On the welfare economics of intergenerational mobility

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Acknowledgements

This lecture builds originally on joint and is inspired work with Stephen P Jenkins

Markus Jäntti and Stephen Jenkins (2015). "Income mobility". In: Handbook of Income Distribution. Ed. by Anthony B Atkinson and François Bourguignon. Vol. 2. Elsevier. Chap. 10, pp. 807–935

and on an unpublished lecture by Tony Atkinson,

Anthony B Atkinson (May 2008). "Mobility, Meritocracy and Markets". Unpublished lecture at Russell Sage Foundation, New York

as well as loose ideas inspired by

Markus Jäntti, Ravi Kanbur, et al. (2014). "Poverty and Welfare Measurement on the Basis of Prospect Theory". In: Review of Income and Wealth 60.1, pp. 182–205. ISSN: 1475-4991. DOI: 10.1111/roiw.12095. URL: http://dx.doi.org/10.1111/roiw.12095

and ongoing work with Jesper Roine

Outline

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Motivation

- much of mobility measurement concerned with the issue of whether one (at least bivariate) distribution has *more/less* mobility than another
- we know and understand much less about whether or not more or less mobility is *socially desirable*; social desirability depending (in economics) on the welfare judgment of mobility (often in comparison with another population)
- my main interest is on *inter-generational* mobility and how welfare judgements about it differ from the *intra* case

Notation

- restrict interest to two "periods", 1 and 2 – 1 being parent and 2 being offspring in the case of intergenerational income mobility / persistence
- let $Y_j, j = 1, 2$ be the variable of interest, "income", in the two periods ($Y_j \geq 0$)
- we initially work with the bivariate distribution $F(Y_1, Y_2)$ (or density f)
- we are concerned with the problem of *comparing* mobility across two "populations", A and B with F_A, F_B
- let $M(F)$ be a (statistical) measure of mobility; our concern is *not* to assess if

$$M(F_A) \precsim M(F_B) \quad \text{or} \quad M(F_A) \succsim M(F_B)$$

- instead, we are concerned with assessing if, given a welfare function(al) W,

$$W(F_A) \precsim W(F_B) \quad \text{or} \quad W(F_A) \succsim W(F_B)$$

Remarks

- the “periods” could at this stage be different time periods (*intra*-generational mobility), or they could be two generations within the same family or dynasty (*inter*-generational mobility)
- “income” could be some income variable, or consumption, or wealth) much of the literature is concerned with permanent income); that choice will be taken as given in what follows
- focussing on only two periods may seem too restrictive, but helps fix ideas – and is highly relevant in the intergenerational case
- the main point of my talk is that while the *measurement* of mobility is *in part* similar in the *intra*- and *inter*generational cases, the*welfare economics*, the social desirability of mobility, is different in the two cases

Outline

Mobility concepts

…the mobility literature does not provide a unified discourse of analysis. This might be because the very notion of income mobility is not well-defined; different studies concentrate on different aspects of this multi-faceted concept. At any rate, it seems safe to say that a considerable degree of confusion confronts a newcomer to the field (Fields and Ok, 1999, p. 557).

Mobility concepts

- focus on the distribution of income in two "periods" (e.g., two years for intra-, generations for inter-generational mobility)
- income distribution of $Y = (Y_1, Y_2)'$ with joint density $f(y_1, y_2)$; f_1 and f_2 being the period-specific marginal distributions
- mobility can be thought of as transformation linking the marginal distribution f_1 with marginal distribution f_2
- sometimes, study of a single (longitudinal) population can be informative...
- but as a rule, mobility is about *comparing* two populations A and B (two countries, two different periods, etc)

Mobility concepts

- concepts:
  - positional change
  - individual income growth
  - mobility as inequality reduction
  - income risk
- social desirability of mobility
  - may differ across within/between
  - may differ across concepts
  - relationship to equality of opportunity

Mobility as positional change

- most easily thought of as defined in terms of not the distribution of income but its inverse (summarize positions not by incomes but by the rank associated with an income)
- abstract from the shape of (and changes in) the marginal distribution (“exchange” as opposed to “structural” mobility)
- for every positional change in one direction there must be a corresponding change in the opposite direction
- “no mobility” occurs when no rank changes take place ($p_{ij} \equiv 1 \forall i = j, p_{ij} \equiv 0 \forall i \neq j$)
- “full” mobility:
  - origin independence ($p_{ij} = p_{kl} = 1/n$; each row of the transition matrix has identical entries)
  - rank reversal ($p_{ij} > 0 \quad i = 1, \ldots, n, j = n, \ldots, 1$; all entries in transition matrix on the anti-diagonal)

Mobility as individual income growth

- aggregation of individual income changes (gains or losses)
- no distinction between exchange and structural mobility:
  - no standardization of the distributions
  - all can be upwardly or downwardly mobile
- immobility: $y_{i1} = y_{2i} \forall i$
- mobility: is greater if $d_i = y_{2i} - y_{1i}$ greater, all else equal
- measures: directional growth (gains vs. losses) as opposed to non-directional growth

Mobility as inequality reduction

- comparison of inequality of marginal with "long-term" distribution; defined in terms of $\frac{1}{2}(y_{1i} + y_{2i})$
- no mobility: income of each person in every period is equal to their longer-term income
- maximum mobility: no inequality in longer-term incomes despite inequality in per-period incomes
- directional mobility not relevant
- related to positional change

Mobility as income risk

- period-specific income is sum of a ‘permanent’ component (the longer-term average) and a ‘transitory’ component (the period-specific deviation from the average)
- transitory components represent unexpected idiosyncratic shocks to income (long-term income interpreted as “permanent” income)
- the greater their dispersion across individuals each period, the greater is income risk for this population
- inequality reduction from longitudinal averaging now re-interpreted as a measure of income risk (and has different normative implications)

Is income mobility socially desirable?

- relation to (in)equality of opportunity (but that relationship is complex)
- differs in the intra- and intergenerational cases
- positional change: mobility [often] good in inter- but not necessarily in intra-case
- income growth: gains good, losses bad
- inequality reduction: good (but for instrumental, not intrinsic, reasons)
- income risk: mobility bad

Outline

Income mobility and social welfare

- important contributions to the social welfare foundations of mobility measurement include Atkinson (1981), Atkinson and Bourguignon (1982), Markandya (1984), and Gottschalk and Spolaore (2002)
- social welfare, W, is the expected value (average) of the utility-of-income functions of individuals.
- in two-period case, the utility-of-income function is $U(Y_1, Y_2)$, and weighted by the joint probability density $f(y_1, y_2)$:

$$W = \int_0^{a_2} \int_0^{a_1} U(y_1, y_2) f(y_1, y_2) dy_1 dy_2 \tag{1}$$

  where $U(Y_1, Y_2)$ is differentiable and a_1 and a_2 are the maximum incomes in periods 1 and 2.
- increases in income in either period assumed desirable (so positive income growth raises utility): $U_1 \geq 0$ and $U_2 \geq 0$.

Income mobility and social welfare

- mostly focus is on case where marginal distributions identical (so close to positional mobility analysis)
- if U additively separable (so $U_{12} = 0$), mobility is irrelevant and only marginal distributions matter
- if $U(Y_1, Y_2)$ is a concave transformation of the sum of the per-period utilities, then $U_{12} < 0$ which is the key in determining the social desirability (welfare-improving characteristic) of mobility

Income mobility and social welfare

- we make no difference between the intra- and intergenerational cases yet
- I would argue that in many cases, what makes sense for the welfare interpretation of intragenerational mobility makes less so for the intergenerational cases
- it seems fundamentally different to consider the wellbeing of a single individual under two alternative sets of income streams than to compare a parent-child pair under two different sets of parent-child incomes
- the "traditional" welfare assessment of intergenerational income is based on taking an *ex ante*-view, i.e., evaluating "dynastic utility" looking at it from the *parent*'s perspective (with or without a "altruism" parameter)
- it is not clear this is an evaluation we as a society prioritize
- I would argue that we are more concerned with the well-being each generation (often with a focus on the current population), but the background of each generation matters for out evaluation

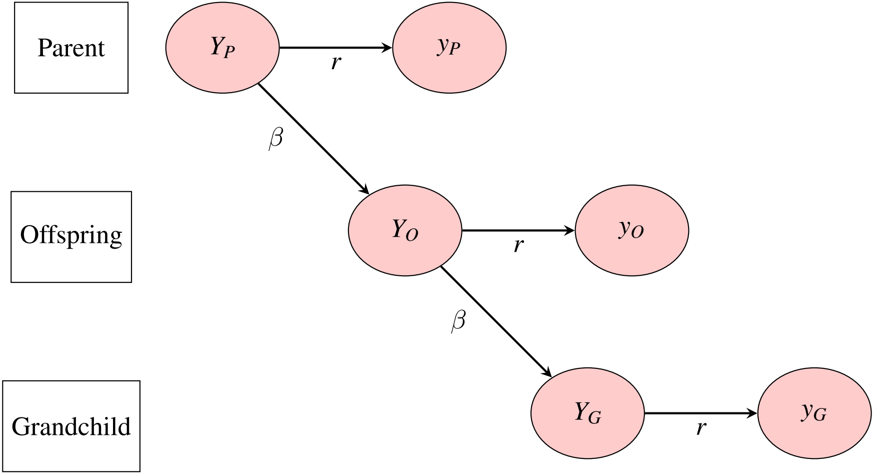
Next

- briefly show an attempt to integrate inter- and intragenerational mobility, extended to also include family income and assortative mating
- discuss an alternative approach
- if time permits, discuss in greater detail “traditional” utilitarian approaches to welfare comparisons involving mobility
  - transition matrices and social welfare
  - welfare dominance
  - origin independence

Intra- or inter-generational mobility

- hitherto, analysis thought to be applicable to both intra- and inter-generational mobility
- the "plasticity" of the framework hides the fact that in intergenerational analysis, individuals experience (welfare-reducing) income fluctuations within generations
- next, we'll look at a simple way of integrating intra- and inter-generational mobility based on Atkinson (2008)

Intra- and inter-generational mobility



Inter- and intragenerational mobility

- focus for now on the 2-generation case, but allow each generation to have annual income that fluctuates around the long-run average such that

$$Y_j = \prod_{t_1}^{T} \tilde{y}_{jt}^{1/T} \text{ and } \ln Y_j = \frac{1}{T} \sum_{t=1}^{T} y_{jt} \quad j = F, S \tag{2}$$

- a parent's utility (or the ex ante evaluation) is

$$U(Y_P, Y_O) = [\ln Y_P + \delta \ln Y_O]/\Delta,\ \Delta = 1 + \delta \tag{3}$$

- we'll measure social welfare by $-\text{Var}[]$, so we need

$$\begin{aligned} \text{Var}[U(Y_P, Y_O)] = & \text{Var}[\ln Y_P] + \delta^2 \text{Var}[\ln Y_O] + \\ & \delta 2 \beta \text{Var}[\ln Y_P]^{1/2} \text{Var}[\ln Y_O]^{1/2} \end{aligned} \tag{4}$$

(β is the intergenerational income *correlation*; δ is the discount rate)

Inter- and intragenerational mobility

- assuming a within-person correlation r_j and stationary transitory error variance $\sigma^2_{v_j}$, the welfare function is

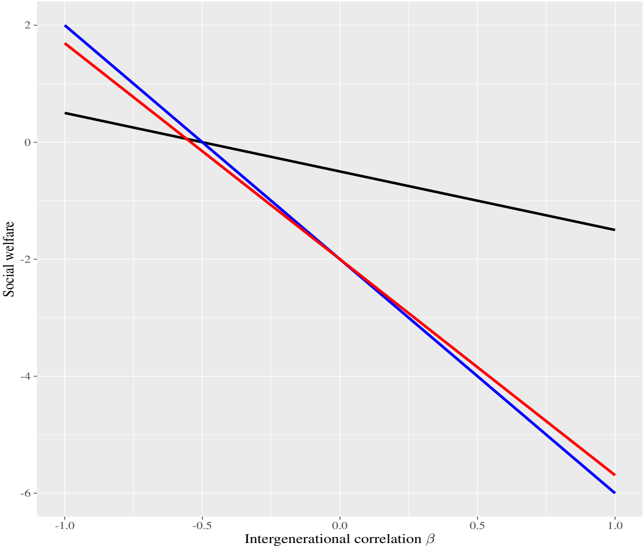
$$
\begin{aligned}
W = -\mathrm{Var}[U(Y_P, Y_O)] = -\Bigg\{ & \sigma_P^2\left(\frac{1}{T} + \frac{T-1}{T} r_P\right) + \frac{\sigma^2_{v_P}}{T} + \\
& \delta^2\left[\sigma_O^2\left(\frac{1}{T} + \frac{T-1}{T} r_O\right) + \frac{\sigma^2_{v_O}}{T}\right] + \\
& \delta 2\beta\sqrt{\sigma_P^2\left(\frac{1}{T} + \frac{T-1}{T} r_P\right) + \frac{\sigma^2_{v_P}}{T}} \times \\
& \sqrt{\sigma_O^2\left(\frac{1}{T} + \frac{T-1}{T} r_O\right) + \frac{\sigma^2_{v_O}}{T}}\Bigg\} / \Delta^2
\end{aligned}
\quad (5)
$$

Inter- and intragenerational mobility

- assume T large and impose stationarity ($\sigma_P = \sigma_O = \sigma; r_P = r_O = r$):

$$W = -\text{Var}[U(Y_P, Y_O)] = -\sigma^2[r(1+\delta^2) + \delta 2\beta]/\Delta^2 \tag{6}$$

Welfare and intergenerational correlation (2-gen)

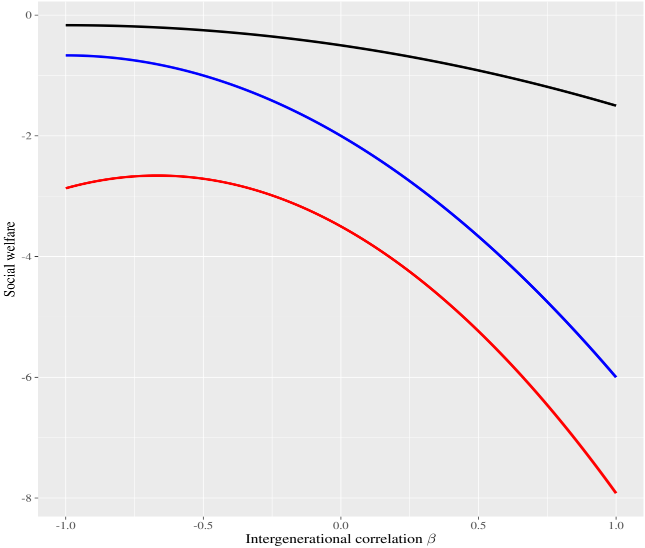


(black=$\sigma^2 = 1; \delta = 1$; blue=$\sigma^2 = 2; \delta = 1$; red=$\sigma^2 = 2; \delta = 1.5$)

Welfare and intergenerational correlation (3-gen)

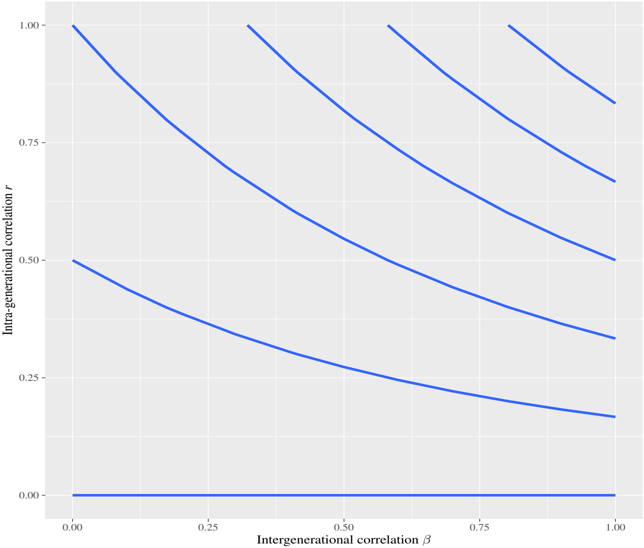
- taking a 3-generation perspective changes this only a little
- welfare is now non-linear (in fact, quadratic) in the intergenerational correlation so it is more sensitive to generational variance and discount factor

Welfare and intergenerational correlation (3-gen)



(black=$\sigma^2 = 1; \delta = 1$; blue=$\sigma^2 = 2; \delta = 1$; red=$\sigma^2 = 2; \delta = 1.5$)

Intra- and intergenerational correlation – trade-off



Family income

- within a generation, long-run family income is

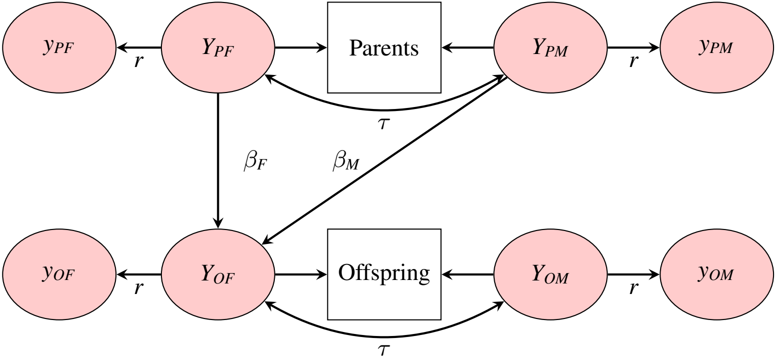
$$Y = Y_M + Y_F \tag{7}$$

- taking logs and imposing $Y_F = \phi Y_M$ we have

$$\ln Y = \ln Y_M + \ln(1+\phi) \approx y_M + y_F := y \tag{8}$$

- for both men and women, there is a measurement model linking annual to ling-run income with intra-person correlation r and transitory shocks (both r and the variance of shocks assumed the same across genders and the shocks orthogonal across time and spouses)
- male and female incomes are correlated with parameter τ

Intra- and intergenerational mobility with assortative mating



Parameters

- β_j: intergenerational correlation (j=*M*ale, *F*emale); simplify to just one parental β
- r: intragenerational correlation
- τ: spousal income correlation (assortative mating)

Welfare with intra- and inter-generational mobility and assortative mating

- assume again large T, variance of transitory shocks to male and female incomes the same and set to unity and imposing stationarity across generations
- the parameters of interest are r, β and τ
- as before, for welfare analysis we must focus on a lineage ("dynasty") since the "triad" of offspring - parent -parents in-law is hard to argue is a unit of welfare analysis (more on this later)
- measuring welfare again by $-\text{Var}[]$, we have (setting σ^2 to unity)

$$W = -\text{Var}[U(Y_P, Y_O)] = -r(1+\tau)(1+\delta^2+2\delta\beta)/\Delta \qquad (9)$$

- thus, social welfare declines in all three parameters – intra- and intergenerational persistence and assortative mating – and all three parameters of interest can be traded off holding welfare constant

Alternative approaches

- equality of opportunity (see e.g. Roemer and Trannoy, 2016)
  - well-structured and strongly-grounded theory of distributional justice (but I will not say more about it here)
- approach inspired by prospect theory (Tversky and Kahneman, 1992),
  - parental income as a reference point?
  - apply ideas developed in Jäntti, Kanbur, et al. (2014) to intergenerational mobility (a sketch)

Prospect theory-based welfare of intergenerational mobility (sketch)

- prospect theory-based inequality and poverty measurement evaluates the well-being of current income relative to a reference point, e.g. $V(Y, \bar{Y}) = V(Y - \bar{Y}) = V(C)$, such that
  - $V' > 0$
  - $V'(-C) \geq V'(C)$
  - $V'' > 0, C < 0$ and $V'' < 0, C > 0$

  with $\bar{Y}$ typically own income in a previous period (see Günther and Maier, 2014; Jäntti, Kanbur, et al., 2014)
- what if we took parental income as the reference point?
- this has a number of advantages:
  - you no longer need to take an *ex ante* view – in practice, looking at this from the point-of-view of a dynastic patriarch
  - "close" to standard inequality measurement theory can be used to summarise and compare populations
  - the interest in "absolute" intergenerational mobility is easily incorporated

Prospect theory-based welfare of intergenerational mobility (sketch)

- use as the "evaluation function" of the child and parent (the "reference point") incomes
$$V(Y_1, Y_2) = V[C(Y_1, Y_2)], \tag{10}$$
where $\partial C(\cdot,\cdot)/\partial Y_2 > 0$ and $\partial C(\cdot,\cdot)/\partial Y_1 < 0$
- to follow applications of prospect theory to inequality measurement, we would have
$$C(Y_1, Y_2) = Y_2 - Y_1 \tag{11}$$
- this formulation assigns little / no value on a person's own income; Jäntti, Kanbur, et al. (2014) apply the hybrid form of reference-dependent utility in Koszegi and Rabin (2007) which would translate in this context to
$$U(Y_1, Y_2) = H(Y_2) + V(Y_2 - Y_1) \tag{12}$$

Prospect theory-based welfare of intergenerational mobility (sketch)

- equivalent income – which could be used to assessing "Atkinsonian" inequality – in the offspring generation can be solved from

$$U(Y_2^*, Y_1) = H(Y_2) + V(Y_2 - Y_1) \tag{13}$$

- the functional from suggested by Koszegi and Rabin (2007) and applied by Jäntti, Kanbur, et al. (2014) to intra-generational mobility is

$$\begin{aligned} U(Y_1, Y_2) =& \frac{Y_2^{1-\eta}}{1-\eta} + \left[\frac{Y_2^{1-\eta}}{1-\eta} - \frac{Y_1^{1-\eta}}{1-\eta}\right]^{\beta}, \; Y_2 > Y_1 \\ U(Y_1, Y_2) =& \frac{Y_2^{1-\eta}}{1-\eta} - a\left[\frac{Y_1^{1-\eta}}{1-\eta} - \frac{Y_2^{1-\eta}}{1-\eta}\right]^{\beta}, \; Y_2 \leq Y_1 \end{aligned} \tag{14}$$

where the paramaters $a > 1$ and $0 < \beta < 1$ are the relative weights of the "loss" of "gain" part of the evaluation function

Prospect theory-based welfare of intergenerational mobility (sketch)

- social evaluation in a finite population is

$$\text{SVF}_A = \sum_i U(Y_{1i}, Y_{2i}) \tag{15}$$

- to compare two populations (consisting for simplicity here of the same individuals) would entail calculating

$$\begin{aligned} \Delta =& \text{SVF}_A - \text{SVF}_B \\ =& \sum_i [H(Y_{A,2i}) + V(Y_{A,2i} - Y_{A,1i}) - H(Y_{B,2}) - V(Y_{B,2} - Y_{B,1})] \end{aligned} \tag{16}$$

- this immediately suggests a decomposition of the change in social welfare into a part that depends on differences in own income alone ($\sum[H_A() - H_B()]$) and difference in income across generations ($\sum[V_A() - V_B()]$)

The utilitarian approach in greater detail

- ▸ transition matrices and social welfare
- ▸ welfare dominance
- ▸ origin independence

Transition matrices and social welfare

Markandya (1984)

- focus first on a discrete distribution of income with identical marginal distributions in both periods, so
  - $f_{1i} \equiv f_{2i} \quad i = 1, \ldots, n$
  - $\boldsymbol{f}_1' \boldsymbol{P} = \boldsymbol{f}_2$
- consider the problem of choosing the transition matrix $\boldsymbol{P}$ that maximizes welfare, given the fixed marginal distribution and a social evaluation function U:

$$\max_{\boldsymbol{P}} W = \sum_i \sum_j U(Y_{1i}, Y_{2i}) p_{ij} f_{1i}$$

$$\text{subject to}$$

$$\sum_i f_{1i} p_{ij} = f_{2j} = f_{1j}, \quad j = 1, \ldots, n \tag{17}$$

$$\sum_j p_{ij} = 1, \quad i = 1, \ldots, n$$

Go back to setup Go back to utilitarian intro

Transition matrices and social welfare

Markandya (1984)

- the crucial "fact" for this problem is the sign of the cross partial derivative, U_{12}:
  - for *positive* ($U_{12} > 0$), welfare is maximized by perfect immobility, i.e. choosing the identity matrix $\boldsymbol{P} = \boldsymbol{I}$
  - for *negative* ($U_{12} < 0$), welfare is maximized by perfect rank reversals (all elements on the anti-diagonal; this solution may not be feasible but transformations that approach it increase welfare)
- note that "origin independence" plays no role here

Exchange and structural mobility

- it is useful to distinguish between changes in mobility that are driven by changes in the marginal distributions ("structural") and those that are driven by the mapping of f_1 to f_2 ("exchange")
- the welfare-based measurement approach allows such a decomposition of mobility
- consider a two different discrete distributions $(\boldsymbol{f}_1^A, \boldsymbol{P}^A, \boldsymbol{f}_2^A)$ and $(\boldsymbol{f}_1^B, \boldsymbol{P}^B, \boldsymbol{f}_2^B)$; the move from $\boldsymbol{P}^A$ to $\boldsymbol{P}^B$ induces both structural and exchange mobility
- one approach would be purely statistical or mathematical;
  - subject to a specific distance measure, find $\widetilde{\boldsymbol{P}}$ that is "closest" to $\boldsymbol{P}^A$ subject to $\widetilde{\boldsymbol{P}}$ being consistent with the marginal distributions
  - then $\boldsymbol{P}^A - \widetilde{\boldsymbol{P}}$ is a measure of exchange mobility; $\widetilde{\boldsymbol{P}} - \boldsymbol{P}^B$ a measure of structural mobility
- an alternative is to rely on the *social evaluation* U to decompose mobility

Exchange and structural mobility – welfare-based

- for each transition matrix $\boldsymbol{P}^A$ there is an equilibrium distribution $\tilde{\boldsymbol{f}}^A$ such that
$$\tilde{\boldsymbol{f}}^{A\prime}\boldsymbol{P}^A = \tilde{\boldsymbol{f}}^A. \tag{18}$$
- a matrix $\widetilde{\boldsymbol{P}}$ is "exchange equivalent" to $\boldsymbol{P}^A$ if
$$\tilde{\boldsymbol{f}}^{A\prime}\widetilde{Pb} = \tilde{\boldsymbol{f}}^A. \tag{19}$$
- the change in *welfare* associated with $(\tilde{\boldsymbol{f}}^A, \boldsymbol{P}^A)$ to that with $(\tilde{\boldsymbol{f}}^B, \boldsymbol{P}^B)$ then considers welfare change induced by $\boldsymbol{P}^A \to \widetilde{\boldsymbol{P}}$ as a measure of exchange mobility; $\widetilde{\boldsymbol{P}} \to \boldsymbol{P}^B$ a measure of structural mobility
- note that $\tilde{f}^k, k = A, B$ is a hypothetical steady-state distribution, not the actual

Exchange and structural mobility – an example

- to examine this more closely, consider $n = 2$ and focus on the case of identical marginal distributions in the two time periods:

$$\boldsymbol{P} = \begin{bmatrix} p_1 & 1 - p_1 \\ 1 - p_2 & p_2 \end{bmatrix} \qquad (20)$$

$$1 > p_i > 0, i = 1, 2; \quad \boldsymbol{f} = (f_1, f_2)' = (f_1, 1 - f_1)'$$

- the welfare function (expected/average utility) for this economy is

$$\begin{aligned} W = & U(Y_1, Y_2) p_1 f_1 + U(Y_1, Y_2)(1 - p_1) f_1 + \\ & U(Y_2, Y_1)(1 - p_2)(1 - f_1) + U(Y_2, Y_2) p_2 (1 - f_1) \end{aligned} \qquad (21)$$

Exchange and structural mobility – an example

- this can re-written as

$$W = [\{U(Y_2, Y_2) - U(Y_2, Y_1)\} - \{U(Y_1, Y_2) - U(Y_1, Y_1)\}]p_1 f_1 + C \quad (22)$$

  (C does not depend on p_1 or p_2)
- to maximize welfare wrt. p_1 we choose a low value when [] is negative (and high when it is positive); the sign of [] equals the sign of the cross-partial derivative (as $Y_1 < Y_2$)

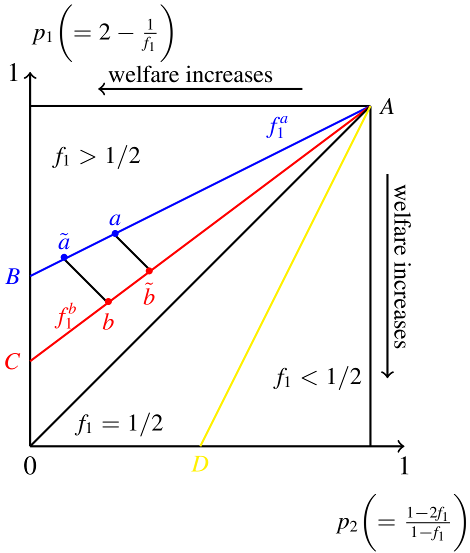
Exchange and structural mobility – an example

- the key here is

$$U(Y_2, Y_2) - U(Y_2, Y_1) \lesseqqgtr U(Y_1, Y_2) - U(Y_1, Y_1) \tag{23}$$

- for a negative cross-partial derivative ($U_{12} < 0$), W is negative
  - the decline in utility from going from high income in both periods to low income in the second is less than the increase in utility from going from low in both periods to high in the second
  - in which case we have a social preference for mobility
  - $p_1 = p_2 = 0$ has here been ruled out on feasibility grounds so complete rank reversal is not a solution

Exchange and structural mobility – graphical representation



- points a and b associated with $\boldsymbol{P}^a$ and $\boldsymbol{P}^b$
- move along f_1^a to $\tilde{a}$ closer to b is the change in mobility with no structural change
- move from $\tilde{a}$ to b preserves welfare but represents structural mobility $f_1^a \to f_2^b$

Exchange and structural mobility – decomposition

Decomposition I

$$\begin{aligned}\text{Total change in welfare} &= W^b - W^a \\ \text{Exchange mobility} &= W^{\tilde{a}} - W^a \\ \text{Structural mobility} &= W^b - W^{\tilde{a}}\end{aligned}$$

Decomposition II

$$\begin{aligned}\text{Total change in welfare} &= W^b - W^a \\ \text{Exchange mobility} &= W^b - W^{\tilde{b}} \\ \text{Structural mobility} &= W^{\tilde{b}} - W^a\end{aligned}$$

Remarks

- one might also take point A (perfect immobility) as the reference for for decomposing, but that would make no use of welfare information.

Welfare dominance in more general bivariate distributions

Atkinson and Bourguignon (1982)

- the problem is still to compare two distributions, f^A and f^B with

$$\Delta f = f^B - f^A \text{ and } \Delta F = F^B - F^A$$

- keeping to the two-period case, the difference in welfare is

$$\Delta W = \int_0^{a_2} \int_0^{a_1} U(y_1, y_2) \Delta f(y_1, y_2) dy_1 dy_2 \tag{24}$$

- we want to know under what conditions $\Delta W > 0$
- restrict interest to the case $U_{12} < 0$

Go back to setup | Go back to utilitarian intro

Welfare dominance in more general bivariate distributions

- equation 24 can be re-expressed as

$$\Delta W = \underbrace{U(a_1, a_2) \int_0^{a_2} \int_0^{a_1} \Delta f(y_1, y_2) dy_1 dy_2}_{=0} - \int_0^{a_1} U_1(y_1, a_2) \Delta F_1(y_1) dy_1 - \int_0^{a_2} U_2(a_1, y_2) \Delta F_2(y_2) dy_2 + \int_0^{a_2} \int_0^{a_1} U_{12}(y_1, y_2) \Delta F(y_1, y_2) dy_1 dy_2 \quad (25)$$

- for all U we are considering, a sufficient condition for $\Delta W > 0$ is that

$$\Delta F(y_1, y_2) \leq 0$$

- Atkinson and Bourguignon (1982) consider other classes of U and derive also higher-order dominance conditions

A closer look at U

- Atkinson and Bourguignon (1982) examine restricted class of utility functions with homothetic preferences
- consider the following evaluation function

$$U(Y_1, Y_2) = [Y_1^{1-\rho} + Y_2^{1-\rho}]^{(1-\epsilon)/(1-\rho)} \tag{26}$$

- the two parameters have the following interpretation (Gottschalk and Spolaore, 2002, p. 295):
  - $\epsilon > 0$ summarizes *aversion to inequality of multi-period utility*,
  - $\rho > 0$ summarizes the degree of *aversion to inter-temporal fluctuations in income*
- $U_{12} < 0$ corresponds to $\epsilon > \rho$, i.e. multi-period inequality aversion offsets aversion to inter-temporal fluctuations (and reversals are socially valued)
- when $\rho = 0$ and perfect substitution of income between periods, one is only interested in the reduction of multi-period inequality

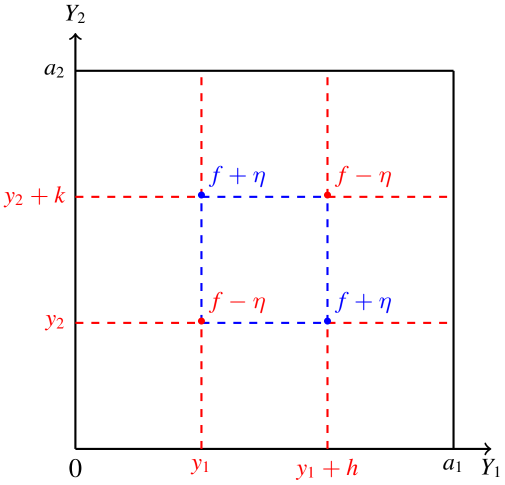
Mobility dominance

- an example that would generate a welfare improvement is a 'correlation-reducing transformation' which leaves the marginal distributions unchanged but reduces the correlation between Y_1 and Y_2 (for $\eta, h, k > 0$):

$$\left\{ \begin{array}{ccc} & y_1 & y_1 + h \\ y_2 & \text{density reduced by } \eta & \text{density increased by } \eta \\ y_2 + k & \text{density increased by } \eta & \text{density reduced by } \eta \end{array} \right\}$$

- mobility dominance powerful in theory but not used much in practice – results apply to simplified situations (identical margins, homothetic preferences, positional mobility)
- Dardanoni (1993) provides an alternative approach to dominance (stochastic dominance results for mobility processes summarised by transition matrices with the same steady-state income distribution)

Mobility dominance – graphical illustration



Mobility dominance – examples

- Go to US transition matrices
- Go to IG mobility dominance Germany, the UK, and USA compared

Welfare dominance with origin independence

Gottschalk and Spolaore (2002)

- origin independence is an important benchmark in non-welfare-based mobility measurement
- origin independence has *no* role in the welfare-based approach
- Gottschalk and Spolaore (2002) introduce origin independence by modifying the evaluation function U
- in particular, let the certainty equivalent of second-period income be

$$\widetilde{Y}_2 = (\mathrm{E}_1[Y_2^{1-\gamma}])^{1/(1-\gamma)}. \tag{27}$$

- the welfare function, using the expectations operator, is then

$$\hat{W} = \{\mathrm{E}_0[Y_1^{1-\rho} + (\mathrm{E}_1[Y_2^{1-\gamma}])^{1/(1-\gamma)})^{1-\rho}]^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)} \tag{28}$$

Go back to setup | Go back to utilitarian intro

Welfare dominance with origin independence

- Gottschalk and Spolaore (2002) prove that time independence is valued if and only if

$$\epsilon \geq \gamma \text{ and } \rho \geq \gamma$$

  i.e., origin independence only matters in the *ex ante* sense that individuals, looking forward, value a sure thing relative to a lottery and that valuation is high enough to dominate aversion to both multiperiod utility (ϵ) and intertemporal variation in income (ρ)
- moreover, in the 2×2 example, setting $p_1 = p_2 = p$, they show that the welfare-maximizing p depends on the relationship between ϵ and ρ

$$p \lesseqqgtr 1/2 \text{ if } \rho \lesseqqgtr \epsilon$$

Measurement of welfare loss

Welfare measures and extended Atkinson indices

Welfare	Index
No mobility preference:	
$W_s = \{E_0[Y_1^{1-\rho} + Y_{12}^{1-\rho}]^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)}$	$A_s = 1 - \frac{W_s}{\bar{Y}}$
Reversals improve welfare:	
$W_r = \{E_0[Y_1^{1-\rho} + Y_2^{1-\rho}]^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)}$	$A_r = 1 - \frac{W_r}{\bar{Y}}$
Origin independence improves welfare:	
$W_o = \{E_0[Y_1^{1-\rho} + (E_1[Y_2^{1-\gamma}])^{1/(1-\gamma)})^{1-\rho}]^{(1-\epsilon)/(1-\rho)}\}^{1/(1-\epsilon)}$	$A_o = 1 - \frac{W_o}{\bar{Y}}$

Note:

Y_{12} is income in period 2 under the assumption of no mobility, i.e., $Y_{12} = F_2^{-1}[F_1(Y_1)]$.

Measurement of welfare loss – empirical illustration

Decomposition of welfare gains from mobility

	$\underbrace{A_o - A_s}_{\text{Overall diff}}$	$=$	$\underbrace{A_o - A_r}_{\text{diff from origin independence}}$	$+$	$\underbrace{A_r - A_o}_{\text{diff from reversals}}$
Germany	$-.096$		$-.041$		$-.055$
US	$-.090$		$-.044$		$-.046$

Source:

Gottschalk and Spolaore (2002), Table 1, p 202

Outline

Concluding comments

- whether intergenerational income mobility is socially desirable is less immediately obvious than commonly believed
- origin independence, which for many looms at “maximal” mobility (but arguably is not), is (only) valued socially if the valuation function prioritizes certainty equivalence ahead of aversion to multiperiod inequality and intertemporal variation in income (which I think makes sense in the intra, but not inter case)
- I advocate moving away from taking the dynastic, ex ante view in favour of evaluating the wellbeing of current populations but taking into account their origins (either by the EqOpp approach or by applying prospect theory-based measures)

Outline

Decile transition matrices: USA, (a) 1979–1988

Note: Income refers to equivalized real annual family disposable income, distributed among all individuals (adults and children). The decile groups are ordered from poorest (1) to richest (10). Source: Hungerford (2011, Tables 2 and 3), based on PSID data. Go back

					Destination					
Origin	1	2	3	4	5	6	7	8	9	10
1979					**1988**					
1	44.3	18.3	12.4	9.2	7.1	3.0	1.8	2.0	0.7	1.3
2	18.1	25.3	21.0	11.7	7.5	5.4	4.7	3.2	1.9	1.1
3	10.6	18.2	15.3	16.8	11.6	9.0	8.8	4.9	3.1	1.7
4	7.2	8.9	14.0	14.0	14.7	15.7	12.0	5.6	6.0	2.1
5	6.1	9.2	10.9	12.8	13.3	16.9	12.3	7.5	7.7	3.4
6	4.1	5.2	8.8	10.3	11.8	10.0	14.2	16.9	12.6	6.2
7	3.5	6.5	6.9	8.6	10.4	13.4	13.3	16.8	13.4	7.2
8	3.1	4.6	3.2	7.7	12.3	9.5	12.6	15.7	17.7	13.6
9	1.2	2.2	4.8	6.3	6.9	10.2	12.2	14.7	18.0	23.5
10	2.1	1.5	2.8	2.5	4.2	7.0	8.5	12.8	18.6	40.0

Decile transition matrices: USA, (b) 1989–1998

Note: Income refers to equivalized real annual family disposable income, distributed among all individuals (adults and children). The decile groups are ordered from poorest (1) to richest (10). Source: Hungerford (2011, Tables 2 and 3), based on PSID data. Go back

					Destination					
Origin	1	2	3	4	5	6	7	8	9	10
1989					**1998**					
1	41.9	21.6	13.7	7.0	4.6	3.7	2.7	2.2	1.9	0.7
2	20.4	22.5	15.4	11.6	11.0	8.1	4.0	4.0	1.7	1.2
3	12.5	20.8	17.1	16.4	10.9	10.3	5.2	3.2	1.7	1.9
4	6.9	11.6	15.5	16.9	14.5	11.4	10.1	7.7	2.3	3.1
5	4.8	6.2	12.2	13.8	16.0	14.2	12.4	7.1	7.5	5.8
6	3.2	3.7	9.1	11.6	16.0	14.4	15.7	11.7	7.7	6.9
7	3.2	4.5	7.6	9.3	8.7	12.2	16.3	15.6	16.8	5.8
8	3.0	4.7	5.2	5.4	7.9	12.1	17.2	17.0	19.3	8.3
9	2.5	3.1	4.0	4.9	7.5	7.1	10.7	18.2	21.8	20.3
10	1.7	1.0	0.4	3.2	3.0	6.3	6.0	13.1	19.3	46.1

Differences in cumulative density: USA, 1979–1988 versus 1989–1998

Source: Authors' calculations from (Hungerford, 2011, Tables 2 and 3), based on PSID data.

Go back

	Destination group									
Origin group	1	2	3	4	5	6	7	8	9	10
1	0.2	–0.1	–0.2	0.0	0.3	0.2	0.1	0.1	–0.1	0.0
2	0.0	0.0	0.4	0.6	0.5	0.2	0.2	0.1	0.0	0.0
3	–0.2	–0.5	–0.2	0.0	0.0	–0.5	–0.1	–0.1	0.0	0.0
4	–0.2	–0.7	–0.6	–0.6	–0.7	–0.7	–0.2	–0.3	0.1	0.0
5	0.0	–0.3	–0.3	–0.5	–0.7	–0.5	0.0	–0.1	0.4	0.0
6	0.1	–0.1	–0.1	–0.4	–1.1	–1.3	–0.9	–0.5	0.4	0.0
7	0.1	0.2	0.0	–0.3	–0.8	–0.9	–0.8	–0.3	0.3	0.0
8	0.1	0.2	–0.2	–0.2	–0.3	–0.7	–1.1	–0.7	–0.3	0.0
9	0.0	–0.1	–0.3	–0.2	–0.4	–0.4	–0.7	–0.6	–0.6	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Intergenerational transition matrices in disposable income among all persons for Germany, the UK and the USA

Source: Authors' calculations from Eberharter (2013, Table 3). Go back

A. Germany

	Offspring				
	1	2	3	4	5
Father					
1	34	29	14	17	7
2	15	23	32	15	16
3	12	16	22	26	24
4	9	11	18	29	33
5	7	11	19	25	39

B. UK

	Offspring				
	1	2	3	4	5
Father					
1	48	22	14	12	5
2	22	26	21	22	10
3	11	18	25	25	21
4	6	16	25	26	25
5	4	16	16	27	36

C. USA

	Offspring				
	1	2	3	4	5
Father					
1	37	31	13	13	5
2	21	23	24	17	15
3	12	23	18	24	24
4	9	11	21	33	26
5	2	10	15	26	46

Cumulated differences in intergenerational transition matrices in disposable income among all persons for Germany, the UK and the USA

Source: Authors' calculations from Eberharter (2013, Table 3). ▸ Go back

A. USA – Germany

	Offspring				
	1	2	3	4	5
Father					
1	3	5	5	1	0
2	9	11	4	2	0
3	9	18	6	2	0
4	9	18	9	9	0
5	4	13	1	2	0

B. USA – UK

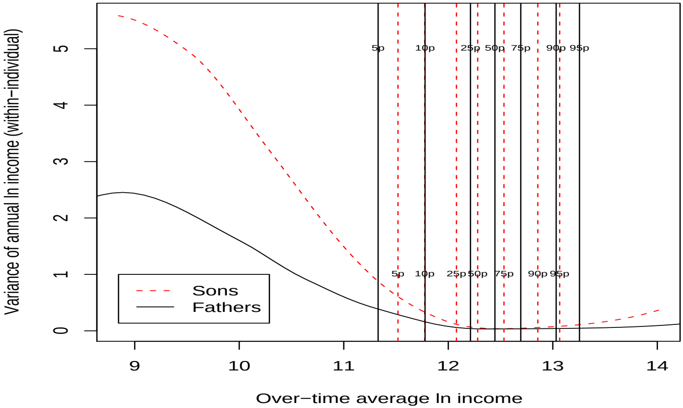
	Offspring				
	1	2	3	4	5
Father					
1	−10	−1	−1	0	0
2	−11	−5	−2	−6	0
3	−11	1	−4	−9	0
4	−8	−3	−12	−10	−1
5	−10	−11	−21	−20	−1

C. UK – Germany

	Offspring				
	1	2	3	4	5
Father					
1	14	6	7	2	0
2	20	16	6	8	0
3	20	18	11	11	0
4	17	20	21	19	1
5	15	24	22	23	1

Transitory errors and long-run income

The variation of annual ln income across over-time mean of ln income – Swedish fathers and sons



Back to the text

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