Integrating mortality into poverty measurement through the Poverty-Adjusted Life Expectancy

Canazei Winter school 2025

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Namur University and World Bank

January, 2025

Motivation

Globally, premature mortality is one of the main source of well-being losses.

	Extreme poverty PY (millions of person-years)	Premature mortality YLL (wrt 50 years) (millions of person-years)
Developing world (2015)	705	402

Motivation

The case for integrating mortality into poverty measurement.

- Mortality reduces lifespan, which has high *intrinsic* value.
- Mortality has *instrumental* impact on poverty

Age in year t / Birth year	0 / t	1 / $t-1$	2 / $t-2$	3 / $t-3$
Poor dynasty A	P	D	D	D
Non-poor dynasty A	NP	NP	NP	NP
Poor dynasty B	P	P	P	D
Non-poor dynasty B	NP	NP	NP	NP

$H(A) = \frac{1}{5} < \frac{3}{7} = H(B)$, mortality paradox (Kanbur & Mukherjee, 2007)

$\Rightarrow$ Poverty measures that ignore mortality may yield counterintuitive comparisons

Motivation

Empirical relevance: a society's mortality is not perfectly correlated with its income

- Wellbeing comparisons substantially affected when accounting for mortality
  - Becker et al (2005), Murphy and Topel (2006), Jones and Klenow (2016)
- Some policies may imply a trade-off between poverty and mortality
  - Public spendings in health VS social protection
- The covid pandemic *reduced* poverty estimates in some countries.

Motivation

No poverty measures accounts for mortality in a way that both

(i) always attributes intrinsic value to longevity

(ii) avoids the mortality paradox

This paper proposes the poverty-adjusted life expectancy (PALE) index:

- PALE satisfies both (i) and (ii)
- PALE is a simplified version of well-being à la Harsanyi
- under some conditions, PALE comparisons are independent on the value selected for its normative parameter
- we quantify the impact of integrating mortality with extreme poverty

Definition of PALE

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\begin{aligned}
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where

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where

- LE is life expectancy at birth in t (quantity of life)
- H is the poverty head-count ratio in t (quality of life)
- $\theta \in [0,1]$ is normative parameter
  - ◇ if $\theta = 0$, then $PALE = LE$, i.e., one PY is neglibible wrt one YLL.
  - ◇ if $\theta = 1$, then $PALE = PFLE$, i.e., one PY is "as bad as" one YLL.
  - ◇ if $\theta > 1$, then "being poor" is assumed worse than "being dead".

PALE is an approximation of Harsanyi's welfare

Rational preferences over consumption streams (Koopmans 1960)

$$U = \sum_{a=0}^{d} \beta^a u(c_a)$$

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- $\beta = 1$: attribute same weight to individuals of all ages
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- $\mathbb{E}[u(c_a)] = \pi(a) u_P + (1 - \pi(a)) u_{NP}$
  - ◇ **Assumption**: $\pi(a)$ is independent on age and mortality, i.e. $\pi(a) = H$

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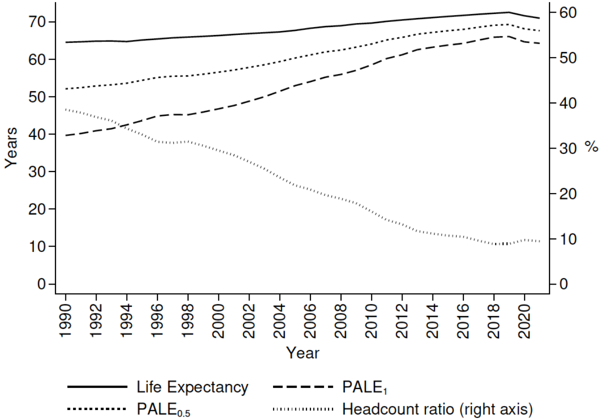
As $LE = \sum_{a=0}^{a^*} S(a)$, we get

$$\frac{EU}{u_{NP}} = LE\left(1 - \underbrace{\frac{u_{NP} - u_P}{u_{NP} - u_D}}_{\theta} H\right).$$

PALE global trend

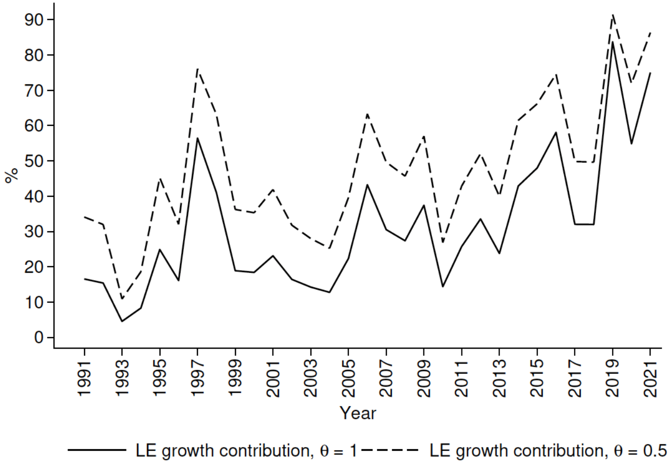
Figure 4: Evolution of $PALE_\theta$ and Life Expectancy, 1990-2021



Reading: in 1990, Poverty-Adjusted Life Expectancy was about 40 years according to $PALE_1$ and 52 years according to $PALE_{0.5}$.

Decomposition of PALE's growth

Figure 5: Share of the growth of LE in the growth of $PALE_1$, 1990-2021



Reading: in 1991, the growth of life expectancy contributed to 17% of the growth of $PALE_1$ and to 34% to that of $PALE_{0.5}$.

PALE comparisons sometimes valid for all $\theta \in [0, 1]$

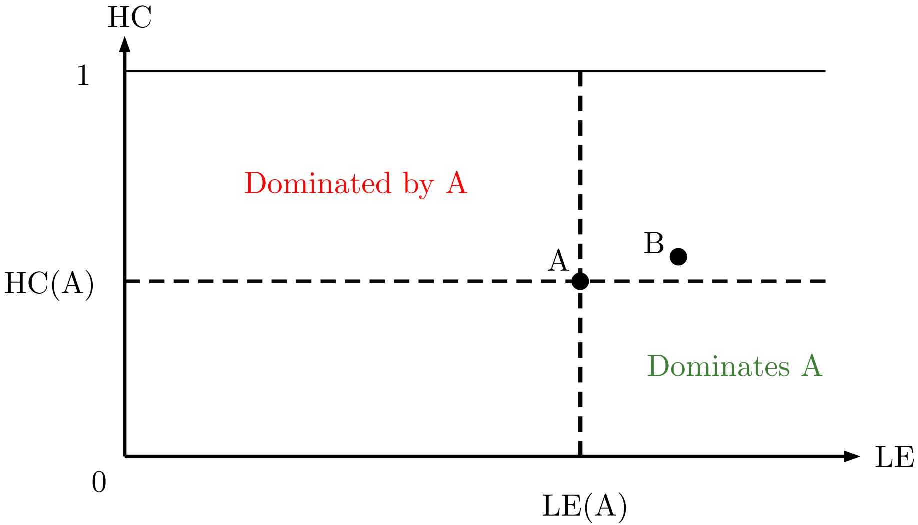


Figure: How does EU of A and B compare?

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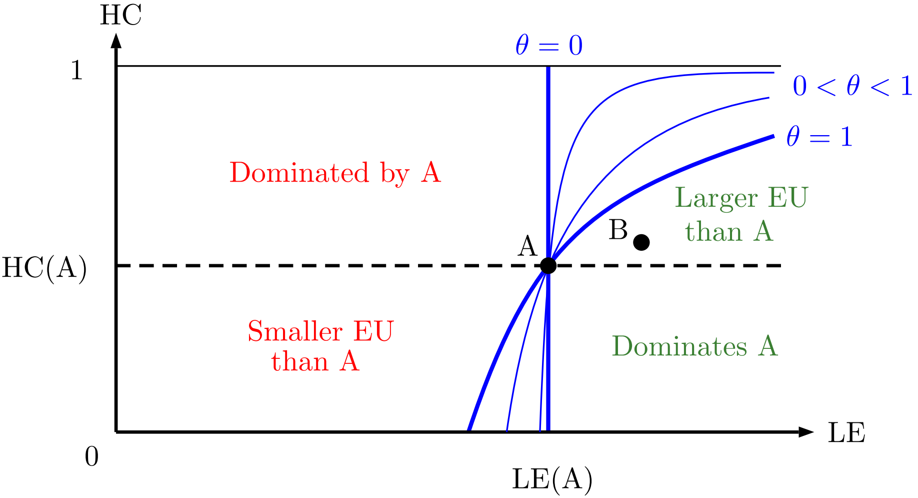


Figure: B has larger EU than A for all $\theta \in [0, 1]$.

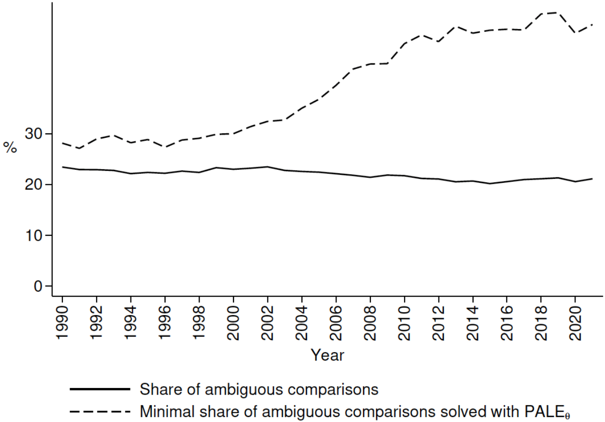
An example of comparisons valid for all $\theta \in [0, 1]$

Table: Pakistan and Bangladesh in 2021.

	Headcount ratio	Life Expectancy	Poverty Expectancy ($LE * H$)	Poverty Free Life Expectancy $LE * (1 - H) = PALE_1$
Pakistan	4.2%	64.0	2.7	61.3
Bangladesh	6.0%	71.4	4.3	67.2

Robust cross-country PALE comparisons

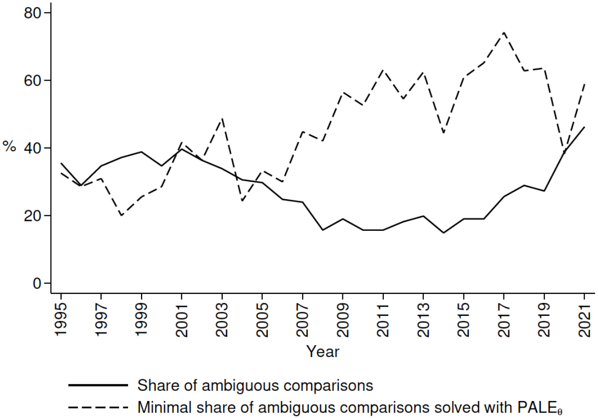
Figure 6: Evolution of the resolution of ambiguous inter-country comparisons, 1990-2021



Reading: in 1990, countries had on average 23% of ambiguous comparisons, out of which at least 28% were unambiguously ranked by $PALE_\theta$.

Robust within-country PALE trends

Figure 7: Evolution of the resolution of ambiguous countries' trajectories, 1990-2021



Reading: in the 1995, 36% of countries' trajectories was ambiguous. Among these, 33% can be assessed with $PALE_\theta$.

Conclusion

We suggest integrating mortality into poverty measures using the PALE index

- PALE has decent theoretical foundations,
- PALE $=$ equivalent number of years of life out of poverty.
- PALE can be computed with readily-available data.
- Even when H and LE are in conflict, PALE comparisons sometimes robust for all $\theta \in [0, 1]$,

Thank you for your attention!

XXXX

Table: Comparison of stationary societies A and B.

Age in year t Birth year	0 t	1 $t-1$	2 $t-2$	3 $t-3$
Poor dynasty A	P	D	D	D
Non-poor dynasty A	NP	NP	NP	NP
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Motivation

Mainstream aggr. welfare indicators are not suited to **inform public debate**

- Composite indices

$$W_{comp} = w(1 - HC) + (1 - w)\frac{LE - LE^{min}}{LE^{max} - LE^{min}}.$$

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  - ◇ black box: its value cannot be interpreted,
  - ◇ comparison depends on parameters values: $W_{comp}(A) > W_{comp}(B)$ if $w \to 1$, but $W_{comp}(A) < W_{comp}(B)$ if $w \to 0$.

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Robustness to Assumption

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If $HC(A) < HC(B)$, $LE(A) < LE(B)$ *and* $PALE_1(A) < PALE_1(B)$, *then*

$$PALE_\theta(A) < PALE_\theta(B) \text{ for all } \theta \in [0,1],$$

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Answer: yes, because $PALE_\theta$ is EU of newborn in stationary population.

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Notation

- life $l = (l_0, l_1, \ldots, l_d) \in L$, e.g. $l = (NP, NP, P, \ldots, P)$,
- n_t individuals born in year t,
- distribution of lives in year t is $\Gamma_t : L \to [0,1]$.

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Stationary population in t: for all $t' \in \{t - a^*, \dots, t\}$

- constant distribution $\Gamma_{t'} = \Gamma_t$,
- constant size $n_{t'} = n_t$,

$\Rightarrow \frac{EU}{u_{NP}} = PALE_\theta$ in stationary population, even without Assumption.

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  - ◇ *LE* is valid aggregation of mortality in t

By analogy:

- Even if population is not stationary, then
  - ◇ *PALE* is welfare expectation of a newborn in t who assumes that society is stationary in t.
  - ◇ *PALE* is valid aggregation of welfare costs in t.

Application to multidimensional poverty

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Mainstream multidimensional poverty indices suffer from same limitations

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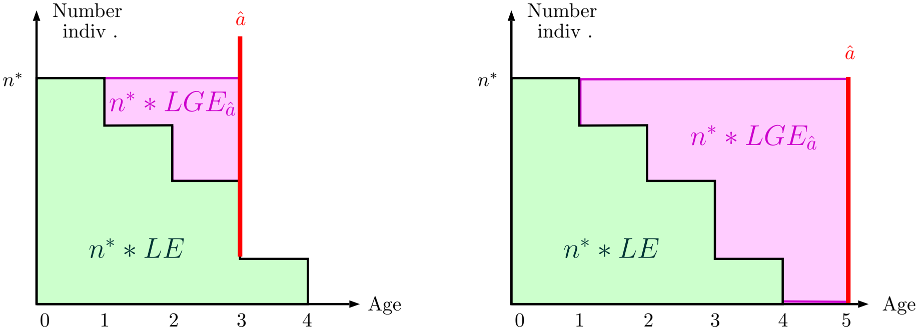

$LGE_{\hat{a}}$ measures the number of years that a newborn expects to lose prematurely (based on mortality rates observed in t)

Application to multidimensional poverty

$$ED_\theta = \underbrace{\frac{LE * HC}{LE + LGE_{\hat{a}}}}_{quality\ \ deprivation} + \frac{1}{\theta} \underbrace{\frac{LGE_{\hat{a}}}{LE + LGE_{\hat{a}}}}_{quantity\ \ deprivation}, \tag{1}$$

where $\theta \in [0, 1]$ and

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- $LE * HC$ is number of years that a newborn expects to spend in poverty.

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If $\hat{a} \geq a^*$, then

$$PALE_\theta(A) \geq PALE_\theta(B) \Leftrightarrow ED_\theta(A) \leq ED_\theta(B).$$

[redacted]

Empirics

Current results:

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- Fraction of pairs for which PALE and ED agree, as a function of $\hat{a}$.
- Fraction of pairs for which ED and closest alternative index (GD) agree, as a function of $\hat{a}$.