Approaches to Inequality Measurement Within and Between Households

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Inequality Measurement

- Set of distributions $\{F_i(y)\}_{i=1}^n$, $n \geq 1$
- Apply measure of inequality/poverty to each $F_i(y)$
- For $n > 1$ compare measures/distributions

Why? Assumption:

- Wellbeing increases monotonically with y.
- Inequality in $y \Rightarrow$ welfare inequality

But if we want to draw welfare implications, problem:

- relevance to policy formulation, design and implementation?
- When y is a measure of household income or consumption
- And policy is concerned with **individual** wellbeing
- Then, as long-recognised, we need individual rather than household y

- If y is income or consumption, this not usually available, so construct it

$$y' \equiv y/d(.) \tag{1}$$

- where for example $d(.)$ claims to correct for household public goods/economies of scale and/or demographic composition
- Implicitly assumes within-household distribution not a matter of concern
- Household distributes income exactly as "planner would like", e.g. equally.

- Haddad and Kanbur 1990: is a quantitative question
- Case study: Standard approach understates individual inequality significantly
- Need to model household more seriously.
- In particular need welfare economics of multi-person household
- Inequality measurement theory and practice has only relatively recently begun to take account of the developments in household modelling of the last 4 or 5 decades.

Lise and Seitz 2010: Use "collective model" to show:

- Initial inequality (1965) greatly understated
- Growth in inequality (1965-2001) greatly overstated when we take account of fall in within-household inequality
- Essentially take the ratio of after-tax wage rates of male and female as measure of within-household inequality
- This measure high in 1960's but fell steadily over the period to 2001

Facts About Households

Composition:

- Singles
- Singles with kids
- Couples $\pm$ kids - families*
- Extended families
- Wohngemeinschaften

Decision variables

- Fertility: number and timing of kids
- Time allocation: labour supplies, child care, household production, leisure
- Consumption, saving/investment (human capital, housing, financial assets)

All over the "Family Life Cycle"

Heterogeneity

- Especially of female (second earner) labour supply over the life cycle
- Roughly 3-way split between full-time, part-time and zero market labour supply
- Begins when couple first has kids
- Important persistence over rest of life cycle

Research Question

How do we construct models of households (families) which

- explain the decisions and their outcomes,
- in a way that is relevant and useful for the measurement of inequality/poverty
- and public policy - especially taxation - more generally?

Modelling the Household

It all starts with Samuelson 1956

- Think of the household as a small economy

(in his case, a pure exchange economy, but this easily generalised)

- Cooperative household can be modelled as maximising a form of Social Welfare Function (HWF)

$$W = W(u_1, u_2, .., u_n) \tag{2}$$

concave and strictly increasing, differentiable as required

- (Becker 1965 assumed this is the utility function of the "head of the household")

- This approach - called the "unitary model" - rejected by later authors - why?
- Simple example:

$$u_i = u_i(x_i, l_i) \quad i = 1, 2 \tag{3}$$

- So the problem becomes

$$\max_{x_i, l_i} W(u_1, u_2) \quad \text{s.t.} \quad \sum_i (x_i + w_i l_i) \leq \sum_i (w_i + \mu_i) \tag{4}$$

Then we have restrictions on household demands $x(w_1, w_2, \mu_1, \mu_2)$, $l_i(w_1, w_2, \mu_1, \mu_2)$:

- Symmetric, negative semidefinite Slutsky matrix
- and

$$\frac{\partial x}{\partial \mu_1} = \frac{\partial x}{\partial \mu_2}; \quad \frac{\partial l_i}{\partial \mu_1} = \frac{\partial l_i}{\partial \mu_2} \quad i = 1, 2 \tag{5}$$

- Misleadingly called "the pooling hypothesis"
- Better would be "anonymity".
- These implications of Samuelson's approach rightly rejected.

Note an important contribution of Samuelson:

- the **Sharing Rule formulation**

Let $v_i(w_i, s_i)$ be the indirect utility function derived from

$$\max_{x_i, l_i} u_i(x_i, l_i) \;\; s.t. \; x_i + w_i l_i \leq s_i \;\; i = 1, 2 \tag{6}$$

and consider the problem

$$\max_{s_i} W[v_1(w_1, s_1), v_2(w_2, s_2)] \;\; s.t. \; \sum_i s_i \leq \sum_i (w_i + \mu_i) \tag{7}$$

- Gives share functions $s_i(w_1, w_2, \mu_1, \mu_2)$ as optimal solutions.

- Then Samuelson shows that solving the problem in (7) with

$$s_i \equiv s_i(w_1, w_2, \mu_1, \mu_2) \tag{8}$$

is equivalent to solving the problem in (4).

- Moreover the existence of such sharing functions - the "sharing rule" - is equivalent to the existence of a HWF.
- Sharing rule of course gives the household income/consumption distribution

- Note further that the functional forms of the HWF and the individual utility functions in general determine the form of the sharing rule.

(See Mas Colell et al ch 4)

- Choice of any two of HWF, utility functions and sharing rule determines form of third.
- The above also generalises to $n > 2$ individuals
- (Samuelson and Mas-Colell et al do it for the entire economy)

The Nash-Bargaining Approach and the Collective Model

- Nash bargaining approach can be thought of as specifying the HWF

$$W = (u_1 - u_1^0)(u_2 - u_2^0) \tag{9}$$

where $u_i^0 = v_i(w_i, \mu_i, \mathbf{e})$ is i's "threat point" utility,

- **e** is a vector of "exogenous environmental parameters" (McElroy)
- or "distributional variables" (Browning and Chiappori)
- that enter neither the utility functions nor budget constraint
- but influence the household utility distribution.

- The "collective model" (of Browning/Chiappori) specifies the HWF

$$W = \lambda(.)u_1 + [1 - \lambda(.)]u_2 \tag{10}$$

where

$$\lambda(.) \equiv \lambda(w_1, w_2, \mu_1, \mu_2, \mathbf{e}) \tag{11}$$

- Note: the "collective model" in general requires more than just "the assumption of Pareto efficiency".

- So why not just generalise Samuelson by assuming

$$W = W(u_1, u_2, .., u_n; \mathbf{p}, \mathbf{w}, \boldsymbol{\mu}, \mathbf{e}) \tag{12}$$

p is a price vector, **w** vector of wage rates, $\boldsymbol{\mu}$ vector of nonwage incomes, and **e** as before

- Samuelson's equivalence results go through, with share functions $s_i(\mathbf{p}, \mathbf{w}, \boldsymbol{\mu}, \mathbf{e})$
- No longer get Slutsky symmetry and anonymity
- Any *HWF* must specify how the elements of $\mathbf{p}, \mathbf{w}, \boldsymbol{\mu}, \mathbf{e}$ affect the marginal rates of substitution $(-W_i / W_1)_{W^0}$ for $i = 2, ..., n$.

- Note that the sharing rule is a function of exogenous nonwage income
- **not earned income** $w_i(1 - l_i)$, since this is **endogenous**
- Making the sharing rule depend on endogenous income requires a different model (e.g. Basu 2006)
- Note finally that all these formulations are examples of "price-dependent preferences"
- Old literature (see Pollak 1977) shows that in this case Slutsky matrix no longer symmetric and negative semidefinite
- Browning/Chiappori 1998 give a special case of this result.

Child Care and Household Production

- As in much of the literature, the above discussion left out children and household production
- Assumed the only uses of adult time were market work and leisure
- Data suggest that this gives a very limited view of the household
- Easy however to extend the model to include these.

- Household (types) $h = 1, 2, .., H$, each consisting of two adults, a primary and a second earner, $i = 1, 2$
- And a child, labelled $i = 3$
- Each consumes a market good x, a household public good X, a domestically produced good y
- the child consumes child care c
- Goods c and y produced by combining parental time inputs t_i^c, t_i^y, $i = 1, 2$, with bought in market goods z^c, z^y.

- Production functions, linear homogenous and strictly quasiconcave

$$c_h = \phi^c(t^c_{1h}, t^c_{2h}, z^c_h, k^c_h) \tag{13}$$

$$y_h = \phi^y(t^y_{1h}, t^y_{2h}, z^y_h, k^y_h) \tag{14}$$

- k^c_h, k^y_h may differ across households - differences in productivities due to differences in endowments of human and physical capital.

- Minimizing the costs of producing one unit of c_h and y_h we get implicit prices

$$p_h^c = \gamma^c(w_{1h}, w_{2h}, q_h^c, k_h^c) \tag{15}$$

$$p_h^y = \gamma^y(w_{1h}, w_{2h}, q_h^y, k_h^y) \tag{16}$$

- q_h is the price of the market input, which may vary with h
- $\gamma(.)$ is a unit cost function, strictly increasing in its arguments

- Adult utility functions are $u^i(x_{ih}, X_h, y_{ih,} l_{ih})$, $i = 1, 2$
- Child's utility function is $u^3(x_{3h}, X, y_{3h}, c_h)$
- The *household full income constraint is*:

$$\sum_{i=1}^{3}(x_{ih} + p_h^y y_{ih}) + \sum_{i=1}^{2} w_{ih} l_{ih} + \pi_h X_h + p_h^c c_h \leq \sum_{i=1,2} (w_{ih} + \mu_{ih}) \quad (17)$$

The household chooses resource allocation x_{ih}, X_h, l_{ih} y_{ih}, c_h to solve:

$$\max W(u^1(.), u^2(.), u^3(.); \mathbf{p}^h, \mathbf{w}^h, \boldsymbol{\mu}^h, \mathbf{e}) \quad \text{s.t. } (17) \tag{18}$$

where $\mathbf{p}^h = [1, \boldsymbol{\pi}_h, p_h^y, p_h^c]$ is the vector of prices, which may be household-specific

Solution gives individual and aggregate demands and time allocations

- The value function of this problem is $V^h = V(\mathbf{p}^h, \mathbf{w}^h, \boldsymbol{\mu}^h, \mathbf{e})$
- This can be called the household's indirect welfare function
- It is a complete representation of the aggregate utility possibilities of the household
- It can be made the basis for household welfare rankings
- It can also be used to define equivalent incomes

Samuelson's sharing rule approach can still be applied

- X_h^* denotes the optimal level of the public good for the household
- $v_{ih}(p_h^y, w_{ih}, X_h^*, s_{ih})$, $i = 1, 2$, and $v_{3h}(p_h^y, p_h^c, X_h^*, s_{ih})$ are indirect utility functions, derived by solving

$$\max_{x_{ih}, y_{ih}, l_{ih}} u^i(x_{ih}, X_h^*, y_{ih}, l_{ih}) \tag{19}$$

$$\text{s.t. } x_{ih} + p_h^y y_{ih} + w_{ih} l_{ih} \leq s_{ih} \quad i = 1, 2 \tag{20}$$

and

$$\max_{x_{3h}, y_{3h}, c_h} u^i(x_{3h}, X_h^*, y_{3h}, c_h) \tag{21}$$

$$\text{s.t. } x_{3h} + p_h^y y_{3h} + p_h^c c_h \leq s_{3h} \tag{22}$$

- Then we solve the problem

$$\max_{s_{ih}} W(v_{1h}(.), v_{2h}(.), v_{3h}(.); \mathbf{p}^h, \mathbf{w}^h, \boldsymbol{\mu}^h, \mathbf{e}) \tag{23}$$

$$\text{s.t.} \ \sum_i s_{ih} \leq \sum_i (w_{ih} + \mu_{ih}) - \pi_h X_h^* \tag{24}$$

for share functions $s_{ih}(\mathbf{p}^h, \mathbf{w}^h, \boldsymbol{\mu}^h, \mathbf{e}, X_h^*)$

- These again give the "within household income distribution"
- Note: although each household member consumes the same amount of the public good, they may not obtain the "same utility" from it.

What does this model tell us about inequality within and between households?

Within household income distribution given by the:

- Total value of market and household good consumptions
- plus, for the adults, the values of leisure consumption, at their market (net) wage rates,
- and for the child, the value of child care, at the implicit child care price.

Intrahousehold distribution of consumption is determined by:

- Prices, wage rates, exogenous nonwage incomes, and distributional variables
- Also depends on optimal expenditure on the household public good
- This determines the amount of household full income available for the "private" forms of consumption

Measuring household inequality by only **one** of the components of consumption, for example x_{ih}, may give a totally misleading picture of real inequality.

Measure of "leisure consumption" relevant for within household income distribution is:

- total time *minus* time spent in market labour supply, household production and child care
- Omitting these last two forms of time use may seriously bias measured inequality
- The optimal amount of the household public good is a weighted average of the individual Lindahl prices
- The weights are the partial derivatives W_i at the household optimum.

Across Household Inequality

- Consider the full income budget constraint (17)
- Its height is determined by the household's real full income
- This is what determines the household's attainable utility possibilities, whatever its actual equilibrium choice of consumptions

This budget constraint will be higher:

- The higher are non-wage incomes
- The higher the wage rates
- The lower are π_h,p_h^y, and p_h^c.

These latter two prices will be lower:

- The higher are productivities k_h^c, k_h^y
- The lower are prices of the bought in market inputs, q_h^c, q_h^y

How good is total household income as an indicator of a household's real full income?

Recall that the household indirect utility function $V^h = V(\mathbf{p}^h, \mathbf{w}^h, \mu^h, \mathbf{e})$ does not contain household income, because of endogeneity of labour supplies

In general problematic, because of:

- Variation across households in prices, especially q_h^c, and productivities
- Variation in second earner labour supply across households

May both cause negative relationship between real full income and household labour income

Example:

- Household A, single earner, income \$58,000, wife works entirely at home
- Household B, two full time earners, income \$60,000.

A could well have higher real full income if value of household production from (potential) second earner sufficiently high.

- Might expect that lower q_h^c, q_h^y would increase second earner market labour supply and therefore household income
- Likewise for productivities k_h^c, k_h^y
- But this does not hold in general because of opposing scale and substitution effects.
- Households with higher real full income could have lower market income because of lower second earner labour supply and higher value of household production and child care.

Conclusion

Purpose of modelling within-household time and consumption allocations in the presence of household production is:

- Not only to analyse within household inequality
- But also to allow us to analyse rigorously the way in which second earner labour supply heterogeneity conditions the relationship between households' labour incomes and their real full income and utility possibilities
- This analysis casts doubt on the idea of a simple monotonically-increasing relationship between household income and wellbeing that underlies conventional inequality measurement approaches
- Suggests we should place far more emphasis on wage rates and price variation across households, especially of child care for households in the relevant phases of the life cycle.

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