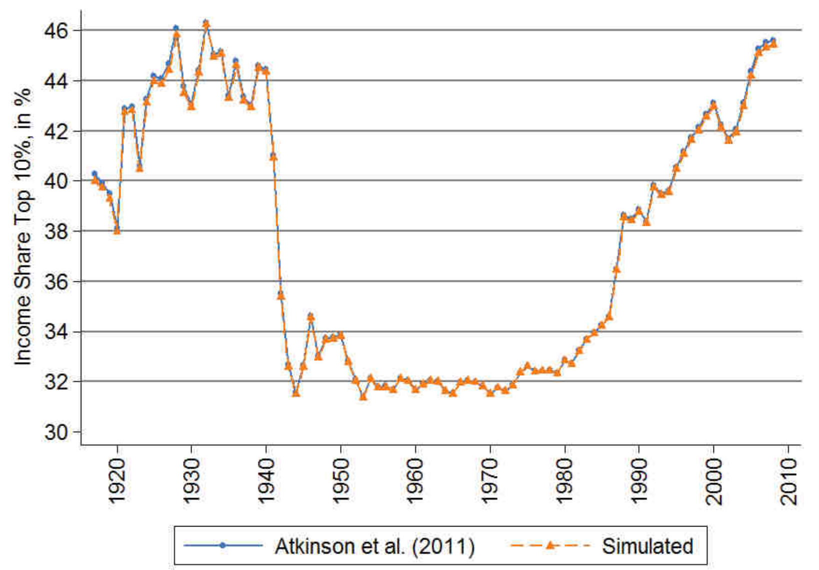
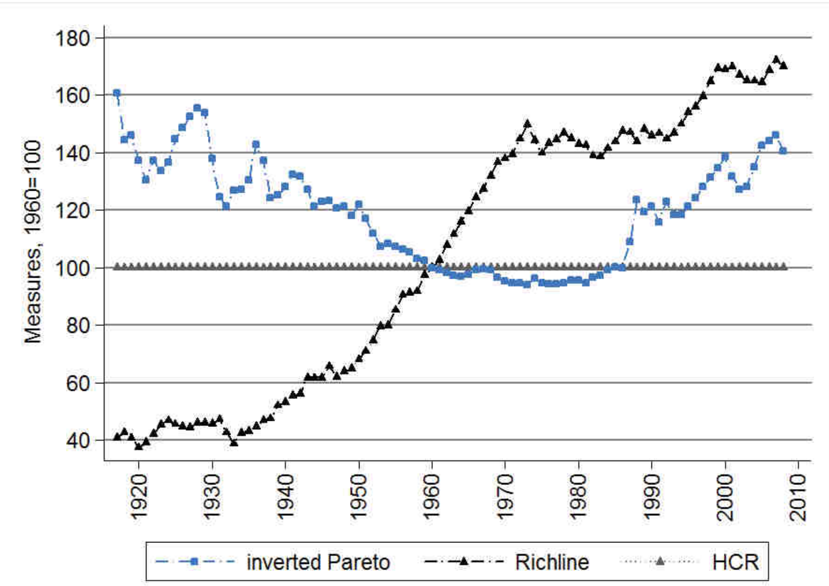
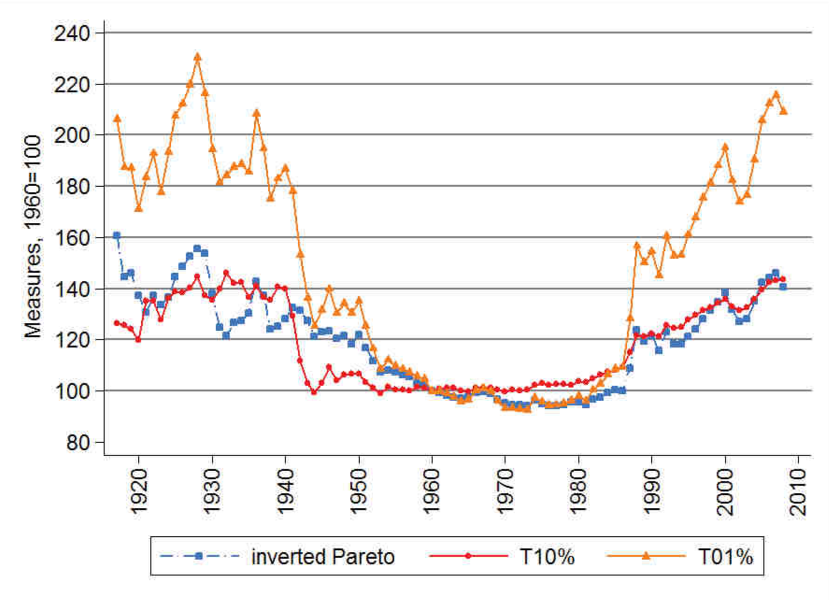
  density: $f(y) = \alpha \frac{k^\alpha}{y^{(1+\alpha)}}$, $(k > 0, \alpha > 1)$
  α: Pareto parameter; k scale parameter
  $\beta = \frac{\alpha}{(\alpha-1)}$: inverted Pareto parameter;
  lower α (higher β): more inequality [fatter upper tail]
- "The World Top Incomes Database": income shares and averages
- α / β and k can be computed from this data
- Assumption: upper tail follows Pareto law (Atkinson/Piketty/Saez)
- Simulate (top) income distribution for each country-year in database
- compute and compare various affluence measures / trends
  - richness line: P90 (T10) threshold ($HCR = 10\%$)


Income Share Top 10%, in %
46
44
42
40
38
36
34
32
30
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
Atkinson et al. (2011)
Simulated


Measures, 1960=100
180
160
140
120
100
80
60
40
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
inverted Pareto
Richline
HCR



Measures, 1960=100
240
220
200
180
160
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1920
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1960
1970
1980
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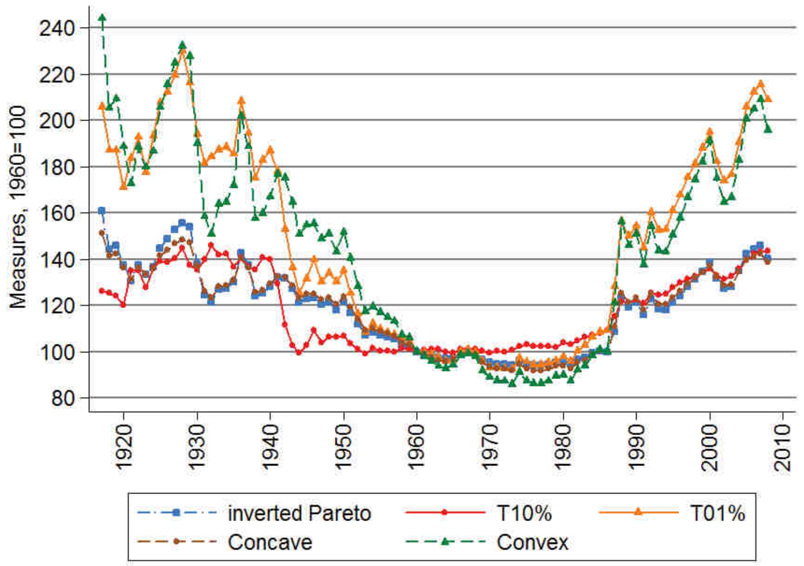
2010
inverted Pareto
T10%
T01%


Measures, 1960=100
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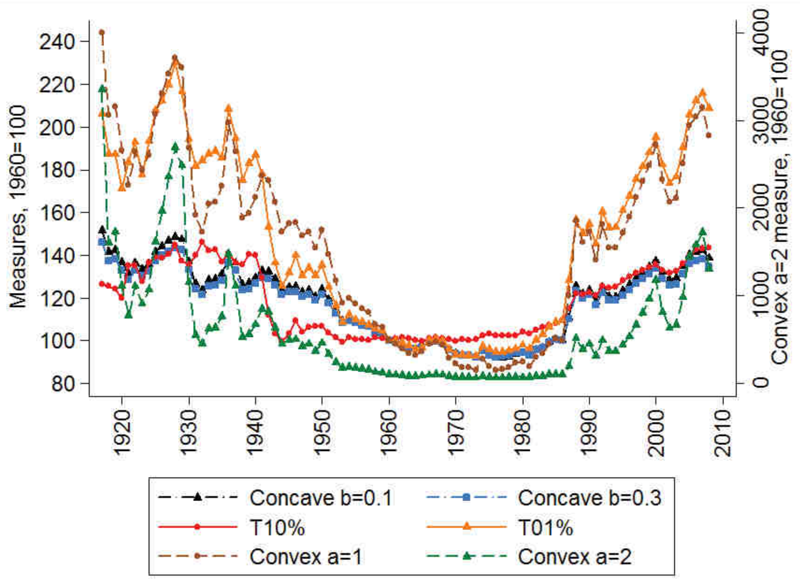
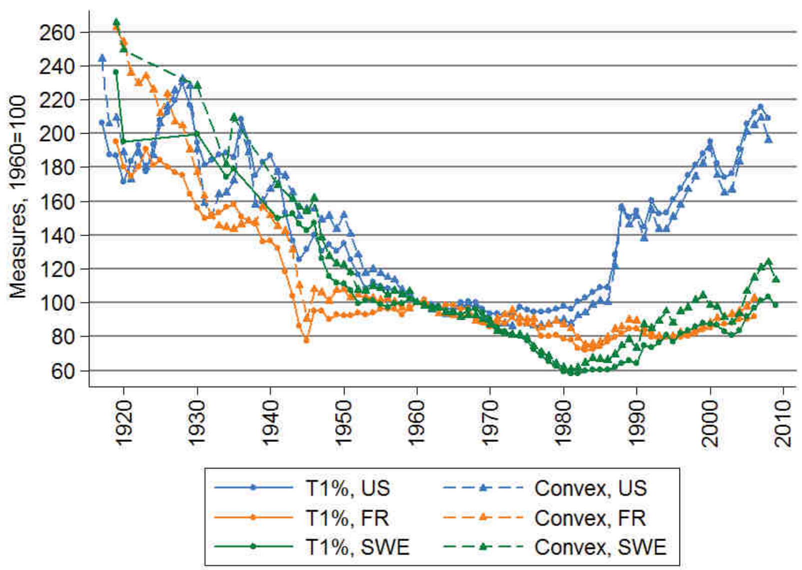
inverted Pareto
T10%
T01%
Concave
Convex


Measures, 1960=100
240
220
200
180
160
140
120
100
80
Convex a=2 measure, 1960=100
4000
3000
2000
1000
0
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
Concave b=0.1
Concave b=0.3
T10%
T01%
Convex a=1
Convex a=2


Measures, 1960=100
260
240
220
200
180
160
140
120
100
80
60
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
T1%, US
T1%, FR
T1%, SWE
Convex, US
Convex, FR
Convex, SWE

correlations all countries

	Concave1	Concave2	Convex1	Convex2	Absolute	T10	T01
Concave1	1.000						
Concave2	0.980	1.000					
Convex1	0.966	0.906	1.000				
Convex2	0.461	0.394	0.627	1.000			
Absolute	-0.020	-0.030	0.015	0.083	1.000		
T10	0.793	0.801	0.741	0.312	-0.101	1.000	
T01	0.948	0.904	0.950	0.506	-0.023	0.905	1.000

5. Multidimensional Affluence

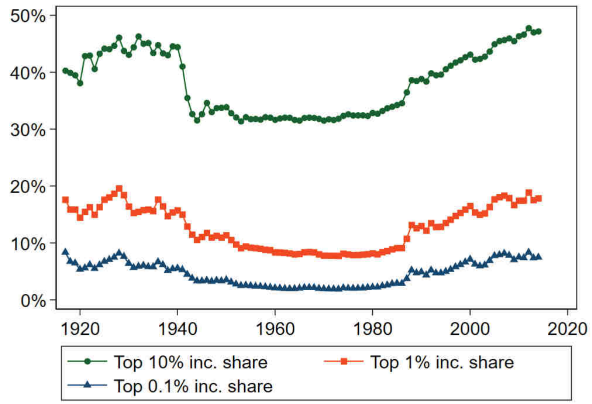
Peichl, A. and N. Pestel (2013): Multidimensional Affluence: Theory and Applications to Germany and the US, Applied Economics 45 (32), 2013, 4591-4601.

- Peichl / Pestel (2013): extend affluence measures (Peichl et al. 2010) to the multidimensional case following Alkire/Foster (2011)
- incorporate wealth as dimension of multidimensional affluence
- empirical application to Germany and the US

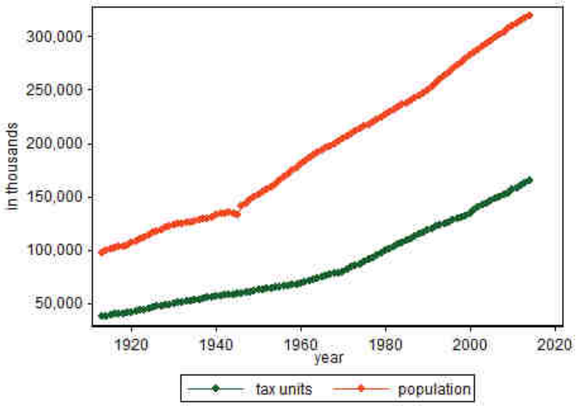
6. Reassessing US Top Income Shares

C. Krolage, A. Peichl, D. Waldenström: Reassessing Trends in U.S. Top Income Shares: The Role of Population and Productivity Growth, work in progress.

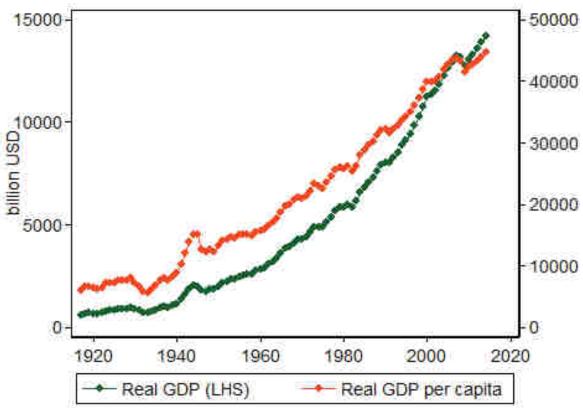
Development of top income shares



Population growth



Economic growth

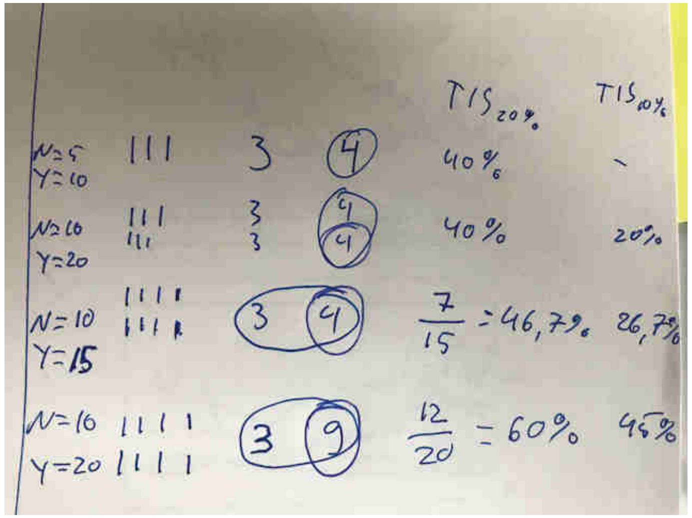


Measurement of top inequality

- Piketty & Saez popularized top income shares to measure inequality
  - TIS capture share of total income for fixed fraction of the population
- This approach does not explicitly account for:
  - Population growth (US population tripled between 1917-2014)
  - Absolute changes – only relative measure. But: Growing TIS may be driven by rich becoming richer, or by poor becoming poorer or both

⇒ We re-investigate long run trends in top income inequality in the US, accounting for population and (differential) economic growth

Problem of top income shares



Related Literature

- Our analysis contributes to several strands of the inequality literature:
  - US inequality trends (e.g. Piketty and Saez, 2003)
  - Long run trends in top income shares (e.g. Piketty, 2001; Atkinson and Piketty, 2007, 2010; Leigh, 2009)
  - Inequality and population growth (e.g. Blau and Kahn, 2015)
  - Different measurement approaches for top shares: e.g. adding unrealized capital gains (Armour, Burkhauser and Larrimore, 2013), using a national accounts-equivalent measure (Piketty, Saez and Zucman, 2016)

⇒ Our analysis keeps the income concept unchanged and focuses on different statistical measures of top shares (and their composition)

Decomposition methods

1. Construct counterfactual top income shares accounting for population and/or productivity growth
   - Income shares above fixed real top thresholds (1917, 1980 and 2014)
   - Growth adjustment: GDP-deflated income thresholds
   - Population size adjustment: Constant number of top tax units
2. Decompose top 1 percent income share S_1 into contributions of population, overall income and top income growth
   - $S_i = \frac{\bar{Y}_i N_i}{\bar{Y} N} \Leftrightarrow \Delta ln S_i = \Delta ln \bar{Y}_i + \Delta ln N_i - \Delta ln \bar{Y} - \Delta ln N$
3. Decompose counterfactual top income shares into the contributions of wage, capital, and entrepreneurial income

Decomposition – approach 1 detailed

- We compute top income shares (TIS) for four different top groups:
  A) *Baseline* as in Piketty/Saez: Fixed pop share, variable group size.
  B) *TIS for those earning above CPI-deflated threshold*: both variable.
  C) *TIS for those earning above GDP-deflated threshold*: both variable.
  D) *Constant number of top earners*: Variable pop share, fixed group size.
- Difference between B & C captures to what extent top incomes have grown faster than the overall economy
- D isolates effect of rising incomes above (fixed) income thresholds

Data

- World Wealth and Income Database (c.f. Piketty and Saez, 2003) based on individual tax statistics from the Statistics of Income (SOI) in the Internal Revenue Service (IRS)
  - Top income shares (top 10% and above)
  - Real income thresholds
  - Population and number of tax units
- Real economic growth
- Some additional SOI data (income shares below P90)
- Interpolation assuming Pareto distribution

Different measures for the top 1 percent

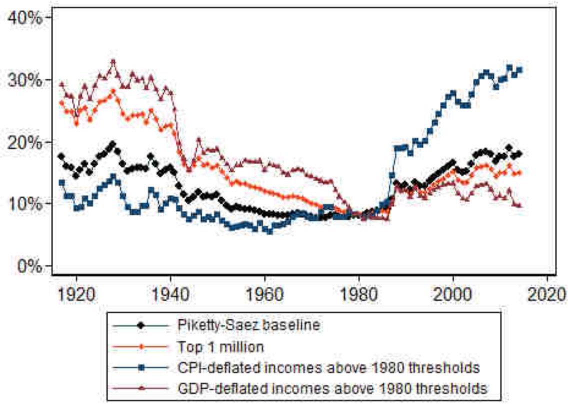


Figure: Income shares

Different measures for the top 1 percent

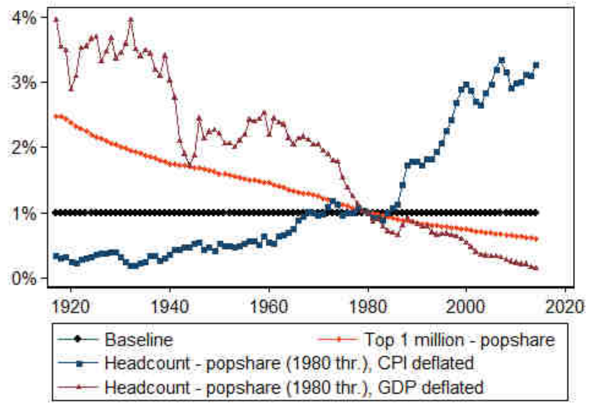


Figure: Population shares

Income shares above real thresholds

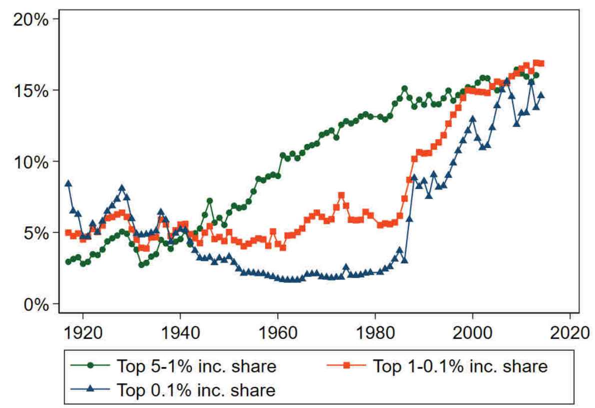


Figure: Income shares above CPI-deflated 1980 thresholds

Income shares above growth-adjusted thresholds

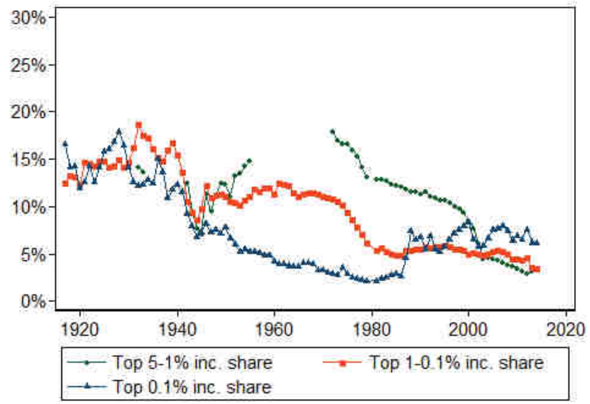


Figure: Income shares above GDP-deflated 1980 thresholds

Accounting for population growth: Fixed number of tax units

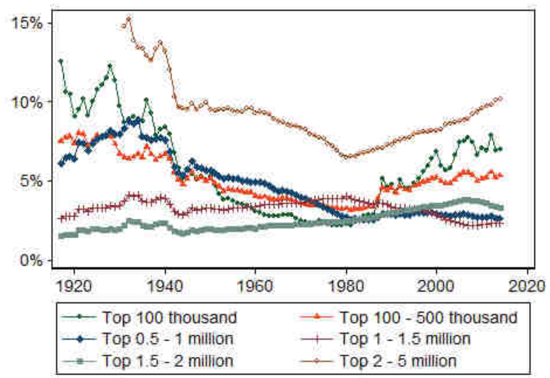


Figure: Income shares of fixed numbers of top tax units

Distributional national accounts measure: Fixed number of tax units

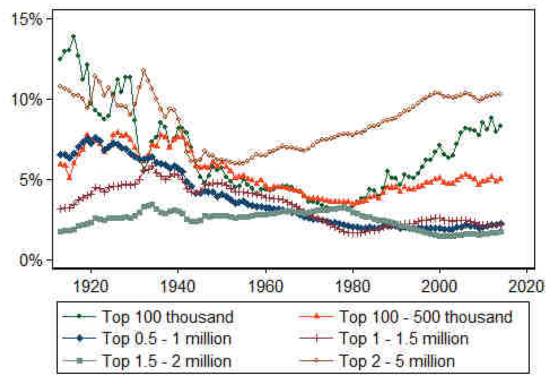


Figure: Income shares of fixed numbers of top tax units

Decomposition of the top 1 percent income share

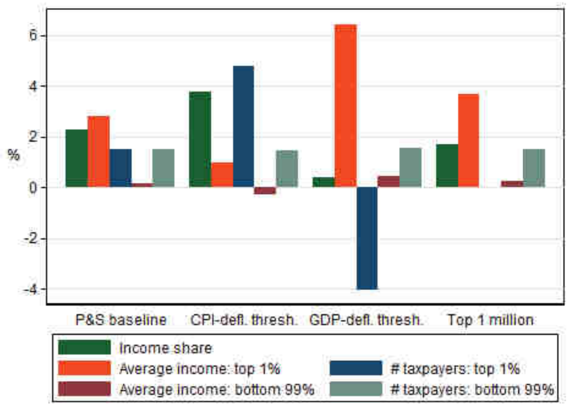


Figure: Top 1 percent log decomposition: 1980-2014

Decomposition of the top 1 percent income share

Annual growth in income and population shares								
Period	Baseline (P&S)		CPI-deflated top thresholds		GDP-deflated top thresholds		Top 1 million	
	Income	Pop.	Income	Pop.	Income	Pop.	Income	Pop.
1917-1929	2.4	0	3.4	3.1	1.7	-0.5	1.4	-1.6
1929-1950	-2.1	0	-1.8	3.0	-1.9	-1.3	-2.3	-1.1
1950-1980	-0.7	0	1.0	2.9	-2.0	-2.1	-1.8	-1.5
1980-2000	4.4	0	7.5	6.2	4.6	-1.8	4.0	-1.5
2000-2014	0.4	0	0.6	0.5	-1.3	-8.4	0.0	-1.5

Table: Average annual growth rates of different top 1 percent measures

Income source decomposition: Fixed number of tax units

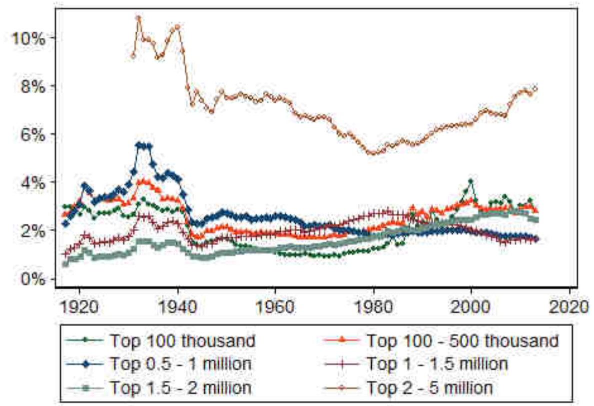


Figure: Wage income: Income shares of fixed numbers of top tax units

Income source decomposition: Fixed number of tax units

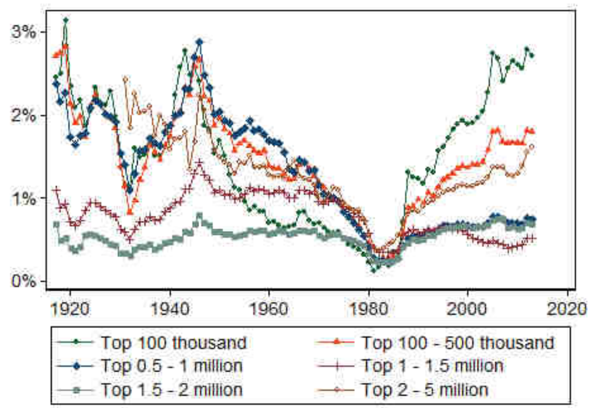


Figure: Entrepreneurial income: Income shares of fixed numbers of top tax units

Income source decomposition: Fixed number of tax units

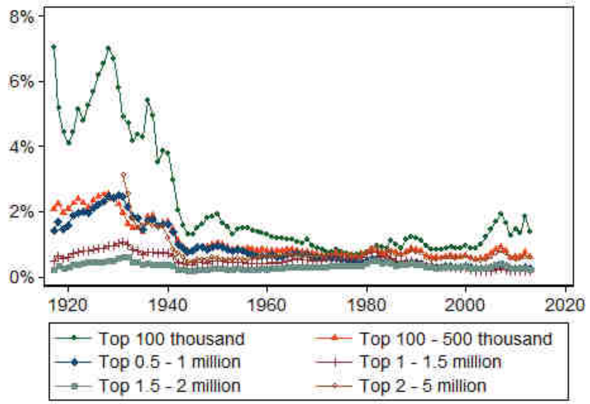


Figure: Capital income: Income shares of fixed numbers of top tax units

Summary & Outlook

- We re-assess top income shares by accounting for population and economic growth
- Results broadly in line with Piketty and Saez, with some notable divergences
- With alternative methods: more strongly diverging developments between top income brackets, not always yielding a U-shaped development
- Income earners at the very top and in the upper middle class experience the most gains
- In contrast: income shares of earners just below the very top remain rather constant for some measures

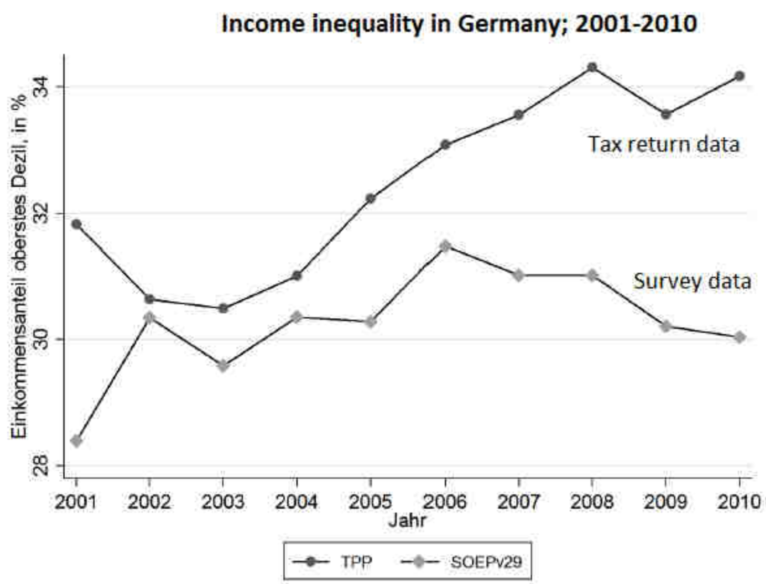
7. Top income inequality over the business cycle

Moritz Drechsel-Grau, Andreas Peichl, Kai D. Schmid: Inequality by Income Source over the Business Cycle, work in progress

Annual GDP Growth in Germany

Figure: Real GDP Growth in Germany, 2000-2014

Inequality trends in DE: data matters!



Motivation & Literature

- How do booms and recessions affect income distribution?
  → decomposition of overall income risk to evaluate cost of recessions

Research Questions

- Does income growth vary with past income (or past income *growth*?)
- Does distribution of income growth vary with the business cycle? And across income sources?
- How does this affect measures of income inequality?
- Differences across different subpopulation (age, gender, regions)?

Related Literature

- Earnings risk over the BC: [Storesletten et al., 2004], [Guvenen et al., 2014], [Busch et al., 2016]
- Earnings risk in general: [Guvenen et al., 2016]
- Role of non-labor income for inequality: [Saez and Zucman, 2016], [Alstadsæter et al., 2016]

  → our contribution: DE (instead of US), other inc-sources

Preview of Results

(Preliminary) Results:

- Income growth depends on previous income
- Variation of income growth also depends on previous income
- Business cycle has differential effect on (variation of) income growth

Earnings Income Growth: DE is not US (!)

- Less spread compared to US (P90-P10 about half as large)
- Lower SD, strongly U-shaped
- Generally no left-skewness; only in 2008 and not as pronounced

Some results consistent with GOS14

1. Left-Skewness procyclical while SD acyclical
2. Increased left-skewness in crisis most pronounced for 45-49 olds

Data and Sample Restrictions

German Tax Payer Panel (TPP)

- administrative data of universe of (personal income) tax returns
- current version: 5% sample; balanced panel from 2001 to 2010
- unit of observation: taxpayer (ie. individual or married couple)
- all information necessary to calculate taxable income, including deductions and income-source specific information!
- no top-coding

Limitations

- limited socio-demographic information
- only tax payers (unemployed, retired people missing)
- (almost) no transfers

Income Sources and Notation

Income Sources

- earnings income: Y_t^{EARN}
- self-employment income: Y_t^{SE}
- business income: Y_t^{BUS}
- income from capital and wealth: Y_t^{CAP}
- total market income: $Y_t^{TO} = Y_t^{EARN} + Y_t^{SE} + Y_t^{CAP} + Y_t^{BUS}$

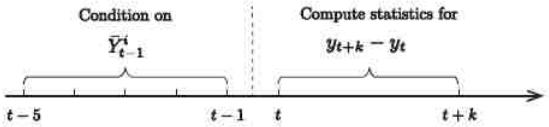
Notation

- upper case: level
- lower case: natural logarithm
- example: $y_t^{CAP} = ln(Y_t^{CAP})$

Overview of Empirical Approach

Object of interest:

- Income growth differentials ...
  - (growth btw. base year t and year $t + n$)
- ... conditional on past income (in year $t - s$)
  - (to account for growth differential along income distribution)



- ... net of life-cycle effects.
  - (Income exhibits pronounced life-cycle pattern (hump-shaped))

- Further complications:
  - Negative income realizations (log change not defined)
  - Low incomes with high volatility → minimum income threshold

Netting Out Life-Cycle Effects

- **Object of interest**: income growth differentials conditional on past income net of life-cycle effects.
- Income exhibits pronounced life-cycle pattern (hump-shaped)
- First: split the sample into six **age groups**: 25-34, 35-39, 40-44, 45-49, 50-54, 55-60
- However, even within these age groups, younger individuals may still receive systematically less income than older individuals.
  $\rightarrow$ scale own income by **age-specific average incomes**
- Regress log income on a full set of age and cohort dummies.

$$y_{it}^X = \alpha + \sum_{h=26}^{60} \beta_h \mathbf{1}(age_{it} = h) + \sum_{k=1940}^{1985} \delta_j \mathbf{1}(cohort_i = j) + u_{it}$$

- Define **age-specific average log income** by adding the age effects onto the average income at age 25:

$$\bar{y}^{X,h} \equiv \bar{y}^{X,25} + \hat{\beta}_h$$

Growth Measures

Log Difference

defined if $Y_t > 0$ **and** $Y_{t+n} > 0$

$$glog_t^{X,n} = \log(Y_{t+n}) - \log(Y_t)$$

Arc Percent Change

defined if $Y_t > 0$ **or** $Y_{t+n} > 0$

$$garc_t^{X,n} = \frac{Y_{t+n} - Y_t}{\frac{1}{2}(Y_t + Y_{t+n})}$$

Negative Income Realizations

While log-measure is not defined for negative realizations, the arc measure is well-defined. However, interpretation is complicated.

Sample Restrictions, Minimum Income Threshold

Sample Restrictions

1. males between 25 and 60 years of age
2. income exceeds a minimum threshold, $Y_{is} \geq Y_s^{min}$ for $s \in \{t, t+1\}$
   $\rightarrow$ reasonable attachment to the labor market
3. income in $t-1$ (and sometimes in at least some years between $t-2$ and $t-5$) have to exceed the respective minimum threshold.
   $\rightarrow$ recent income can be computed in a sensible way

Minimum Earnings Threshold

= 3 months of part-time work (20h) at the federal minimum wage

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2015
w_t^{min}	6.32	6.32	6.28	6.35	6.39	6.63	6.85	6.92	6.53	6.80	7.36
Y_t^{min}	1643	1643	1632	1650	1662	1724	1780	1800	1698	1768	1912

Note: All values are expressed in 2005 prices.

Empirical Approach

Non-Parametric Approach

Estimate various moments of the conditional distribution of income growth given recent income.

- **Moments** to be considered:
  - Percentiles: 1, 5, 10, 20, . . . , 80, 90, 95, 99
  - Standard Deviation
  - Skewness
  - Kurtosis
  - Upper tail (P90-P50)
  - Lower tail (P50-P10)
- Repeat analysis for different **subpopulations**
  - age groups
  - male vs. female (not yet)
  - etc.

Measures of Skewness/Asymmetry

Third (Standardized) Central Moment

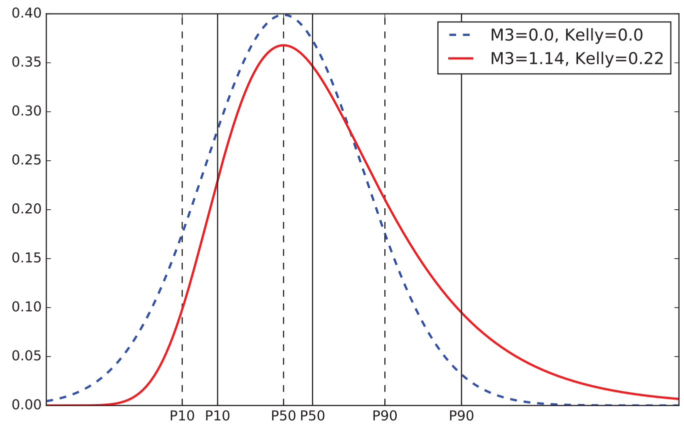
$$SKEW_{M3} = \mathbb{E}\big[(x_i - \mathbb{E}[x_i])^3\big]/\sigma^3 \tag{1}$$

Kelly (1947) Skewness

$$SKEW_{Kelly} = \frac{[P90 - P50] - [P50 - P10]}{P90 - P10} \tag{2}$$

$$SKEW_i \quad \begin{cases} < 0 & \Rightarrow \text{negative skewness (left-skewed, left-tailed)} \\ = 0 & \Rightarrow \text{symmetrical} \\ > 0 & \Rightarrow \text{positive skewness (right-skewed, right-tailed)} \end{cases}$$

Illustration Skewness/Asymmetry



- $SKEW_{M3} = \mathbb{E}\left[(x_i - \mathbb{E}[x_i])^3\right]/\sigma^3$
- $SKEW_{Kelly} = \frac{[P90 - P50] - [P50 - P10]}{P90 - P10}$

Measures of Excess Kurtosis (Peakedness and Tailedness)

Excess Fourth (Standardized) Central Moment

$$KURT_{M4} = \mathbb{E}\left[(x_i - \mathbb{E}[x_i])^4\right]/\sigma^4 - \underbrace{KURT_{M4}(Gaussian)}_{=3}$$

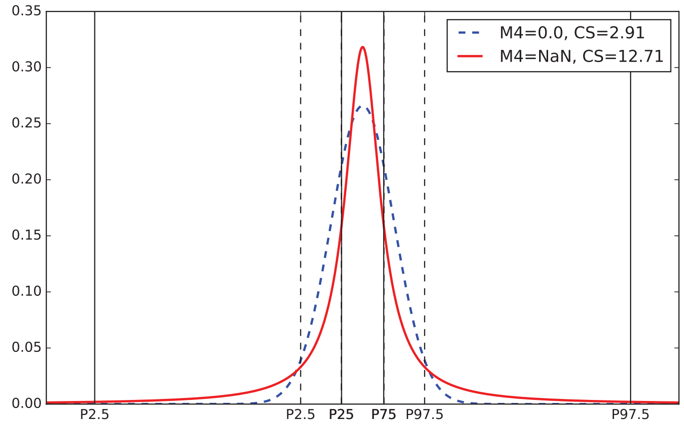
$$KURT_{M4} \quad \begin{cases} < 0 & \Rightarrow \text{platykurtic (thin-tailed, sub-Gaussian)} \\ = 0 & \Rightarrow \text{mesokurtic (Gaussian)} \\ > 0 & \Rightarrow \text{leptokurtic (fat-tailed, super-Gaussian)} \end{cases}$$

Crow-Siddiqui (1967) Measure

$$KURT_{CS} = \frac{P97.5 - P2.5}{P75 - P25}$$

for Gaussian: $KURT_{CS} = \frac{1.960}{0.674} = 2.908$

Illustration Excess Kurtosis



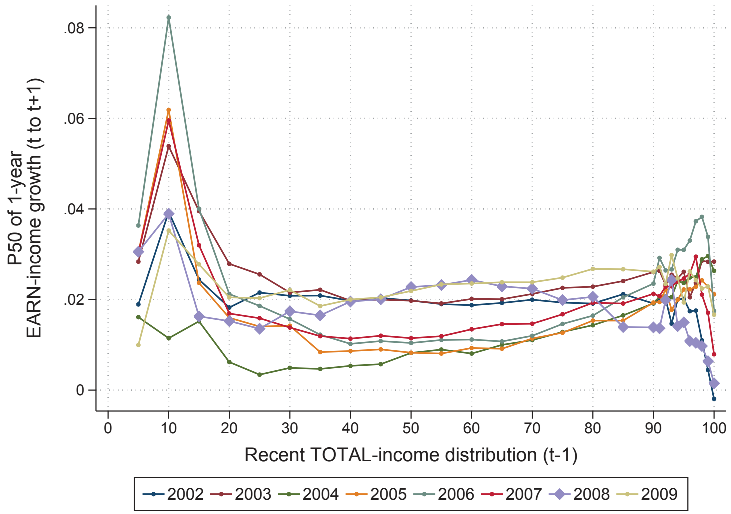
- $KURT_{M4} = \mathbb{E}\left[(x_i - \mathbb{E}[x_i])^4\right]/\sigma^4$
- $KURT_{CS} = \frac{P97.5 - P2.5}{P75 - P25}$

Preliminary Results

Baseline choices

- only log-measure
- $n = 1$ (one-year growth rates)
- only $s = 1$ (conditional on last year's income)
- 5 % sample $\rightarrow$ noisy
- average over age groups
- focus on earnings

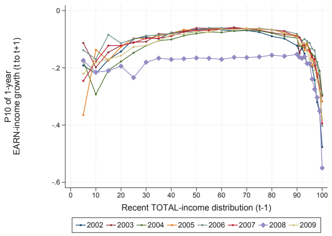
Illustration (P50 of earnings GROWTH cond. on total inc)



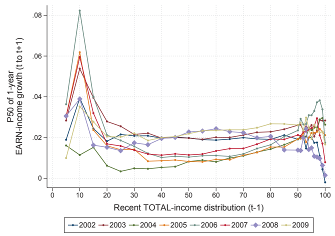
Earnings Income (EARN)

EARN-Inc Growth Cond. on TOTAL-Inc

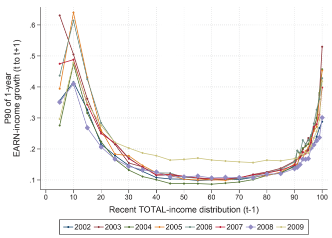
(a) P10



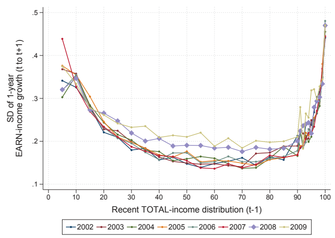
(b) P50



(c) P90

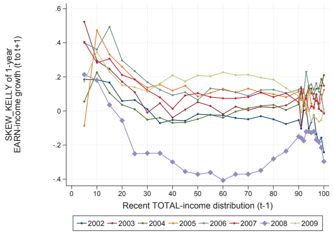


(d) Standard Deviation

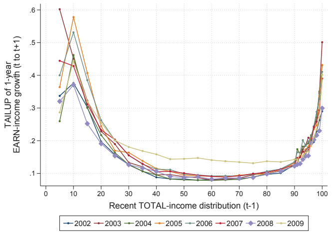


EARN-Inc Growth Cond. on TOTAL-Inc

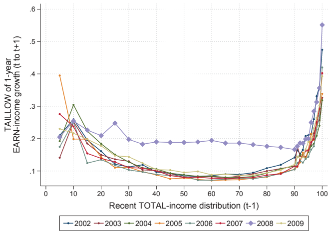
(e) Kelly's Skewness



(f) P90 - P50

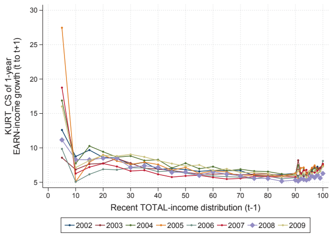


(g) P50 - P10

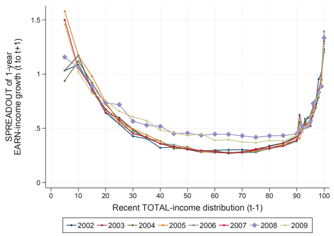


EARN-Inc Growth Cond. on TOTAL-Inc

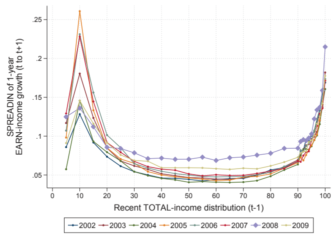
(h) CS-Kurtosis



(i) P97.5 - P2.5

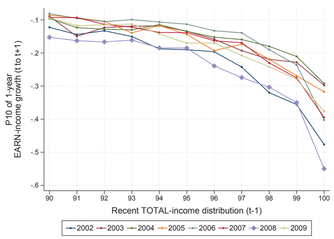


(j) P75 - P25

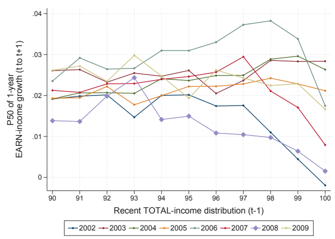


Top 10: EARN-Inc Growth Cond. on TOTAL-Inc

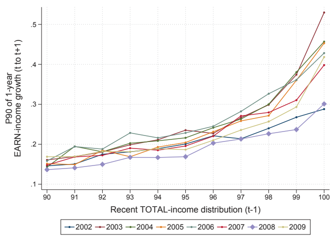
(a) P10



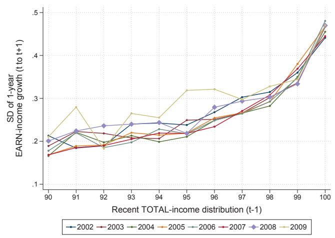
(b) P50



(c) P90

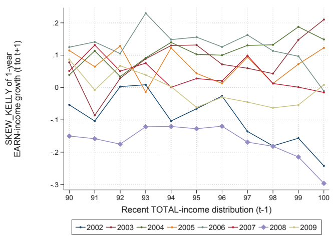


(d) Standard Deviation

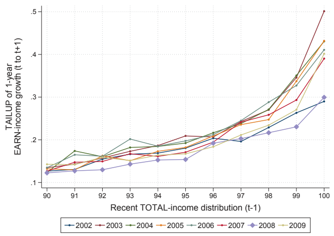


Top 10: EARN-Inc Growth Cond. on TOTAL-Inc

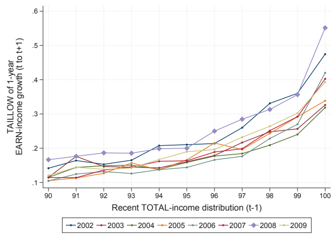
(e) Kelly's Skewness



(f) P90 - P50

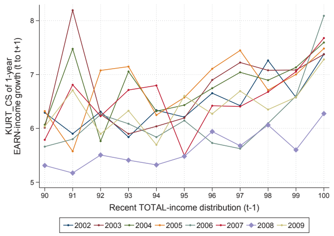


(g) P50 - P10

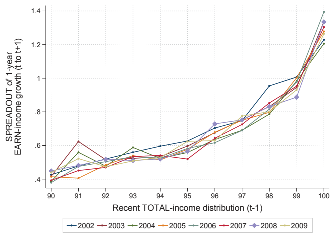


Top 10: EARN-Inc Growth Cond. on TOTAL-Inc

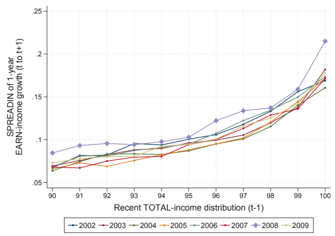
(h) CS-Kurtosis



(i) P97.5 - P2.5



(j) P75 - P25



Earnings Income Growth: Summary

General Comments

- Less spread compared to US (P90-P10 about half as large)
- Lower SD, strongly U-shaped
- Slightly increased SD in 2008, 2009

Skewness

- Strong left-skewness in 2008; U-shaped, most pronounced for percentiles 40 to 80
- Left-skewness in 2008 driven by increase in lower tail (P50-P10)
- Slight right-skewness in 2009 (same shape as left-skewness in 2008), driven by increase in upper tail (P90-P50)
- No significant skewness in 2002-2007
- Acyclical for age group 6 (55-60)

Comparison to Guvenen et al. (2014)

Consistent with GOS14

1. Left-Skewness procyclical while SD acyclical
2. Increased left-skewness in crisis most pronounced for 45-49 olds
3. U-shape of SD
4. Left-skewness increases up to 80th percentile, then decreases quickly
5. Left-skewness in 2008 most pronounced for percentiles 40-80

Inconsistent with GOS14

1. Distribution less spread out (P90-P10 about half as large, lower SD)
2. Generally no left-skewness; only in 2008 and not as pronounced

Business Income (BUS)

SE-Income Growth: Summary

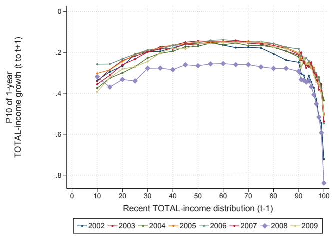
General Comments

- Results only available for top 5 percentiles of RI_t^{SE}-distribution
- Spread (P90-P10) and SD decrease with recent income (both bases)
- Upper and lower tail shrink with recent income (both bases)
- No clear skewness pattern
- 2008, 2009?

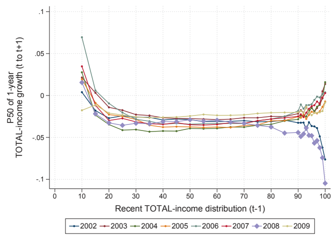
Total Income (TOT)

TOTAL-Inc Growth Cond. on TOTAL-Inc

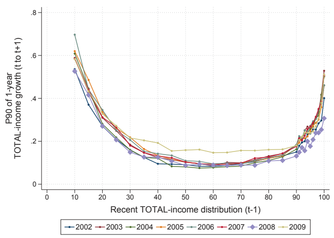
(a) P10



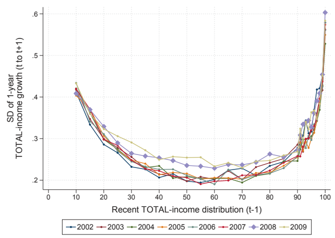
(b) P50



(c) P90

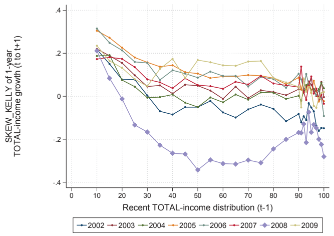


(d) Standard Deviation

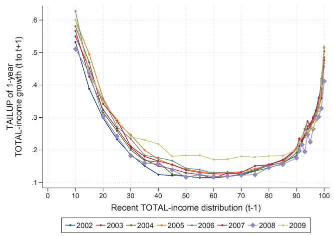


TOTAL-Inc Growth Cond. on TOTAL-Inc

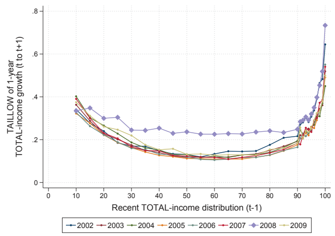
(e) Kelly's Skewness



(f) P90 - P50

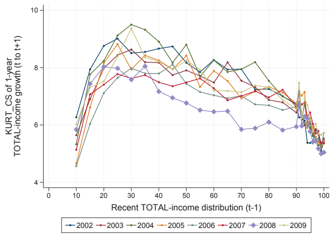


(g) P50 - P10

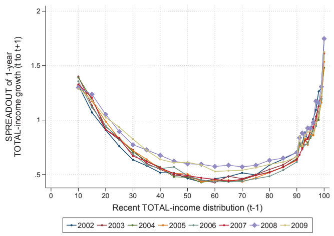


TOTAL-Inc Growth Cond. on TOTAL-Inc

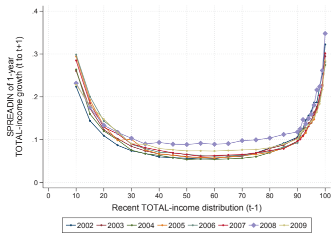
(h) CS-Kurtosis



(i) P97.5 - P2.5



(j) P75 - P25



Conclusion and Outlook

Summary

1. Less spread compared to US (P90-P10 about half as large, lower SD)
2. Generally no left-skewness; only in 2008 and not as pronounced

Outlook

1. More to come
2. Full data (not only sample / balanced panel)
3. More on income sources
4. More on heterogeneity
5. Inequality measures

8. Conclusion

- Increasing inequality at top since 1970s
- Top income shares: simple descriptive stats; but powerful
- Different (normative) measurement choices can lead to slightly different conclusions
- Correlation between measures relatively high
- Multidimensional measurement allows taking into account correlation between dimensions

Inequality at the top related to

- Macroeconomics: (big) recessions; financial crisis, inflation, war
- Roine / Vlachos / Waldenström (2009): e.g. financial development
- Executive remuneration: tournament / superstar theories, bargaining
- Progressive taxation: elasticity of income w.r.t. net-of-tax rate (Saez / Slemrod / Giertz, 2011): supply side, income shifting and bargaining
- Political Economy: partisanship?
- Globalization, (skill-biased) technol. change (how relevant at top?)

- World Wealth and Income Database (WID.world) http://wid.world/data/
- From top income shares to Distributional National Accounts (DINA)
- Data available and can be used for research, some examples:
  - Julia Tanndal & Daniel Waldenström: Does Financial Deregulation Boost Top Incomes? Evidence from the Big Bang, Economica, forthcoming,
  - Clemens Fuest, Andreas Peichl & Daniel Waldenström Pikettys r-g model: wealth inequality and tax policy, CESIfo Forum, No. 1, 1-10, 2015.
  - J Roine & Daniel Waldenström: Common Trends and Shocks to Top Incomes: A Structural Breaks Approach, Review of Economics and Statistics, 93(3), 832846, 2011.
  - J Roine, J Vlachos & Daniel Waldenström: The Long-Run Determinants of Inequality: What Can We Learn from Top Income Data?, Journal of Public Economics 93(78), 974988, 2009.

Thank you for your attention!

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Dual cutoff method

- *so far:* affluence w.r.t. single dimensions separately (*1st cutoff*)
- *now:* individual (multidimensionally) affluent if affluence counts at least at certain threshold (*2nd cutoff*)

Measures:

- dimension adjusted "headcount ratio"
- dimension adjusted multidimensional richness measures

- German Socio-Economic Panel Study (SOEP)
- Survey of Consumer Finances 2007 (SCF)
- Income
  - market income from all sources and household members
  - substract asset income (*interest, dividends, gains etc.*)
- Wealth
  - household net worth (assets - debt)
- Cutoffs
  - distinguish affluent person from a non-poor but non-affluent
  - 80%-quantile of age group (*head aged* $<$*30, 30–59, 60*$+$)
- Adjustments
  - equivalence weighting $\rightarrow$ square root scale
  - currency $\rightarrow$ values expressed in 2007 PPP $US

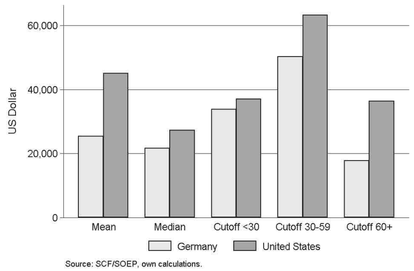


Figure: Income

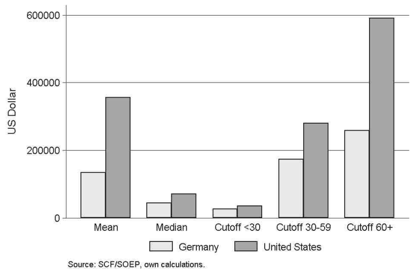


Figure: Wealth

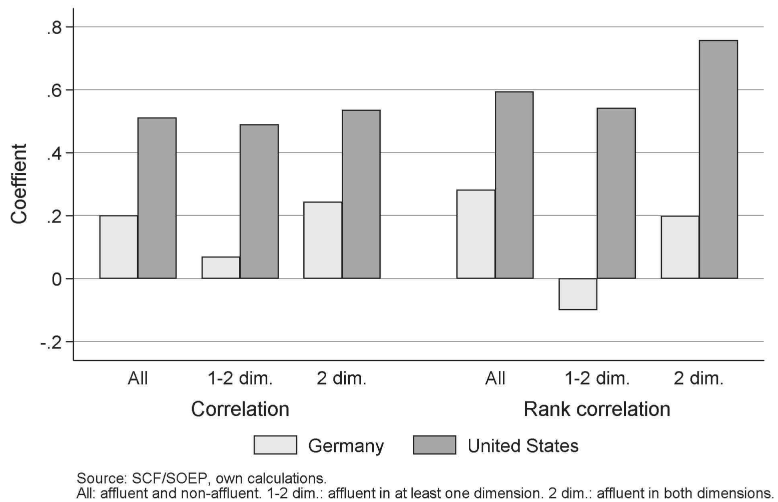


Figure: Correlations between income and wealth

k	R^M_{HR}	$R^M_{\alpha=1}$	$R^M_{\alpha=2}$	$R^M_{\beta=1}$	$R^M_{\beta=3}$
			United States 2007		
1	0.199	0.133	9.143	0.020	0.030
2	0.111	0.103	8.446	0.012	0.016
			Germany 2007		
1	0.200	0.104	0.997	0.030	0.049
2	0.081	0.051	0.457	0.013	0.020

Note: k denotes the second cutoff threshold. Source: SCF/SOEP, own calculations.

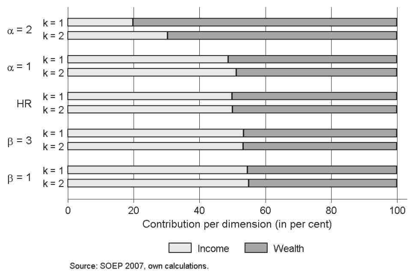


Figure: Germany

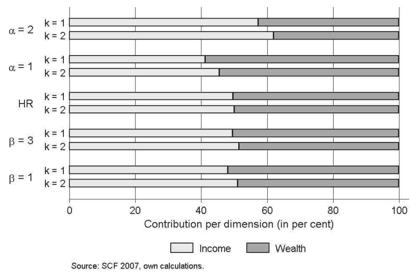


Figure: US

Robustness

- survey data vs. administrative tax data (for Germany)
- different cutoff thresholds (larger quantiles, % of median)

Discussion

- data requirements (availability of *all* dimensions)
- pension wealth and further dimensions

Summing up

- propose multidimensional affluence measures (convex and concave)
- conclusions from GE-US comparison depends on (normative) view
- importance of dimensions at the top different

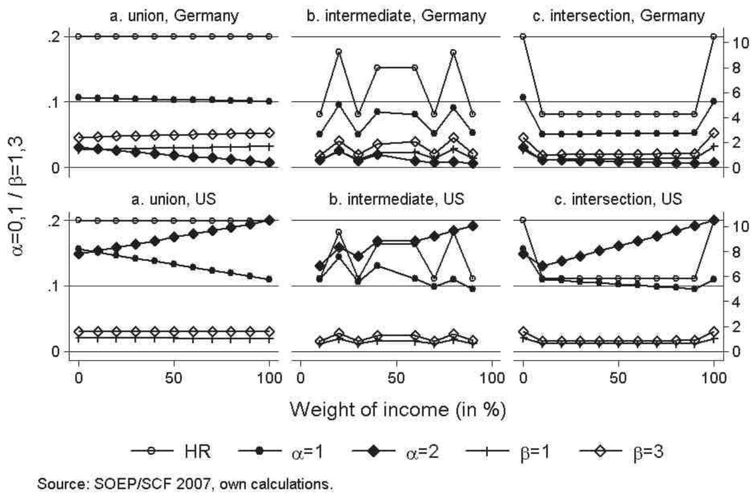


Figure: Multidimensional affluence for different weights

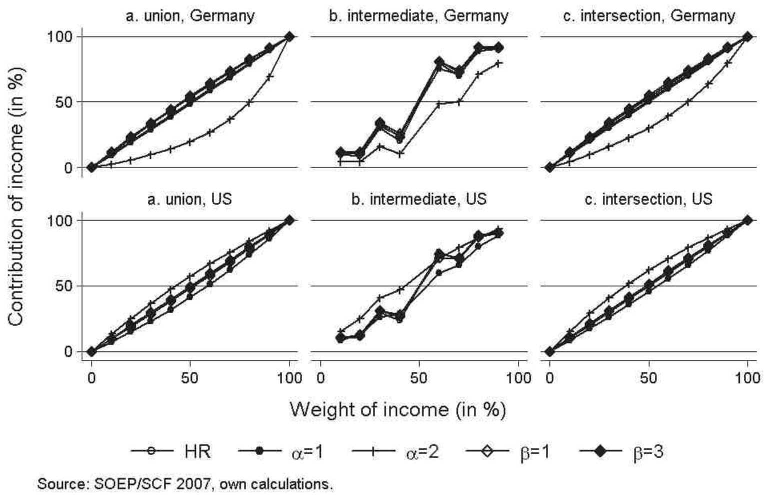


Figure: Multidimensional affluence for different weights

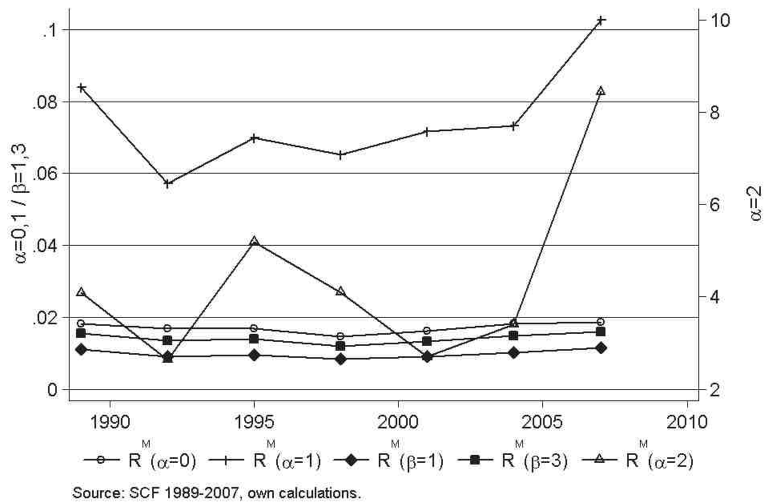


Figure: Multidimensional affluence (United States, 1989–2007)

- *Focus axiom:* a richness index shall be independent of the incomes of the non-rich.
- *Continuity axiom:* the index shall be a continuous function of incomes, i.e. small changes in the income structure shall not lead to discontinuously large changes in the richness index.
- *Monotonicity axiom:* a richness index shall increase if c.p. the income of a rich person increases.
- *Subgroup decomposability axiom:* the overall degree of richness may be decomposed into the (population) weighted sum of subgroup richness indices.

Transfer axiom in poverty: index shall decrease with rank-preserving progressive transfer from a poor person to someone who is poorer.

⇒ Translation to richness?:

- *Transfer axiom T1 (concave):* richness index shall **increase** with rank-preserving progressive transfer between two rich persons.
- *Transfer axiom T2 (convex):* richness index shall **decrease** with rank-preserving progressive transfer between two rich persons.

Question behind these two opposite axioms: shall richness index increase if

- (i) a billionaire gives an amount x to a millionaire,
- (ii) the millionaire gives the same amount x to the billionaire.

Concave:

- FGT index satisfying T1:

$$R_\alpha^{FGT,T1}(\mathbf{x},\rho) = \frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{x_i-\rho}{x_i}\right)_+\right)^\alpha, \quad \alpha \in (0,1)\,.$$

- index analogous to the poverty index of Chakravarty (1983):

$$R_\beta^{Cha}(\mathbf{x},\rho) = \frac{1}{n}\sum_{i=1}^{n}\left(1-\left(\frac{\rho}{x_i}\right)^\beta\right)_+, \beta > 0.$$

Convex:

- FGT index satisfying T2:

$$R_\alpha^{FGT,T2}(\mathbf{x},\rho) = \frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{x_i-\rho}{\rho}\right)_+\right)^\alpha, \alpha > 1$$

Consider two populations with income distribution

$$\mathbf{x} = (5, 5, 5, 11, 11) \text{ and } \mathbf{y} = (5, 5, 5, 100, 100) .$$

Let $\rho_{\mathbf{x}}$, $\rho_{\mathbf{y}}$ be 200% of the median income. Then $\rho_{\mathbf{x}} = \rho_{\mathbf{y}} = 10$ and we obtain

$$R^{HC}(\mathbf{x},\rho = 10) = R^{HC}(\mathbf{y},\rho = 10) = 0.400 ,$$

and

$$R^{Cha}_{\beta=1}(\mathbf{x}) = 0.036 \quad \text{and} \quad R^{Cha}_{\beta=1}(\mathbf{y}) = 0.360,$$
$$R^{FGT,T2}_{\alpha=2}(\mathbf{x}) = 0.004 \quad \text{and} \quad R^{FGT,T2}_{\alpha=2}(\mathbf{y},) = 32.4.$$

$$\mathbf{x} = (5, 5, 5, 11, 9989) \text{ and } \mathbf{y} = (5, 5, 5, 1000, 9000)\,,$$

where $\mathbf{y}$ is obtained from $\mathbf{x}$ by a progressive transfer of 989 monetary units between the two rich persons. Again we obtain

$$R^{HC}(\mathbf{x}) = R^{HC}(\mathbf{y}) = 0.400\,,$$

but different results for the intensity measures:

$$R^{Cha}_{\beta=1}(\mathbf{x}) = 0.218 \quad \text{and} \quad R^{Cha}_{\beta=1}(\mathbf{y}) = 0.398,$$
$$R^{FGT,T2}_{\alpha=2}(\mathbf{x}) = 19,916,088 \quad \text{and} \quad R^{FGT,T2}_{\alpha=2}(\mathbf{y}) = 16,360,039.$$

Technical reasons:

- possibility to standardize the index (unit interval)
- use of survey data

Normative judgements:

- "equiprobability model for moral value judgments" (Harsanyi, 1977): a concave value function with diminishing marginal utility
- "polarization view", i.e. richness is increasing when the homogeneity of the top of the distribution increases
- people are rather envious of a rich dentist living next door but admire superstars gaining several millions
- progressive tax system where the (marginal) tax payment is a concave function of taxable income.

- n individuals, $d \geq 2$ dimensions and matrix $\mathbf{Y} = [\ y_{ij}\ ]_{n \times d}$
- for each dimension j some cutoff value γ_j

$$\theta_{ij}(y_{ij};\gamma) = \begin{cases} 1 & \text{if } y_{ij} > \gamma_j, \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

- $0 - 1$ matrix of dimension-specific affluence:

$$\boldsymbol{\Theta}^{\mathbf{0}} = [\ \theta_{ij}\ ]_{n \times d} \tag{3}$$

  - vector of affluence counts $\mathbf{c} = (c_1, \ldots, c_n)'$ with $c_i = \sum_j \theta_{ij}$

- matrix $\mathbf{\Theta^0}$ only provides binary information
- instead: evaluate *intensity* of affluence (Peichl et al. 2010):
  - **convex case**:

$$\mathbf{\Theta}^{\alpha} = \left[\left(\frac{y_{ij} - \gamma_j}{\gamma_j} \right)_+^{\alpha} \right]_{n \times d} \quad \text{for } \alpha \geq 1 \tag{4}$$

  - **concave case**:

$$\mathbf{\Theta}^{\beta} = \left[\left(1 - \left(\frac{\gamma_j}{y_{ij}} \right)^{\beta} \right)_+ \right]_{n \times d} \quad \text{for } \beta > 0 \tag{5}$$

- for *larger* (*smaller*) values of α (β) more weight on the “very” rich

Dual cutoff method

- **so far:** affluence w.r.t. single dimensions separately (*1st cutoff*)
- **now:** individual (multidimensionally) affluent if affluence counts at least at certain threshold (*2nd cutoff*)
  - see Alkire/Foster (2011)

- identification for integer $k \in \{1, \ldots, d\}$:

$$\phi_i^k(y_i, \gamma) = \begin{cases} 1 & \text{if } c_i \geq k, \\ 0 & \text{if } c_i < k \end{cases} \tag{6}$$

  - number of the affluent: $s = \mid \Phi^k \mid$
- replace affluence counts ($\mathbf{c}$) with zero when $\phi_i^k = 0$ (*focus axiom*):

$$c_i^k = \begin{cases} c_i & \text{if } c_i \geq k, \\ 0 & \text{if } c_i < k \end{cases} \tag{7}$$

  - vector of affluence counts $\mathbf{c^k} = (c_1^k, \ldots, c_n^k)'$ with $c_i^k = c_i \cdot \phi_i^k$

- **dimension adjusted "headcount ratio":**

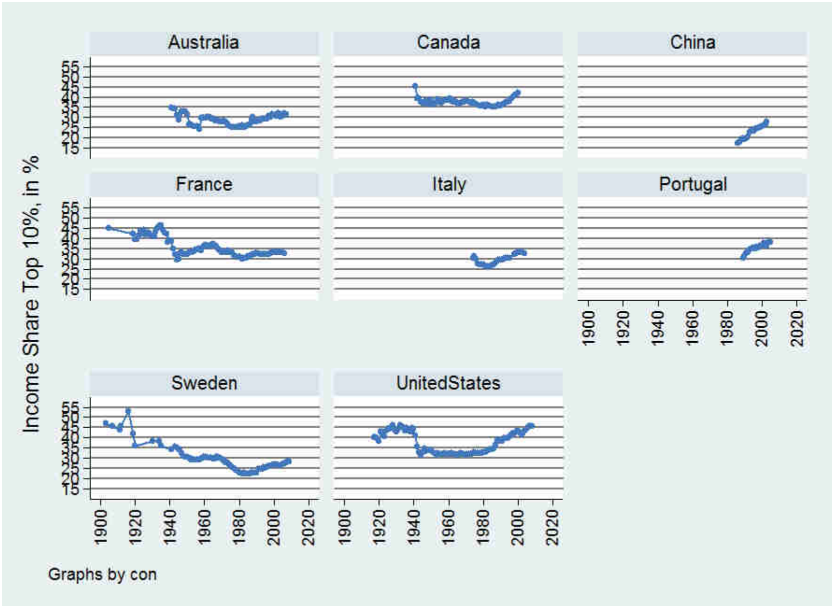
$$R^M_{HR} = \frac{|\ \mathbf{c^k}\ |}{n \cdot d} \tag{8}$$

  - satisfies *dimensional monotonicity*, but not *monotonicity*

- **dimension adjusted multidimensional richness measures:**

$$R^M_c = R^M_{HR} \cdot \frac{|\ \mathbf{\Theta^c(k)}\ |}{|\ \mathbf{c^k}\ |} = \frac{|\ \mathbf{\Theta^c(k)}\ |}{n \cdot d} \tag{9}$$

  - for $c = \alpha$ (convex case) and for $c = \beta$ (concave case)
  - measures satisfy *monotonicity*


Australia
Canada
China
France
Italy
Portugal
Sweden
UnitedStates
Income Share Top 10%, in %
55
50
45
40
35
30
25
20
15
1900
1920
1940
1960
1980
2000
2020
Graphs by con

correlations US

	Concave1	Concave2	Convex1	Convex2	Absolute	T10	T01
Concave1	1.000						
Concave2	0.999	1.000					
Convex1	0.992	0.986	1.000				
Convex2	0.849	0.828	0.906	1.000			
Absolute	0.212	0.210	0.203	0.124	1.000		
T10	0.830	0.824	0.829	0.697	0.322	1.000	
T01	0.955	0.948	0.960	0.844	0.274	0.952	1.000