Welfare and poverty comparisons

axiomatic 'prioritarian' procedures

Erwin Ooghe [& Kristof Bosmans & Luc Lauwers]

Motivation

- **aim of this part of the lecture:**
  - to look at an *axiomatic* framework
  - for welfare and poverty comparisons
  - in case of multiple attributes

Motivation

- **aim of this part of the lecture:**
  - to look at an *axiomatic* framework
  - for welfare and poverty comparisons
  - in case of multiple attributes
- some **problems**:
  - how deal with ordinal attributes?
  - (only for poverty:) how identify the poor?
  - how give priority to the worse off?

How deal with different attribute types?

- MD poverty and welfare measurement; **typically**:
  - all attributes cardinal
  - 1 cardinal & 1 ordinal attribute (index of needs)
  - 1 cardinal & many ordinal (usually binary) attributes

How deal with different attribute types?

- MD poverty and welfare measurement; **typically**:
  - all attributes cardinal
  - 1 cardinal & 1 ordinal attribute (index of needs)
  - 1 cardinal & many ordinal (usually binary) attributes
- a **unifying** framework; notation:
  - set of attributes $J = C \cup O$

How deal with different attribute types?

- MD poverty and welfare measurement; **typically**:
  - all attributes cardinal
  - 1 cardinal & 1 ordinal attribute (index of needs)
  - 1 cardinal & many ordinal (usually binary) attributes
- a **unifying** framework; notation:
  - set of attributes $J = C \cup O$
  - attribute bundles $x = (x_C, x_O) \in B$

How deal with different attribute types?

- MD poverty and welfare measurement; **typically**:
  - all attributes cardinal
  - 1 cardinal & 1 ordinal attribute (index of needs)
  - 1 cardinal & many ordinal (usually binary) attributes
- a **unifying** framework; notation:
  - set of attributes $J = C \cup O$
  - attribute bundles $x = (x_C, x_O) \in B$
  - a distribution $X = (x^1, x^2, \ldots) \in D$

How deal with different attribute types?

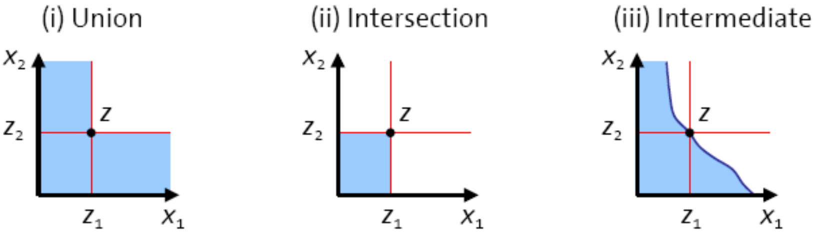
- MD poverty and welfare measurement; **typically**:
  - all attributes cardinal
  - 1 cardinal & 1 ordinal attribute (index of needs)
  - 1 cardinal & many ordinal (usually binary) attributes
- a **unifying** framework; notation:
  - set of attributes $J = C \cup O$
  - attribute bundles $x = (x_C, x_O) \in B$
  - a distribution $X = (x^1, x^2, \ldots) \in D$
  - a 'better-than' ranking $\succsim$ on D: $X \succsim Y$ & $x \succsim y$

How deal with different attribute types?

- MD poverty and welfare measurement; **typically**:
  - all attributes cardinal
  - 1 cardinal & 1 ordinal attribute (index of needs)
  - 1 cardinal & many ordinal (usually binary) attributes
- a **unifying** framework; notation:
  - set of attributes $J = C \cup O$
  - attribute bundles $x = (x_C, x_O) \in B$
  - a distribution $X = (x^1, x^2, \dots) \in D$
  - a 'better-than' ranking $\succsim$ on D: $X \succsim Y$ & $x \succsim y$
- note: some axioms will be **tailored** to attribute type

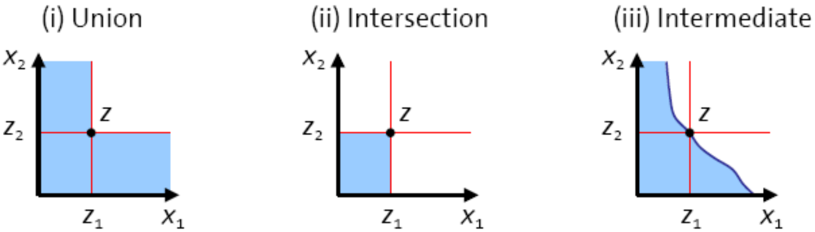
How identify the poor?

- Given a poverty bundle $z \in B$, **who is poor**?



How identify the poor?

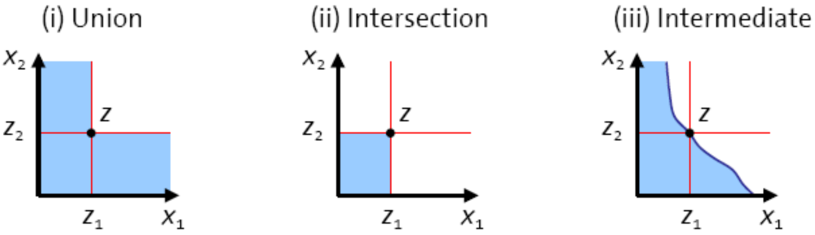
- Given a poverty bundle $z \in B$, **who is poor**?



- **minimalistic:** $P = \{x \in B | x \prec z\}$ & $R = \{x \in B | x \succsim z\}$

How identify the poor?

- Given a poverty bundle $z \in B$, **who is poor**?



- **minimalistic:** $P = \{x \in B | x \prec z\}$ & $R = \{x \in B | x \succsim z\}$
- note: poverty frontier—as in (iii)—but defined by axioms

How give priority to the worse off?

- Example 1; consider:
  - two individuals, 1 cardinal and 1 binary attribute
  - a MD welfare index $W = \sqrt{x_1^1/m(x_2^1)} + \sqrt{x_1^2/m(x_2^2)}$
  - if $m(0) = 2 > m(1) = 1$, then $x^1 = (4,0) \prec (4,1) = x^2$

How give priority to the worse off?

- Example 1; consider:
  - two individuals, 1 cardinal and 1 binary attribute
  - a MD welfare index $W = \sqrt{x_1^1/m(x_2^1)} + \sqrt{x_1^2/m(x_2^2)}$
  - if $m(0) = 2 > m(1) = 1$, then $x^1 = (4,0) \prec (4,1) = x^2$
- what to do with an **extra** unit of the cardinal attribute?
  - if we give it to the worse off, then $\Delta W \cong 0.17$
  - if we give it to the better off, then $\Delta W \cong 0.24$

How give priority to the worse off?

- Example 1; consider:
  - two individuals, 1 cardinal and 1 binary attribute
  - a MD welfare index $W = \sqrt{x_1^1/m(x_2^1)} + \sqrt{x_1^2/m(x_2^2)}$
  - if $m(0) = 2 > m(1) = 1$, then $x^1 = (4,0) \prec (4,1) = x^2$
- what to do with an **extra** unit of the cardinal attribute?
  - if we give it to the worse off, then $\Delta W \cong 0.17$
  - if we give it to the better off, then $\Delta W \cong 0.24$
- note: 'old' problem = Sen's (1973) critique on utilitarianism

How give priority to the worse off?

- Example 2; consider:
  - two individuals and two cardinal attributes
  - a MD welfare index $W = \sqrt{x_1^1 x_2^1} + \sqrt{x_1^2 x_2^2}$
  - $x^1 = (4, 6) \prec (6.5, 4) = x^2$

How give priority to the worse off?

- Example 2; consider:
  - two individuals and two cardinal attributes
  - a MD welfare index $W = \sqrt{x_1^1 x_2^1} + \sqrt{x_1^2 x_2^2}$
  - $x^1 = (4,6) \prec (6.5,4) = x^2$
- what to do with an **extra** unit of dimension 2?
  - if we give it to worse off, then $\Delta W \cong 0.39$
  - if we give it to better off, then $\Delta W \cong 0.60$

How give priority to the worse off?

- **priority** = give priority to the worse off
  - with 'worse off' defined in a consistent way
  - i.e., according to the ranking $\succsim$ itself

How give priority to the worse off?

- **priority** = give priority to the worse off
  - with 'worse off' defined in a consistent way
  - i.e., according to the ranking $\succsim$ itself
- **cardinal** version of priority: for all X in D
  - for all [poor] i and j with $x^i \succsim x^j$
  - for all $\delta = (\delta_C, \delta_O)$ in B with $\delta_C > 0$ & $\delta_O = 0$
  - $(\ldots, x^i, \ldots, x^j + \delta, \ldots) \succsim (\ldots, x^i + \delta, \ldots, x^j, \ldots)$

How give priority to the worse off?

- **priority** = give priority to the worse off
  - with 'worse off' defined in a consistent way
  - i.e., according to the ranking $\succsim$ itself
- **cardinal** version of priority: for all X in D
  - for all [poor] i and j with $x^i \succsim x^j$
  - for all $\delta = (\delta_C, \delta_O)$ in B with $\delta_C > 0$ & $\delta_O = 0$
  - $(\ldots, x^i, \ldots, x^j + \delta, \ldots) \succsim (\ldots, x^i + \delta, \ldots, x^j, \ldots)$
- **ordinal** version of priority: for all X in D
  - for all [poor] i and j with $x^i \succsim x^j$
  - for all $\delta = (\delta_C, \delta_O)$ in B with $\delta_C = 0$ & $\delta_O > 0$
  - with $\delta_k(x^i_k - x^j_k) = 0$ for all k in J
  - $(\ldots, x^i, \ldots, x^j + \delta, \ldots) \succsim (\ldots, x^i + \delta, \ldots, x^j, \ldots)$

Axioms

- AR: Additive representation, i.e., $W = \frac{1}{n_X} \sum_{i=1}^{n_X} U\left(x^i\right)$

Axioms

- AR: Additive representation, i.e., $W = \frac{1}{n_X} \sum_{i=1}^{n_X} U\left(x^i\right)$
- M: Monotonicity ($\Rightarrow U$ strictly increasing)

Axioms

- AR: Additive representation, i.e., $W = \frac{1}{n_X} \sum_{i=1}^{n_X} U\left(x^i\right)$
- M: Monotonicity ($\Rightarrow U$ strictly increasing)
- CP: cardinal priority & OP: ordinal priority

Results

$\succsim$ on D satisfies AR, M and CP iff there exist

1. $w_j > 0$, for each j in C & s.i. $g : \mathbb{N}^{|O|} \to \mathbb{R}$
2. s.i. and concave f

such that for all X, Y in D, we have $X \succsim Y$ iff

$$\frac{1}{n_X} \sum_{i=1}^{n_X} f\left(\sum_{j \in C} w_j x_j^i + g(x_O^i)\right) \geq \frac{1}{n_Y} \sum_{i=1}^{n_Y} f\left(\sum_{j \in C} w_j y_j^i + g(y_O^i)\right) \quad (*)$$

Results

$\succsim$ on D satisfies AR, M and CP iff there exist

1. $w_j > 0$, for each j in C & s.i. $g : \mathbb{N}^{|O|} \to \mathbb{R}$
2. s.i. and concave f

such that for all X, Y in D, we have $X \succsim Y$ iff

$$\frac{1}{n_X} \sum_{i=1}^{n_X} f\left(\sum_{j\in C} w_j x_j^i + g(x_O^i)\right) \geq \frac{1}{n_Y} \sum_{i=1}^{n_Y} f\left(\sum_{j\in C} w_j y_j^i + g(y_O^i)\right) \quad (*)$$

$\succsim$ on D satisfies AR, M and CP + OP iff there exist

1. same as before, except
2. s.i. $g_j : \mathbb{N} \to \mathbb{R}$ for each j in O (rather than g)

such that for all X, Y in D, we have $X \succsim Y$ iff

$(*)$ holds, with $g(x_O^i) = \sum_{j\in O} g_j(x_j^i)$ & $g(y_O^i) = \sum_{j\in O} g_j(y_j^i)$

Cardinal attributes only

- if $|J| = |C|$, then W reduces to $\frac{1}{n_X} \sum_{i=1}^{n_X} f\left(\sum_{j \in J} w_j x_j^i\right)$
  - problematic for index?
  - less so for dominance ...

Cardinal attributes only

- if $|J| = |C|$, then W reduces to $\frac{1}{n_X} \sum_{i=1}^{n_X} f(\sum_{j \in J} w_j x_j^i)$
    - problematic for index?
    - less so for dominance ...
- if $|J| = |C|$, there is an equivalence between
    - Kolm's (1977) budget dominance criterion,
    - Koshevoy and Mosler's (1999) inverse GL-curve, and
    - unanimity among rankings satisfying AR, M, & CP

1 cardinal & 1 ordinal attribute

- if $|C| = 1 = |O|$, then W reduces to $\frac{1}{n_X} \sum_{i=1}^{n_X} f(x_1^i + g(x_2^i))$
  - we knew that absolute scales can solve Sen's conflict
  - our result tells us that it is the only way to solve it

1 cardinal & 1 ordinal attribute

- if $|C| = 1 = |O|$, then W reduces to $\frac{1}{n_X} \sum_{i=1}^{n_X} f(x_1^i + g(x_2^i))$
  - we knew that absolute scales can solve Sen's conflict
  - our result tells us that it is the only way to solve it
- if $|C| = 1 = |O|$, there is an equivalence between
  - Bourguignon's (1989) dominance criterion, and
  - unanimity among rankings satisfying AR, M, & CP

1 cardinal & 1 ordinal attribute

- if $|C| = 1 = |O|$, then W reduces to $\frac{1}{n_X} \sum_{i=1}^{n_X} f(x_1^i + g(x_2^i))$
  - we knew that absolute scales can solve Sen's conflict
  - our result tells us that it is the only way to solve it
- if $|C| = 1 = |O|$, there is an equivalence between
  - Bourguignon's (1989) dominance criterion, and
  - unanimity among rankings satisfying AR, M, & CP
- note: similar to FHT

Axioms

- AR: Additive representation, i.e., $\Pi = \frac{1}{n_X} \sum_{i=1}^{n_X} \pi_z \left(x^i \right)$

Axioms

- AR: Additive representation, i.e., $\Pi = \frac{1}{n_X} \sum_{i=1}^{n_X} \pi_z \left(x^i \right)$
- F: focus, i.e., only the poor—$\{ i | x^i \text{ in } P \}$—matter

Axioms

- AR: Additive representation, i.e., $\Pi = \frac{1}{n_X} \sum_{i=1}^{n_X} \pi_z \left(x^i\right)$
- F: focus, i.e., only the poor—$\{i | x^i \text{ in } P\}$—matter
- M: Monotonicity for the poor

Axioms

- AR: Additive representation, i.e., $\Pi = \frac{1}{n_X} \sum_{i=1}^{n_X} \pi_z \left(x^i \right)$
- F: focus, i.e., only the poor—$\{i | x^i \text{ in } P\}$—matter
- M: Monotonicity for the poor
- CP: cardinal priority & OP: ordinal priority for the poor

Results

$\succsim$ on D satisfies AR, F, M and CP iff there exist

1. $w_j > 0$, for each j in C & s.i. $g : \mathbb{N}^{|O|} \to \mathbb{R}$ with $g(0) = 0$
2. continuous f with
   1. $f(a) = f(\omega)$ whenever $a \geq \omega := \sum_{j \in C} w_j z_j + g(z_O)$
   2. f strictly decreasing and convex on $[0, \omega)$

such that for all X, Y in D, we have $X \succsim Y$ iff

$$\frac{1}{n_X} \sum_{i=1}^{n_X} f\left(\sum_{j \in C} w_j x_j^i + g(x_O^i)\right) \leq \frac{1}{n_Y} \sum_{i=1}^{n_Y} f\left(\sum_{j \in C} w_j y_j^i + g(y_O^i)\right) \quad (*)$$

Results

$\succsim$ on D satisfies AR, F, M and CP iff there exist

1. $w_j > 0$, for each j in C & s.i. $g : \mathbb{N}^{|O|} \to \mathbb{R}$ with $g(0) = 0$
2. continuous f with
   1. $f(a) = f(\omega)$ whenever $a \geq \omega := \sum_{j \in C} w_j z_j + g(z_O)$
   2. f strictly decreasing and convex on $[0, \omega)$

such that for all X, Y in D, we have $X \succsim Y$ iff

$$\frac{1}{n_X} \sum_{i=1}^{n_X} f\left(\sum_{j \in C} w_j x_j^i + g(x_O^i)\right) \leq \frac{1}{n_Y} \sum_{i=1}^{n_Y} f\left(\sum_{j \in C} w_j y_j^i + g(y_O^i)\right) \quad (*)$$

$\succsim$ on D satisfies AR, F, M and CP + OP similar ...

1 cardinal & several binary attributes

- if $|J| = |C|$ or if $|C| = 1 = |O|$, similar remarks as before

1 cardinal & several binary attributes

- if $|J| = |C|$ or if $|C| = 1 = |O|$, similar remarks as before
- 1 cardinal & several binary, and choose $z = (z_0, 0)$
  - Π becomes $\frac{1}{n_X} \sum_{i=1}^{n_X} f(x_1^i + \alpha \cdot x_O^i)$
  - (generalized) counting approach

1 cardinal & several binary attributes

- if $|J| = |C|$ or if $|C| = 1 = |O|$, similar remarks as before
- 1 cardinal & several binary, and choose $z = (z_0, 0)$
    - Π becomes $\frac{1}{n_X} \sum_{i=1}^{n_X} f(x_1^i + \alpha \cdot x_O^i)$
    - (generalized) counting approach
- there is an equivalence between
    - $\cap$ rankings satisfying AR, M, & CP+OP & $z_0 \leq \bar{z}_0$
    - $\sum_{t \in B_O} \int_0^{z_t} \{p_t F_t(y) - q_t G_t(y)\} dy \leq 0$ for z_t such that
        - $0 \leq z_0 \leq \bar{z}_0$
        - for all $t \in B_O : z_t \geq z_{t'}$ if $t \leq t'$

1 cardinal & several binary attributes

- if $|J| = |C|$ or if $|C| = 1 = |O|$, similar remarks as before
- 1 cardinal & several binary, and choose $z = (z_0, 0)$
  - Π becomes $\frac{1}{n_X} \sum_{i=1}^{n_X} f(x_1^i + \alpha \cdot x_O^i)$
  - (generalized) counting approach
- there is an equivalence between
  - $\cap$ rankings satisfying AR, M, & CP+OP & $z_0 \leq \bar{z}_0$
  - $\sum_{t \in B_O} \int_0^{z_t} \{p_t F_t(y) - q_t G_t(y)\} dy \leq 0$ for z_t such that
    - $0 \leq z_0 \leq \bar{z}_0$
    - for all $t \in B_O : z_t \geq z_{t'}$ if $t \leq t'$
- application ...

- Atkinson, 2003, Multidimensional deprivation: contrasting social welfare and counting approaches, *Journal of Economic Inequality* 1, 51-65.
- Atkinson & Bourguignon, 1982, the comparison of multi-dimensioned distributions of economic status, *Review of Economic Studies* XLIX, 183-201.
- Bosmans, Lauwers & Ooghe, 2009, A consistent multidimensional Pigou-Dalton transfer principle, *Journal of Economic Theory* 144, 1358-1371.
- Bourguignon, 1989, Family size and social utility: income distribution dominance criteria, *Journal of Econometrics* 42, 67-80.
- Duclos, Sahn & Younger, Robust multidimensional poverty comparisons, *The Economic Journal* 116, 943-968.
- Ebert, 1997, Social welfare when needs differ: an axiomatic approach, *Economica* 64, 233-244.

- Fleurbaey, Hagneré & Trannoy, 2003, Welfare comparisons with bounded equivalence scales, *Journal of Economic Theory* 110, 309-336.
- Hammond, 1979, Dual interpersonal comparisons of utility and the welfare economics of income distribution, *Journal of Public Economics* 7, 51-71.
- Kolm, 1977, Multidimensional egalitarianisms, *Quarterly Journal of Economics* 91, 1-13.
- Koshevoy & Mosler, 1999, Price majorization and the inverse Lorenz function, DP 3/99, University of Cologne.
- Sen, 1973, *On Economic Inequality*, Oxford University Press.
- Shorrocks, 1995, Inequality and welfare evaluation of heterogeneous income distributions, DP 447, University of Essex, also published in *Journal of Economic Inequality* 2 (2004), 193-218.

- Trannoy, 2006, Multidimensional egalitarianism and the dominance approach: a lost paradise? in Farina & Savaglio, eds., *Inequality and Economic Integration*, Routledge.
- Weymark, 2006, The normative approach to the measurement of multidimensional inequality, in Farina & Savaglio, eds., *Inequality and Economic Integration*, Routledge.