Winter School IT6 January 2011 Welfare and poverty evaluations for heterogeneous populations

Claudio Zoli

Peter Lambert, Erwin Ooghe

Aims of the talk

- Overview of selected issues underlying the theory of measurement of inequality, welfare and poverty for populations composed of heterogenous units.

1. **Unidimensional:** Individuals/households are homogeneous in all ethically relevant characteristics except consumption or income.
2. **Multidimensional:** heterogeneous individuals exhibiting differences in a number of "characteristics" (transferable and not transferable) e.g. income, health, housing, bundles of goods, education, household size, level of needs.....

Aims of the talk

- Overview of selected issues underlying the theory of measurement of inequality, welfare and poverty for populations composed of heterogenous units.
- Two broad perspectives:

1. **Unidimensional:** Individuals/households are homogeneous in all ethically relevant characteristics except consumption or income.
2. **Multidimensional:** heterogeneous individuals exhibiting differences in a number of "characteristics" (transferable and not transferable) e.g. income, health, housing, bundles of goods, education, household size, level of needs.....

Aims of the talk 2

- A number of interrelated perspectives of evaluation:

1. Inequality (focus on dispersions across agents),
2. Welfare (taking into account also the size of the cake)
3. Poverty (focussing on deprived agents, size and dispersion matters but only focussing on those deprived)
4. Well being: multidimensional perspective focussing on size and dispersion. (Erwin)

Aims of the talk 2

- A number of interrelated perspectives of evaluation:

1. Inequality (focus on dispersions across agents),
2. Welfare (taking into account also the size of the cake)
3. Poverty (focussing on deprived agents, size and dispersion matters but only focussing on those deprived)
4. Well being: multidimensional perspective focussing on size and dispersion. (Erwin)

- Evaluations: complete rankings (i.e. indices) or partial rankings?

Our Concern

- To provide some intuitions on the interrelations between household characteristics and resources in determining criteria for comparing heterogenous populations........

QUESTIONS

- Units: households, individuals or equivalent adults?
- Comparability assumptions: under what conditions different levels of income may be considered comparable across household with differing needs?

Assumptions on (i) comparability of utility, marginal utility .... and (ii) weigth of the unit of measure are relevant to shape the overall evaluations

$$\begin{aligned} W_F &= \int_0^{\bar{x}} u(x)f(x)dx = -\int_0^{\bar{x}} u'(x)F(x)dx + u(\bar{x}) \\ &= \int_0^{\bar{x}} u''(x)\left[\int_0^{x} F(t)dt\right]dx - u'(\bar{x})\cdot[\bar{x}-\mu_F] + u(\bar{x}) \end{aligned}$$

$\Delta W = W_F - W_G$, $\Delta F = F(x) - G(x)$

$$\begin{aligned} \Delta W &= -\int_0^{\bar{x}} u'(x)\Delta F(x)dx \\ &= \int_0^{\bar{x}} u''(x)\left[\int_0^{x} \Delta F(t)dt\right]dx + u'(\bar{x})\cdot(\Delta\mu) \end{aligned}$$

we will consider mainly households (or individuals) with differing needs.

Unidimensional set up.

- n homogeneous individuals $i = 1, 2, ..., n \geq 2$
- $F_X(\mathbf{x})$ or $F(x)$ c.d.f.
- $\mu(\mathbf{x}) = \sum_{i=1}^{n} x_i / n$ average of distribution $\mathbf{x}_{.j}$ of attribute j (e.g. income)
- $\hat{\mathbf{x}}$ ordered distribution of income: $\hat{x}_{(1)} \leq \hat{x}_{(2)} \leq ... \leq \hat{x}_{(i)} ... \leq \hat{x}_{(n)}$
- $I(X) : \mathbb{R}_+^n \rightarrow \mathbb{R}$ Inequality index,
- $W(X) : \mathbb{R}_+^n \rightarrow \mathbb{R}$ Social Evaluation Function (SEF)

The Unidimensional case

Definition (Cumulative Distribution Function)

$F : \mathbb{R}_+ \to [0,1]$ Function $F(x)$ plotting the proportion of income units within the population with income at most x.

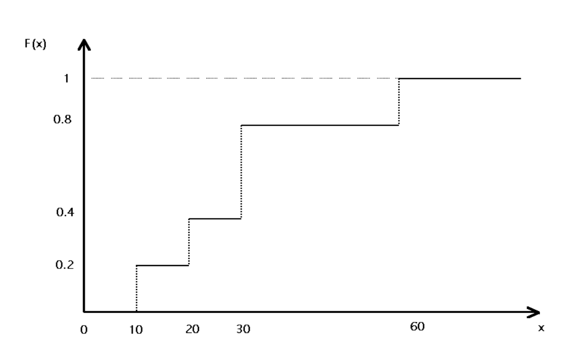

Figure: C.d.f. $F_{\mathbf{x}}(x)$ for $\mathbf{x} = (10, 20, 30, 30, 60)$

Definition (Inverse Distribution Function)

$F^{-1}:[0,1]\rightarrow\mathbb{R}_{+}$. Function $F^{-1}(p)=\inf\{x\in\mathbb{R}_{+}:F(x)\geq p\}$. plotting the income level corresponding to the p^{th} quantile of the population once incomes are ranked in ascending order:

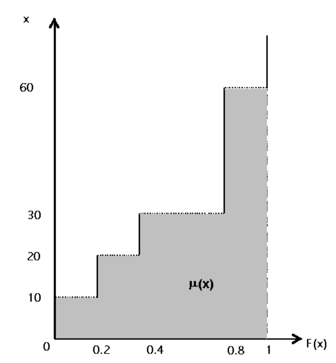

Figure: Inv.d.f. $F_{\mathbf{x}}^{-1}(p)$ for $\mathbf{x}=(10,20,30,30,60)$

Implementing inequality comparisons:

The most common tools applied in inequality analysis to compare income distributions are indeed the partial orders induced by the stochastic dominance conditions (direct and inverse).

Definition (Lorenz Dominance)

Define the **Lorenz curve** for X:

$$L_X(p) := \int_0^p \frac{F_X^{-1}(t)}{\mu(X)} dt.$$

Income profile X *Lorenz dominates* income profile Y, $X \succcurlyeq_L Y$, if and only if

$$L_X(p) \geq L_Y(p) \text{ for all } p \in [0,1] .$$

Note that the Lorenz curve is obtained integrating the graph of the inverse distribution function an dividing by the average income.

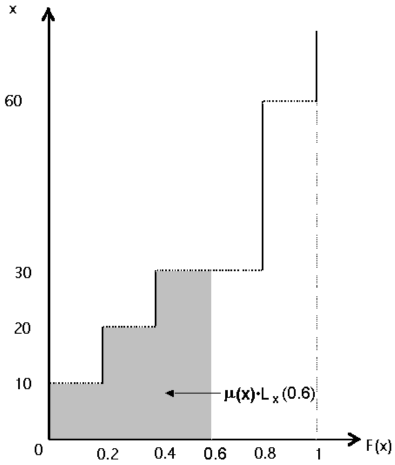

Figure: Lorenz curve derived from inverse distribution.

Definition (Generalized Lorenz Dominance)

Define the **generalized Lorenz curve** for X:

$$GL_X(p) := \mu(X) \cdot L_X(p).$$

Income profile X generalize *Lorenz dominates* income profile Y, $X \succcurlyeq_{GL} Y$, if and only if

$$GL_X(p) \geq GL_Y(p) \text{ for all } p \in [0, 1].$$

Kolm (1969), Shorrocks (1983).

- If $\mu(X) = \mu(Y)$, $\succcurlyeq_{GL} \Longleftrightarrow \succcurlyeq_L$;
- $X/\mu_X \succcurlyeq_{GL} Y/\mu_Y \Longleftrightarrow X \succcurlyeq_L Y$.

$$GL_x(i/n) = \frac{1}{n}\sum_{j=1}^{i} \hat{x}_{(j)} \quad \text{where } \hat{x}_{(i)} \leq \hat{x}_{(i+1)}$$

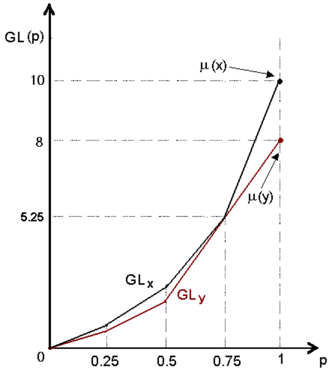

Figure: Generalized Lorenz Curves for $\mathbf{y} = (3, 7, 11, 11)$, $\mathbf{x} = (4, 8, 9, 19)$

no-dominance if the GL curves intersect

Theorem (Hardy, Littlewood & Polya 1934 (HL&P))

Consider a fixed number of individuals n, let $\mu(\mathbf{x}) = \mu(\mathbf{y})$, *the following statements are equivalent:*

(1) For all $k \leq n$, $\sum_{i=1}^{k} \hat{x}_i \geq \sum_{i=1}^{k} \hat{y}_i$ *with at least one strict inequality* $(>)$.

(2) $\hat{\mathbf{x}}$ *can be obtained from* $\hat{\mathbf{y}}$ *through a finite sequence of progressive transfers.*

(3) Let $W_u(\mathbf{x}) = \sum_{i=1}^{n} u(x_i)$ *the Utilitarian Social Evaluation Function,* $W_u(\mathbf{x}) \geq W_u(\mathbf{y})$ *for all* $W_u(\mathbf{x})$ *such that* $u(.)$ *is non decreasing and concave.*

(4) Let $I_\phi(\mathbf{x}) = \sum_{i=1}^{n} \phi(x_i)$ *the additive inequality index* $I_\phi(\mathbf{x}) \leq I_\phi(\mathbf{y})$ *for all* $I_\phi(\mathbf{x})$ *such that* $\phi(.)$ *is convex.*

Theorem

(5) $\mathbf{x} = \Pi\mathbf{y}$ *where* Π *is a* $n \times n$ *bistochastic matrix.*

Links between Inequality & Welfare

We consider Social Evaluation Function (SEF) $W(\mathbf{x}) : \mathbb{R}^n_+ \rightarrow \mathbb{R}$

Theorem (Shorrocks (1983); Kolm (1969))

Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n_+$ *the following statements are equivalent:*
(i) $\mathbf{x} \succcurlyeq_{GL} \mathbf{y}$
(ii) $W(\mathbf{x}) \geq W(\mathbf{y})$ *for all increasing SEFs* $W(\mathbf{x})$ *satisfying Symmetry,and Principle of Transfers.*
(iii) $\frac{1}{n}\sum_{i=1}^{n} u(x_i) \geq \frac{1}{n}\sum_{i=1}^{n} u(y_i)$ *for all Average Utilitarian SEFs where* $u(.)$ *is non decreasing and concave.*

Definition (First Order Stochastic Dominance (SD1))

$F \succcurlyeq_1 G$ if and only if $G(x) \geq F(x)$ for all $x \in [0, \bar{x}]$.

Definition (Rank Dominance RD)

$F \succcurlyeq_R G$ if and only if $F^{-1}(p) \geq G^{-1}(p)$ for all $p \in [0, 1]$.

Lemma

$F \succcurlyeq_1 G \Leftrightarrow F \succcurlyeq_R G$.

Definition (Second Degree Stochastic Dominance: SD2)

$F \succcurlyeq_2 G$ if and only if $\int_0^x F(y)dy \leq \int_0^x G(y)dy$ for all $x \in [0, \bar{x}]$

Lemma

$F \succcurlyeq_2 G \Leftrightarrow F \succcurlyeq_{GL} G$.

Links with poverty

Absolute poverty gap

$$\gamma_i(x, z) = \begin{cases} z - \hat{x}_i & \text{if } z \geq \hat{x}_i \\ 0 & \text{if } z < \hat{x}_i \end{cases}$$

Absolute poverty gap of the agent in position i in the ranked income distribution **x**.

$\gamma_i(x, z)$ is ranked in decreasing order.

The absolute deprivation curve or absoluteTIP (Three I's of Poverty) curve of **x** is:

Definition (TIP Curve)

TIP curve of distribution **x** for the proportion i/n of population:

$$TIP_x(i/n) = \frac{1}{n} \sum_{j=1}^{i} \gamma_i(x, z).$$

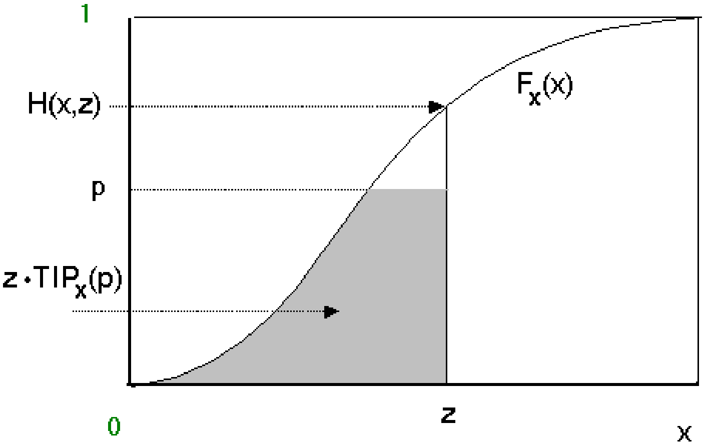

Figure: Geometric derivation of TIP Curve

Similar to $GL_x(i/n)$, we have $\gamma_i(x,z)$ instead of $\hat{x}_i$!
Low poverty for curves with lower height.

Stochastic dominance and poverty evaluations (1)

$$P_\alpha(\mathbf{x},z) = \frac{1}{n}\sum_{i=1}^{q}\left(\frac{z-\hat{x}_i}{z}\right)^\alpha \text{ for } \alpha \geq 0$$

intimately related to the stochastic dominance

Theorem (Foster-Shorrocks (1988))

Let $\mathbf{x}, \mathbf{y} \in X$ such that the higher income is below $\bar{x} > 0$. The following statements are equivalent:
(i) $P_{\alpha-1}(\mathbf{x},z) \leq P_{\alpha-1}(\mathbf{y},z)$ for all poverty lines $z \leq \bar{x}$.
(ii) $\mathbf{x} \succcurlyeq_\alpha \mathbf{y}$ for all incomes in $[0, \bar{x}]$ where $\alpha \in \{1, 2, 3\}$.

Stochastic dominance and poverty evaluations (2)

Dominance for additively decomposable indices

$$AP(\mathbf{x},z) = \frac{1}{n}\sum_{i=1}^{q} p\left(\hat{x}_i, z\right)$$

for all poverty lines within a range $z \in \left[z_{\min}, z^{\max}\right]$.

Theorem (Atkinson (1987))

Let $\mathbf{x}, \mathbf{y} \in X$. *The following statements are equivalent:*
(i) $AP(\mathbf{x},z) < AP(\mathbf{y},z)$ *for all poverty lines* $z \in \left[z_{\min}, z^{\max}\right]$ *and all* $AP(.,z)$ *satisfying Focus, Symmetry, Monotonicity, [Principle of Transfers]*
(ii) $\mathbf{x} \succ_{1[2]} \mathbf{y}$ *for all incomes in* $\left[0, z_{\min}\right]$ *and* $\mathbf{x} \succsim_{1[2]} \mathbf{y}$ *for all incomes in* $\left(z_{\min}, z^{\max}\right]$.

NOW PETER TALK

A dual apporach to poverty dominance for heterogeneous populations

We analyze dominance conditions for:
(I) additively decomposable poverty indices
(II) over heterogeneous populations
(III) when between groups comparisons are made at fixed poverty gap levels instead of fixed income levels.

- Dual because the perspective is shifted from income levels to poverty gaps.
- When groups' poverty lines do not coincide we get different results from the standard ones in Atkinson (1992).

the setting

Population (discrete) partitioned into non overlapping and exhaustive subgroups $i = 1, 2, ..., n$
$x \geq 0$ income: $\mathbf{x}^i = (x_1^i, x_2^i, ..., x_l^i, ..., x_{m_i}^i)$.
$F(x) \in \mathcal{F}$: c.d.f. of an income profile with finite mean $\mu(F) = \int_0^{+\infty} x dF(x)$.
$F^{-1}(p) = \inf \{x : F(x) \geq p\}$, $p \in [0, 1]$: Left continuous inverse distribution function.
$F_i(x)$: c.d.f. of subgroup i of population F
q_i^F : share of individuals belonging to group i in F,
$z > 0$: poverty line, (z_i) of group i.
$\mathbf{z} = (z_1, ..., z_i, ..., z_n)$ where $z_1 \geq z_2 .. \geq z_i \geq .. \geq z_n$,
$P(F, z) : \mathcal{F} \times \mathbb{R}_{++} \to \mathbb{R}$: poverty index

Definition (Absolute Poverty Gap (APG))

$$\gamma^F(p,z) = z - F^{*-1}(p)$$

poverty gap evaluated at quantile p of distribution F (censored at z) of the **total population**.
$\gamma^F(p,z)$ is ranked in decreasing order,
$\gamma^F(p,z) = 0$ if $p \geq F(z)$.

Definition (APG Curve)

$$\mathcal{PG}_F(p,z) = \int_0^p \gamma^F(q,z)dq.$$

AXIOM WEAK MONOTONICITY WM: $F \rightarrow G$ reducing the income of a poor individual $\Longrightarrow P(G, z) \geq P(F, z)$. Sen (E.trica 1976)

AXIOM WEAK PRINCIPLE OF TRANSFERS WPT: $F \rightarrow G$ applying regressive transfer involving poor income units $\Longrightarrow$ $P(G, z) \geq P(F, z)$. Sen (E.trica 1976)

Theorem

For a fixed z, $P(F,z) \leq P(G,z)$ *for all symmetric poverty indices* $P(.,z)$ *satisfying WM and WPT if and only if*

$$\mathcal{PG}_F(p, z) \leq \mathcal{PG}_G(p, z) \quad \forall p \in [0, 1].$$

Spencer & Fisher (Sankya 1992) Jenkins & Lambert (OEP 1997), Shorrocks (1998)

Additive poverty measures, heterogeneous populations

Compare the statements for 2 poor units:

(A) The social marginal utility of income is higher for the needier unit if experiencing the same income as the less needy unit. Atkinson (1992).

(B) The social marginal utility of income is higher for the needier unit if experiencing the same poverty-gap as the less needy unit.

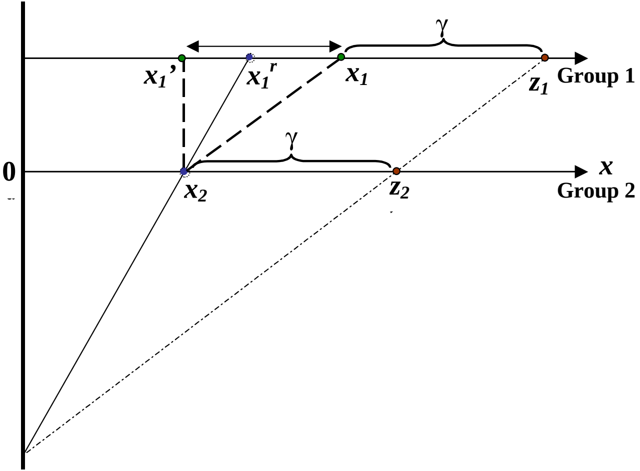

Figure: Comparability Sets for 2 Groups

Poverty lines $\mathbf{z} = (z_1, z_2, ..z_i, ..z_n)$ ranked in non-increasing order, i.e. $z_i \geq z_{i+1} > 0$.
Aggregate poverty index:

$$P(F) = \sum_{i=1}^{n} q_i^F \int_0^{z_i} p_i(x, z_i) dF_i(x).$$

$p_i(x, z_i)$ is individual deprivation function

Let, for groups $i = 1, 2, ..., n$

$$\Delta_i^{[1]}(x) := q_i^F F_i(x) - q_i^G G_i(x), \quad \Delta_i^{[2]}(x) := \int_0^x \Delta_i^{[1]}(t)dt$$

Definition (Sequential Poverty Dominance)

For all $\mathbf{z}$, all $F, G \in \mathcal{F}$:

(1) $F \succcurlyeq_{SPD(1)[\mathbf{z}]} G \iff \sum_{i=1}^{k} \Delta_i^{[1]}(x) \leq 0$ for all $x \leq z_k$, for all $k = 1, 2, ..., n$,

(2) $F \succcurlyeq_{SPD(2)[\mathbf{z}]} G \iff \sum_{i=1}^{k} \Delta_i^{[2]}(x) \leq 0$ for all $x \leq z_k$, for all $k = 1, 2, ..., n$.

Atkinson (Economica 1992), Jenkins and Lambert (RI&W 1993), Chambaz and Maurin (RI&W 1999), Duclos and Makdissi (1999)

SPD not consistent with most common poverty indices:

$H(F, z) = F(z)$ Head Count ratio, proportion of poors.
$H(F, z)I(F, z) = F(z) \cdot [z - \mu(F^*)]$ Average (w.r.t. entire population) poverty gap .

Example

Income profiles $\mathbf{x}$ and $\mathbf{y}$, with distribution functions be F, and G:

$$\begin{aligned} \mathbf{x}^1 &= (1, 3, 6), \quad \mathbf{x}^2 = (0, 4, 4); \\ \mathbf{y}^1 &= (0, 2, 4), \quad \mathbf{y}^2 = (1, 4, 6) \\ z_1 &= 7 \quad , \quad z_2 = 5 \end{aligned}$$

$F \succcurlyeq_{SPD(1)} G$ but $H(G) = 5/6 < 1 = H(F)$.

Analogous examples can be constructe for $F \succcurlyeq_{SPD(2)} G$ once considering $H \cdot I$

Aggregate poverty comparisons

Aggregate poverty of distribution F

$$P(F) = \sum_{i=1}^{n} q_i^F \int_0^{z_i} u_i(z_i - x) dF_i(x)$$

average well being shortfall from a minimum standard of living.
$u_i(\gamma)$ may depend on z_i

Properties

- $u_i(\gamma)$ is continuous and twice differentiable.
- AXIOM A*: $u_i(\gamma) \geq 0$ for all $\gamma > 0$, and $u_i(0) = 0$, for all $i = 1, 2, ..., n$.
- AXIOM A1*: $u_i'(\gamma) \geq u_{i+1}'(\gamma) \geq 0$ for all $\gamma > 0$, and all $i = 1, 2, ..n - 1$.
- AXIOM A2*: $u_i''(\gamma) \geq u_{i+1}''(\gamma) \geq 0$ for all $\gamma > 0$, and all $i = 1, 2, ..n - 1$.

Sequential Poverty Gap Dominance

$\mathbf{z}^k = (z_1, ...z_i, .. \ z_k)$ poverty lines of first k groups,
$\mathbf{z} = (z_1, ...z_i, ..., \ z_n)$ poverty lines of all groups.
$F^{(k)}$ income distribution of first k groups where individuals are ranked according to poverty gaps given $\mathbf{z}^k$

Definition

$\gamma^{F^{(k)}}(p, \mathbf{z}^k)$: absolute poverty gap of individual ranked at the p quantile of the income distribution $F^{(k)}$ of first k groups.

Definition

$\mathcal{PG}_F^k(p, \mathbf{z}^k) = \int_0^p \gamma^{F^{(k)}}(t, \mathbf{z}^k)dt$: absolute poverty gap curve of the distribution including only groups $i = 1, 2, ..., k$.
$\mathcal{PG}_F(p, \mathbf{z}) = \int_0^p \gamma^F(t, \mathbf{z})dt$: absolute poverty gap curve of the overall distribution.

Definition (Sequential Poverty-Gap Dominance)

For all $\mathbf{z}$, all $F, G \in \mathcal{F}$ $s.t.$ $q_i^F = q_i^G$ for all i:

(1) $F \succsim_{SPGD(1)[\mathbf{z}]} G \iff \gamma^{F^{(k)}}(p, \mathbf{z}^k) \leq \gamma^{G^{(k)}}(p, \mathbf{z}^k)$ for all $p \in [0, 1]$, for all $k = 1, 2, ..., n$,

(2) $F \succsim_{SPGD(2)[\mathbf{z}]} G \iff \mathcal{PG}_F^k(p, \mathbf{z}^k) \leq \mathcal{PG}_G^k(p, \mathbf{z}^k)$, for all $p \in [0, 1]$, for all $k = 1, 2, ..., n$.

Theorem

If $q_i^F = q_i^G$ for all i then, for any fixed $\mathbf{z}$,

(i) $P(F) \leq P(G)$ for all u_i satisfying A^* and $A1^*$
$\iff F \succsim_{SPGD(1)[\mathbf{z}]} G$.

(ii) $P(F) \leq P(G)$ for all u_i satisfying A^*, $A1^*$ and $A2^*$ $\iff$
$F \succsim_{SPGD(2)[\mathbf{z}]} G$.

Last stage $k = n$ of
(1) $F \succcurlyeq_{SPGD(1)[\mathbf{z}]} G$ requires:

$$\gamma^F(p, \mathbf{z}) \leq \gamma^G(p, \mathbf{z}) \text{ for all } p \in [0, 1].$$

(2) $F \succcurlyeq_{SPGD(2)[\mathbf{z}]} G$ requires:

$$\mathcal{PG}_F(p, \mathbf{z})) \leq \mathcal{PG}_G(p, \mathbf{z}) \text{ for all } p \in [0, 1].$$

This condition is neither obtained in Atkinson (1992) & related papers, nor for rank-dependent poverty evaluations.

Variable poverty lines

Poverty lines changing within ranges $z_i \in \left[0, z_i^+\right]$ with $z_i \geq z_{i+1} \geq 0$
$Z^n(\mathbf{0}, \mathbf{z}^+)$: set of all ordered poverty lines $z_1 \geq z_2 \geq ... \geq z_n \geq 0$ satisfying $z_i \in \left[0, z_i^+\right]$
Let $\Delta_i^{[1]}(x) := q_i^F F_i(x) - q_i^G G_i(x)$,
and $\Delta_i^{[2]}(x) := \int_0^x \Delta_i^{[1]}(t)dt$ for groups $i = 1, 2, ..., n$.
Algorithm: For $i = 0, 1, 2, ..., n-1$ let

$$\tilde{\Delta}_{i+1}^{[j]}(x; \mathbf{0}, \mathbf{z}^+) := \Delta_{i+1}^{[j]}(x) + \hat{\Delta}_i^{[j]}(x; \mathbf{0}, \mathbf{z}^+) \quad \text{for all } x \in [0, z_{i+1}^+], \tag{1}$$

where $\hat{\Delta}_0^{[j]}(x; \mathbf{0}, \mathbf{z}^+) := 0$ for all x, while for $i = 1, 2, ..., n$ define

$$\hat{\Delta}_i^{[j]}(x; \mathbf{0}, \mathbf{z}^+) := \max_{t \in [x; z_i^+)} \{\tilde{\Delta}_i^{[j]}(t; \mathbf{0}, \mathbf{z}^+)\}. \tag{2}$$

Theorem

Let $j = 1, 2$, and $z_i^- = 0$ for all i. The following statements are equivalent:
(i) $F \succsim_{SPGD(j)[\mathbf{z}]} G$ for all $\mathbf{z} \in Z^n(\mathbf{0}, \mathbf{z}^+)$,
(ii) $\hat{\Delta}_i^{[j]}(0; \mathbf{0}, \mathbf{z}^+) \leq 0$ for all $i = 1, 2, ..., n$.

if $\mathbf{z}^+ = \bar{z}\mathbf{1}$ then the algorithm corresponds to *SPGD* of order $j = 1, 2$ for all ranked poverty lines in $[0, \bar{z}]$. If $j = 2$ and $q_i = q_i^F = q_i^G$ for all i it coincides with the algorithm suggested in Bourguignon (1989) for welfare dominance.

Definition (Bourguignon-Dominance)

For all F, G $s.t.$ $q_i^F = q_i^G = q_i$

$$F \succcurlyeq_{BD} G \Longleftrightarrow \sum_{i=1}^{n} q_i \int_0^{x_i} [F_i(t) - G_i(t)]dt \leq 0$$

for all $x_1, x_2, ..x_n$ s.t. $\bar{x} \geq x_1 \geq x_2 \geq ... \geq x_n \geq 0$. Bourguignon (J.o.Econometrics 1989)

If $q_i = q_i^F = q_i^G$, $SPGD(2)$ is equivalent to BD when considering only $x_1, x_2, ..x_n$ $s.t.$ for a given vector of ordered poverty lines $\mathbf{z}$:

$$\begin{aligned} z_i - x_i &= c \geq 0, \quad x_j = 0 \\ \text{for all } c &\geq 0, \text{ all } i \leq k, \text{ all } j > k, \text{ and all } k = 1, 2, ..n. \end{aligned}$$

If $q_i = q_i^F = q_i^G$, $SPD(2)$ is equivalent to BD when considering only $x_1, x_2, ..x_n$ $s.t.$ for a given vector of ordered poverty lines $\mathbf{z}$:

$$\begin{aligned} 0 &\leq x_i = x_j \leq z_k, \quad x_t = 0 \\ \text{for all } i,j &\leq k, \text{ all } t > k \text{ and all } k = 1, 2, ..n. \end{aligned}$$

NOW ERWIN TALK