# *Theory and Survey Testing for Intra- and Inter-household Welfare Comparisons*

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January 10, 2011

# Intra- and Inter-household Welfare Comparisons

- **Difficulties** when we try to connect:
  - **Individual** well-being with **household-level** well-being
  - **Household-level** well-being with **economy-wide** well-being

# Difficulties in connecting decisions to exogenous characteristics

- Demand functions (also labor-supply functions) depend on:
  - prices, incomes and characteristics of each and every individual in the household
- Data (typically) available at the household level
  - We must understand how we go from individial incentives to the household-level incentinves if we want to understand economy-wide incentives and policy evaluation

# Problems caused by such difficulties in Market Clearing

- **Individual** or household-level demand functions may not add up well
  - Anything-goes theorem Mantel-Sonnenschein-Debreu
- Even aggregate demand functions are well-behaved, welfare comparisons of alternative policies is fragile
  - Social indifference curves may intersect as policies change the underlying distribution

# Where to read about these problems

- Samuelson (QJE 1956): non-intersecting social indifference curves are "rare birds" (see Scitovsky's community indifference contours)
- The same problems apply when we try to move from the individual level to the household level
- Jerison (REStud 1994): one must restrict the shape of income distributions in order to restore non-intersecting social indifference curves (see "optimal income distribution rules")

# But earlier than that:

- Gorman (ECMA 1953): exact linear aggregation is logigally equivalent ("if and only if") to non-intersecting indifference curves **without any distributional restrictions**

- The idea is that a **representative consumer (RC)** can be constructed from a community preference profile

# RC's Existence

- Hinges upon the structure of preferences (Gorman polar form)

- Assuming that there are $\mathcal{I}$ types of utility functions, $\left\{U^i\right\}_{i\in\mathcal{I}}$, in a community,

  - the idea is

$$\text{use} \quad \left\{U^i\right\}_{i\in\mathcal{I}} \quad \overset{\text{construct}}{\longrightarrow} \quad U^{RC}\Big|_{\left\{U^i\right\}_{i\in\mathcal{I}}}$$

# RC's Appeal

- Aggregate data addressed through RA models may be quite informative about economy-wide incentives

- Rationalizes how agents plan in HA models

# What to do with the RC Concept

- It is a very appealing concept: resolves all demand-aggregation considerations, and policy-comparability concerns
- It is **exceptionally precise**
- Research the concept theoretically in order to build a **falsification test**. Invent an empirical method in order to perform the falsification test.

# Focus in Koulovatianos (2005) and Koulovatianos, Schröder and Schmidt (2010)

- Individuals living in multi-member households share goods within the household
- Multi-member households plan ahead counting on household-size economies
- The objective function of multi-member households $\neq$ this of one member households
- Taking demographics seriously $\Rightarrow$ we understand heterogeneity in household-type objectives

# Goals

- **Theory**: "How much" preference heterogeneity can be survived by RC?
  - Care not only about the **functional forms** of $\left\{U^i\right\}_{i\in\mathcal{I}}$ $\xrightarrow{\text{construct}}$ $U^{RC}\big|_{\left\{U^i\right\}_{i\in\mathcal{I}}}$, but also about the **degree of parametric heterogeneity** in $\left\{U^i\right\}_{i\in\mathcal{I}}$ : **necessary and sufficient conditions**
  - Derive **testable implication that can falsify RC**
- **Empirics**: Give a very hard time to RC!

# Theory

- **Dynasties of unitary households**

- preferences of each $i \in \mathcal{I}$:

$$U^i\left(\left(c^i(t)\right)_{t\geq 0}\right) = \int_0^\infty e^{-\int_0^t \rho(\tau)d\tau} u^i\left(c^i(t), t\right) dt \,.$$

  with $\rho : \mathbb{R}_+ \to \mathbb{R}_{++}$.

- budget constraint:

$$\dot{a}^i(t) = r(t)\, a^i(t) + \theta^i(t)\, w(t) - c^i(t) \ ,$$

# Theory

- **Theorem 2** *Under Assumptions 2, 3, and 5 through 7, a representative consumer exists iff*

$$u^i(c,t) = \begin{cases} \dfrac{\left(\alpha c + \beta^i(t)\right)^{1-\frac{1}{\alpha}} - 1}{\alpha\left(1-\frac{1}{\alpha}\right)} & \text{with } \alpha > 0 \text{ and } \beta^i(t) \in \mathbb{R} \\ & \text{or } \alpha < 0 \text{ and } \beta^i(t) \in \mathbb{R}_{++} \\ \text{or} & \\ -e^{-\frac{1}{\beta_i G(t)} c} & \text{with } \beta_i \in \mathbb{R}_{++} \text{ and } G : \mathbb{R}_+ \to \mathbb{R}_{++} \end{cases}$$

*for all $i \in \mathcal{I}$, with functions $\beta^i(t)$ such that Assumptions 6 and 7 are met.*

# Theory

- **Theorem 2 (cont'd)** *...The representative consumer has*

$$U^{RC}\left(\left(c\left(t\right)\right)_{t\geq 0},t\right)=\int_{0}^{\infty}e^{-\int_{0}^{t}\rho(\tau)d\tau}u^{RC}\left(c\left(t\right),t\right)dt\,,$$

with,

$$u^{RC}\left(c,t\right)=\begin{cases}\dfrac{\left(\alpha c+\beta^{RC}(t)\right)^{1-\frac{1}{\alpha}}-1}{\alpha\left(1-\frac{1}{\alpha}\right)} & \text{for } \alpha\neq 0\,,\ \beta^{RC}\left(t\right)=\int_{\mathcal{I}}\beta^{i}\left(t\right)d\mu\left(i\right)\\ -e^{-\frac{1}{\beta_{RC}G(t)}c} & \text{else},\ \beta_{RC}=\int_{\mathcal{I}}\beta_{i}d\mu\left(i\right)\end{cases}$$

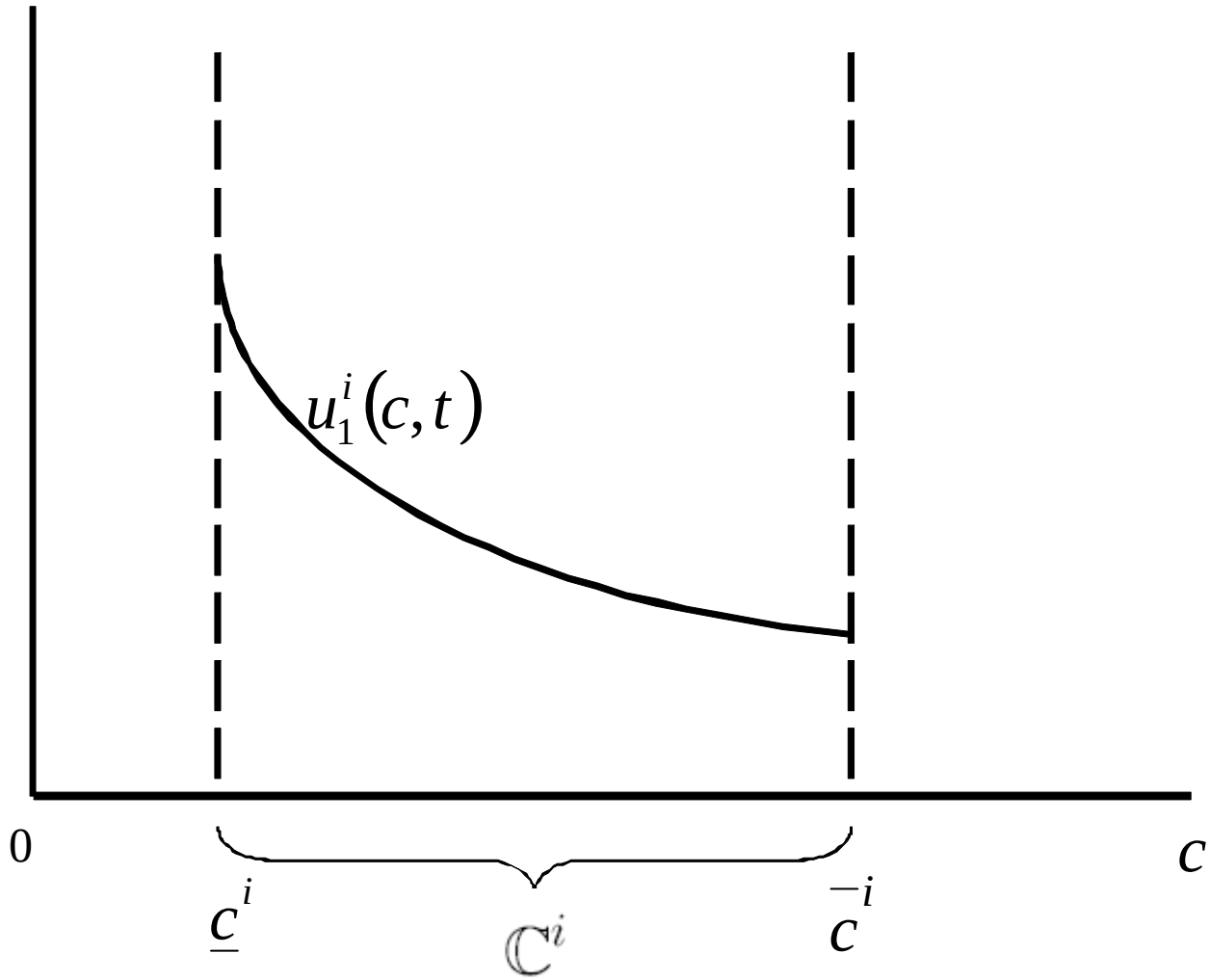
# Theory

- *Consumption decision rules* of all household types, $i \in \mathcal{I}$:

$$c^i(t) = b(t)\, a^i(t) + \zeta^i(t) \ ,$$

- *always linear in financial wealth,* $a^i(t)$*, and parallel across all households*

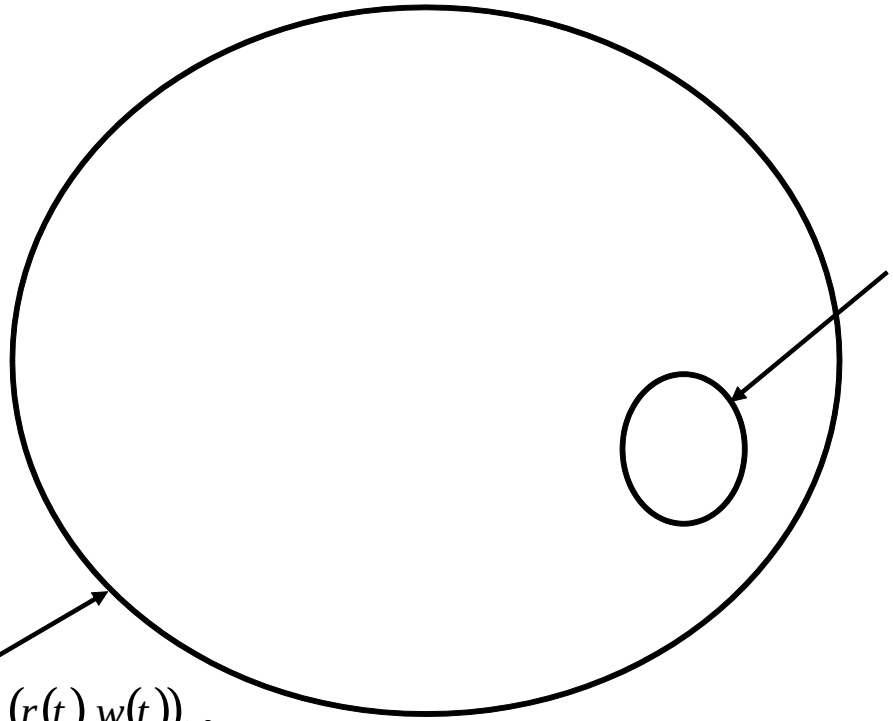
# Assumptions



# Proof Strategy

**Theorems 1 & 2**: Necessity part

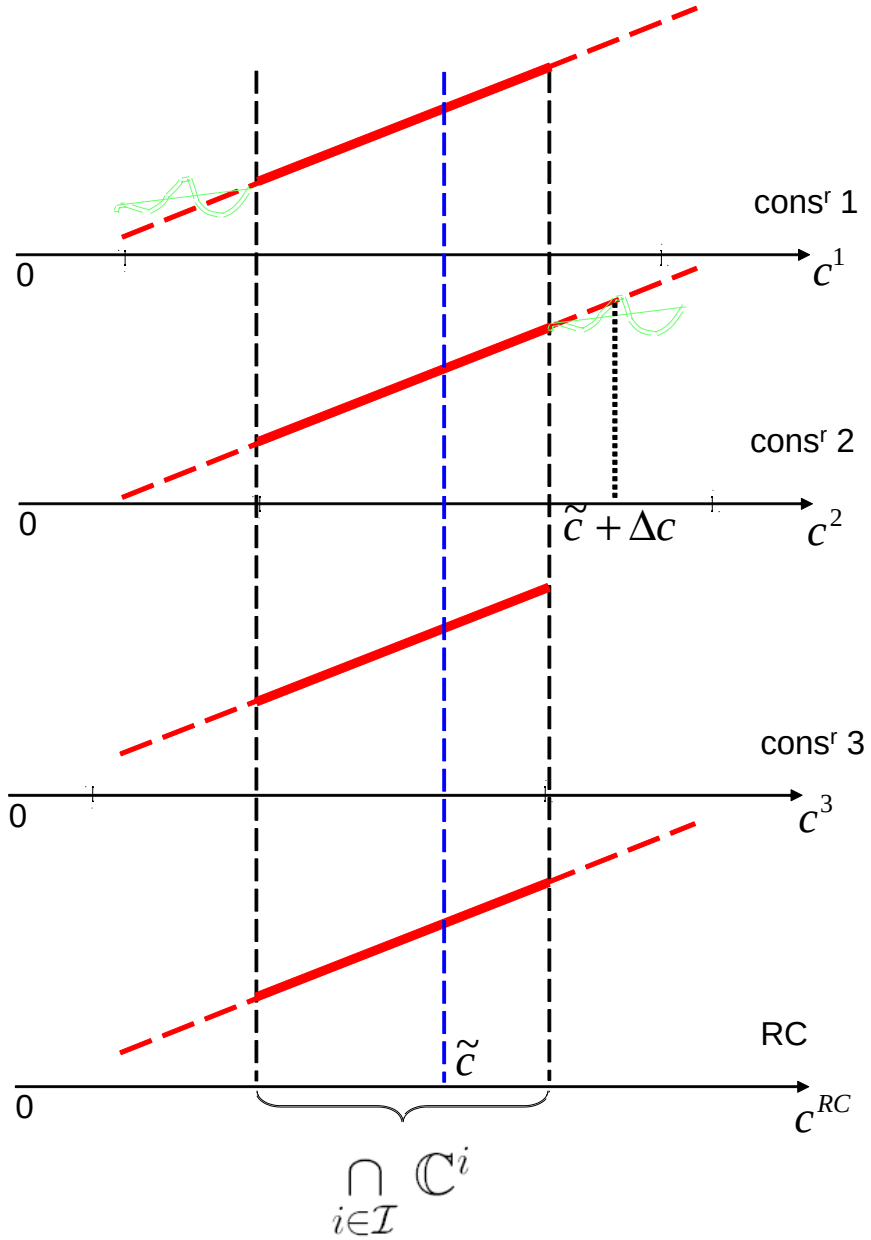
Fix any $(\rho(t))_{t\geq 0}$ and $(u^i)_{i\in I}$



Elements restricted by
$r(t)=\rho(t), t\geq 0$

Domain of $(r(t), w(t))_{t\geq 0}$,
$(a_0^i)_{i\in I}, (\theta^i(t))_{i\in I}$
where all consumers $i$
have interior solutions

# Proof Strategy



# Empirical Implication

- **Permanent-income scenario**:
  - let $\beta^i(t) = \beta_i$ (by fixing a family type over time),
  - let $r(t) = r = \rho = \rho(t)/t$
  - give each household its permanent labor income

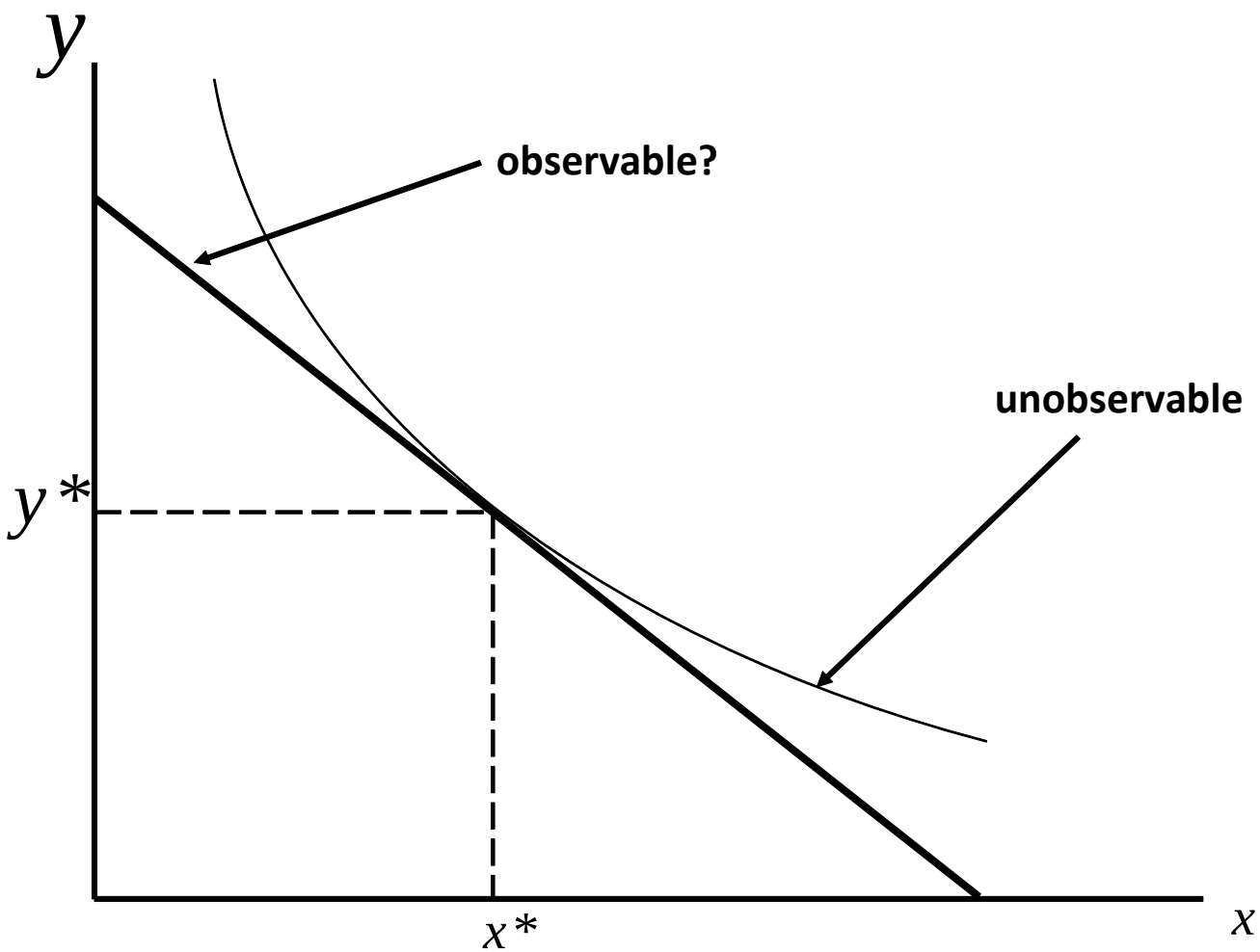
- Then:

$$EPI_i = \psi_{i,j} EPI_j + \chi_{i,j}$$

- where **EPI = equivalent permanent income**: income that equates the material comfort of household members across different family types (see that Donaldson and Pendakur (2006) use such a relationship)

# Empirics

- **Why a survey and not a demand system:**



# Questionnaire structure

| | | | |
|---|---|---|---|
| Single adult household without a child | Reference income, e.g. 1000 Euros per month | Two adult household without a child | ? |
| One parent household with 1 child | ? | Two parent household with 1 child | ? |
| One parent household with 2 children | ? | Two parent household with 2 children | ? |
| One parent household with 3 children | ? | Two parent household with 3 children | ? |

# Empirical Investigation

- **Pilot Samples**
  - Germany 1999: 167 respondents
  - Cyprus 2000: 130 respondents
  - France 2002: 223 respondents
  - China 2004: 196 respondents
  - India 2005: 214 respondents
  - Botswana 2005: 159 respondents

# Raw responses

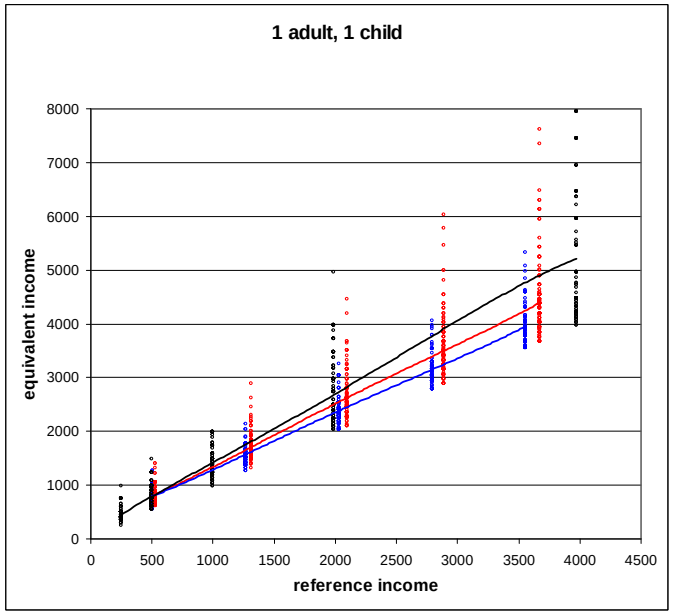
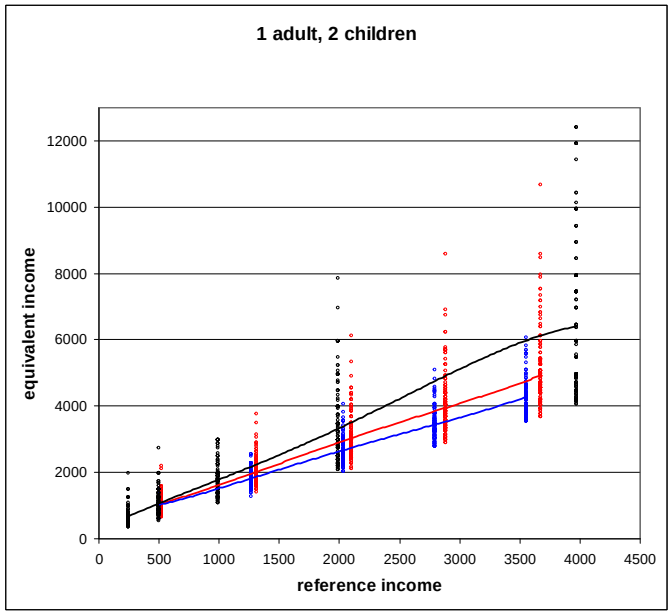
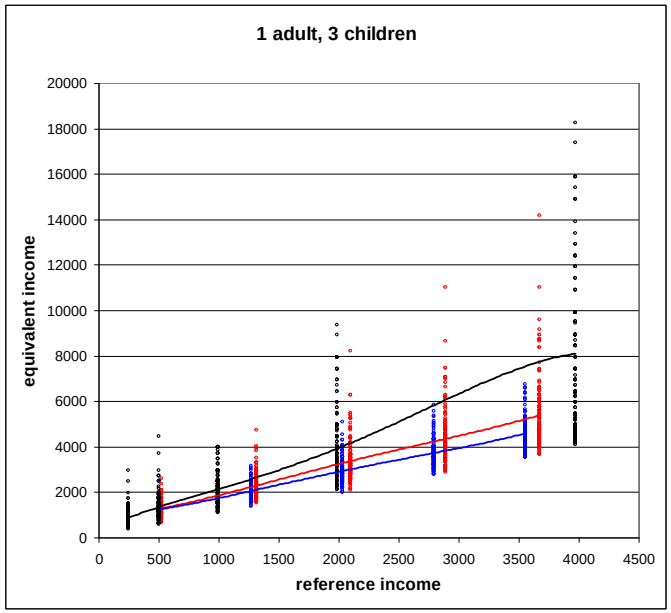
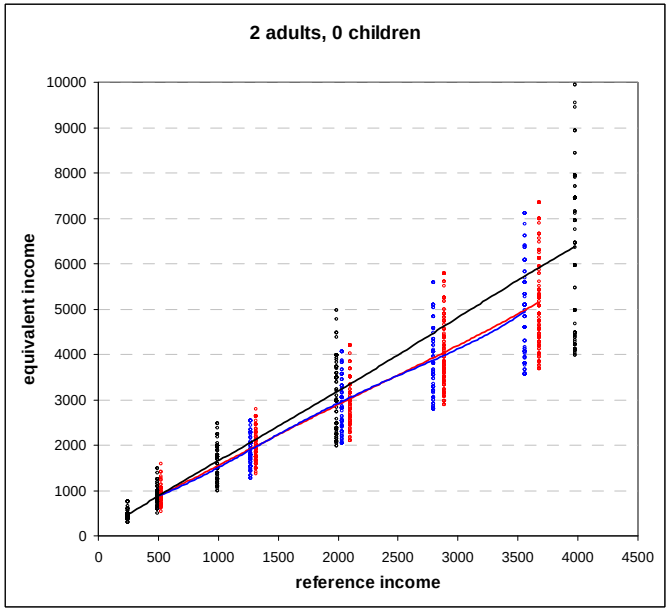


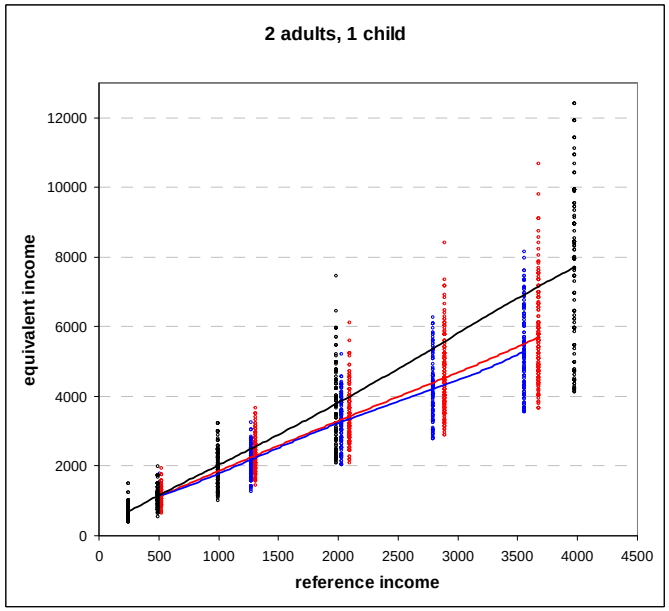

**Figure 1b.** Scatter plots of stated equivalent incomes.
$6^{th}$ degree polynomial fit
France
China
Germany

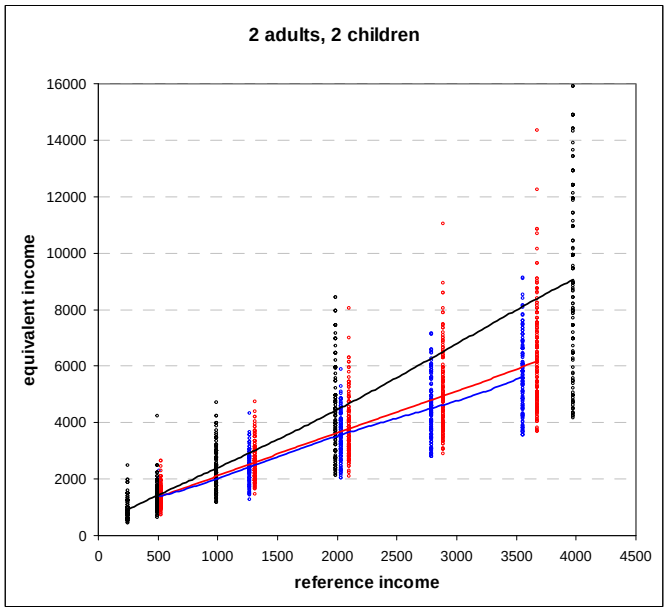


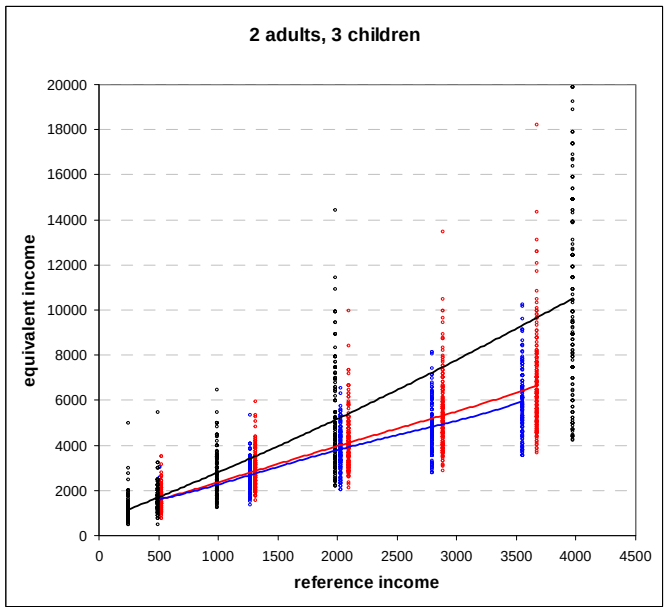



# Raw responses



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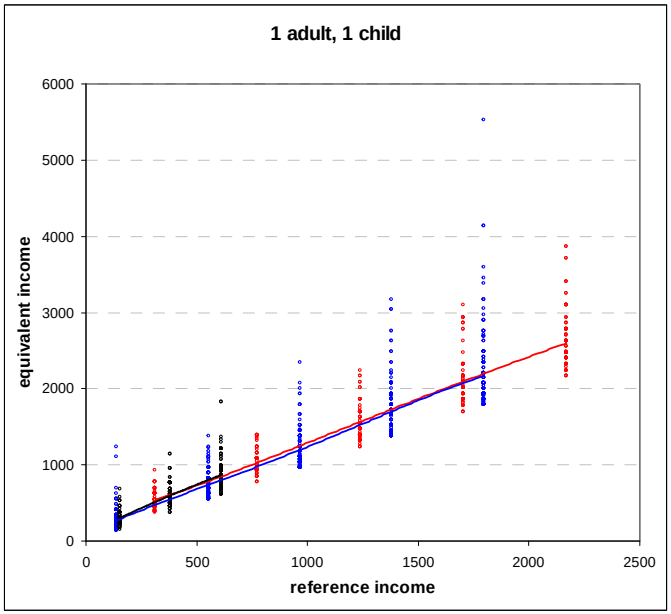
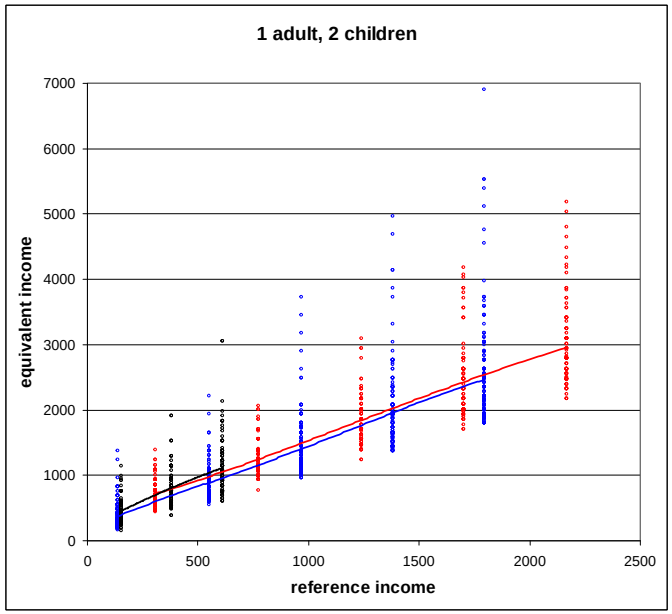
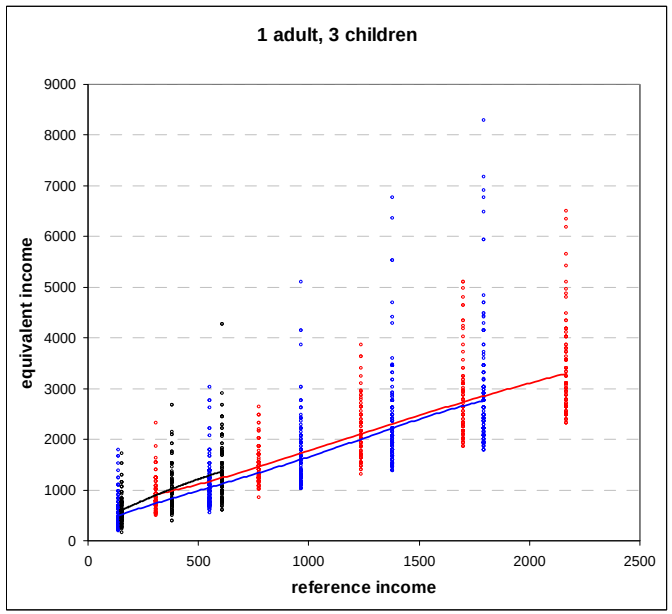



**Figure 1a.** Scatter plots of stated equivalent incomes.
$6^{th}$ degree polynomial fit
Cyprus
Botswana
India



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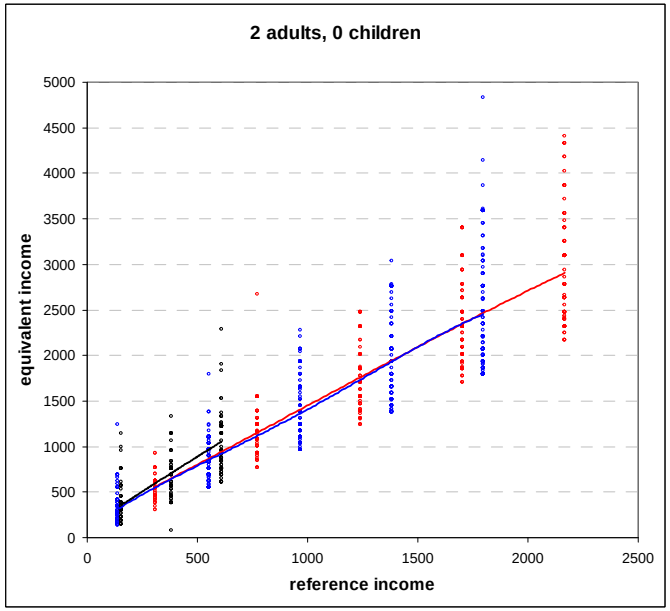
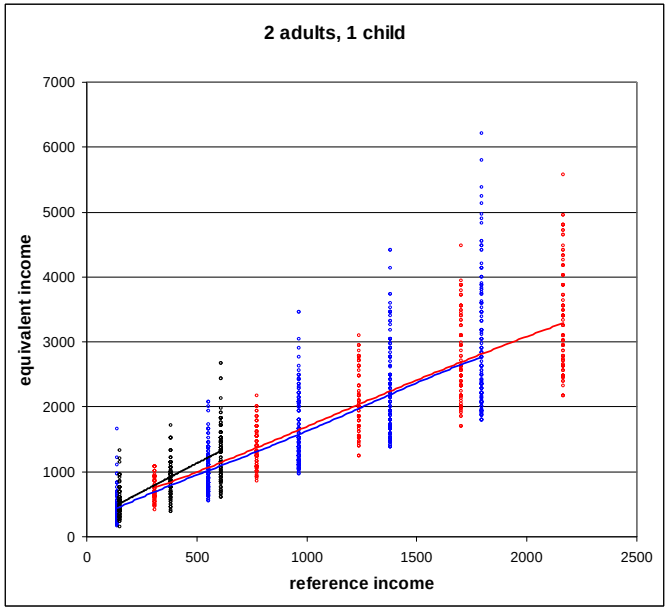
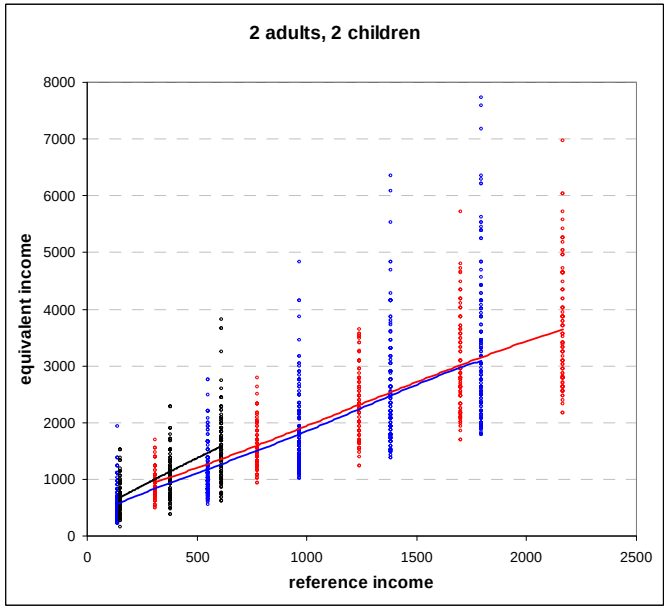
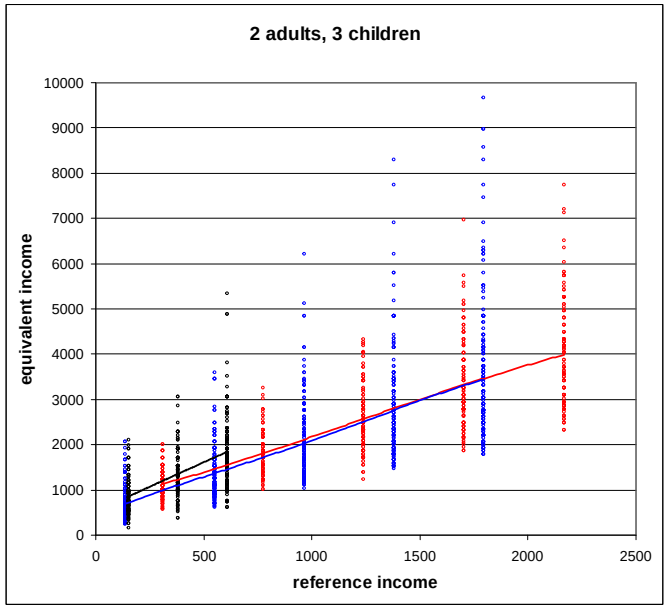


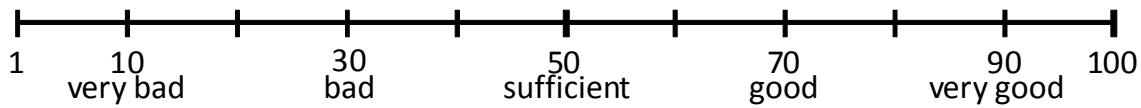



# The German Representative Sample

- Collected by FORSA in late 2006
- We randomly provided **only one reference income to each respondent**
- We examine the **effectiveness of the survey instrument**

# What we do

- Equivalent assessment task




Level of material comfort

# How it works

- Negative correlation

| | 1A0C | 1A1C | 1A2C | 1A3C | 2A0C | 2A1C | 2A2C | 2A3C |
|---|---|---|---|---|---|---|---|---|
| EI | 500 | … | … | … | … | 1,200 | … | … |
| $LS_i$ | 20 | … | … | … | … | 30 | … | … |

$NLSE_i^{2A1C} = ln(30/20)$

$\rightarrow$ If *i* "**means what he/she says**", then $EI_i^{2A1C} < 1{,}200$.

# Raw responses

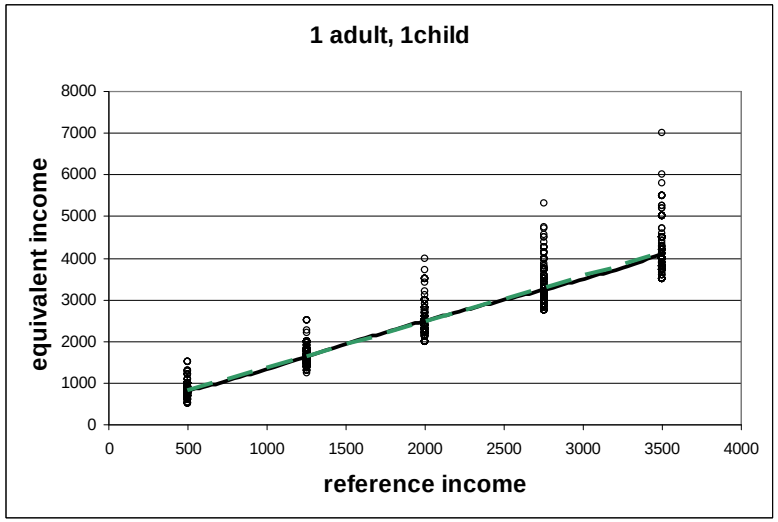
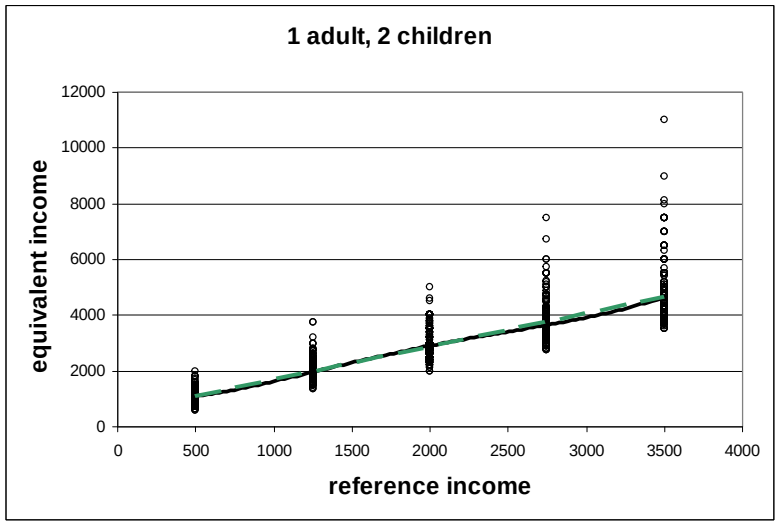
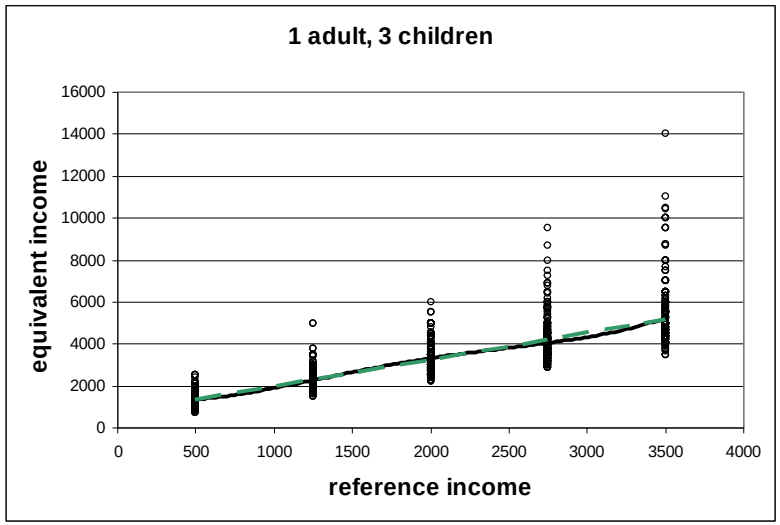



**Fig. 1.** Scatter plots of stated EIs in Part A of the survey for each RI and each family type.

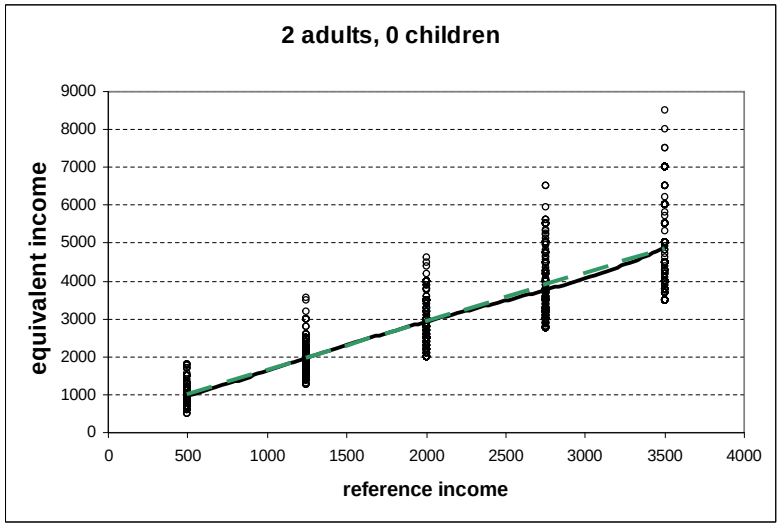
—— $6^{th}$ degree polynomial fit.

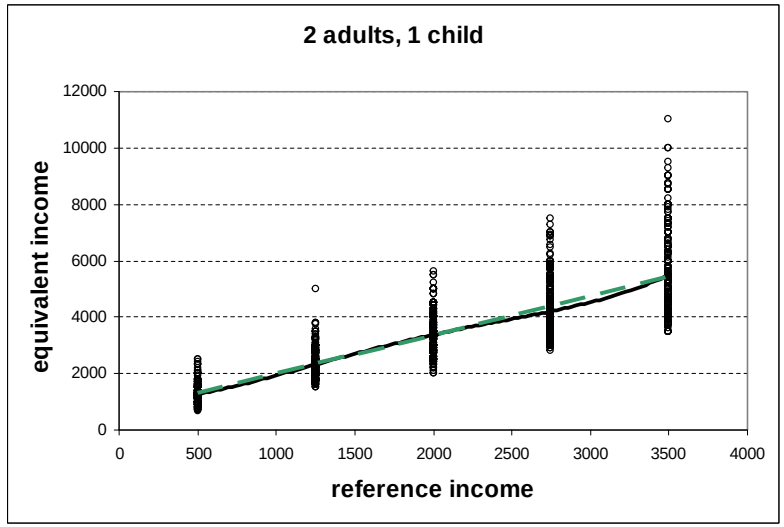
- - - linear regression.

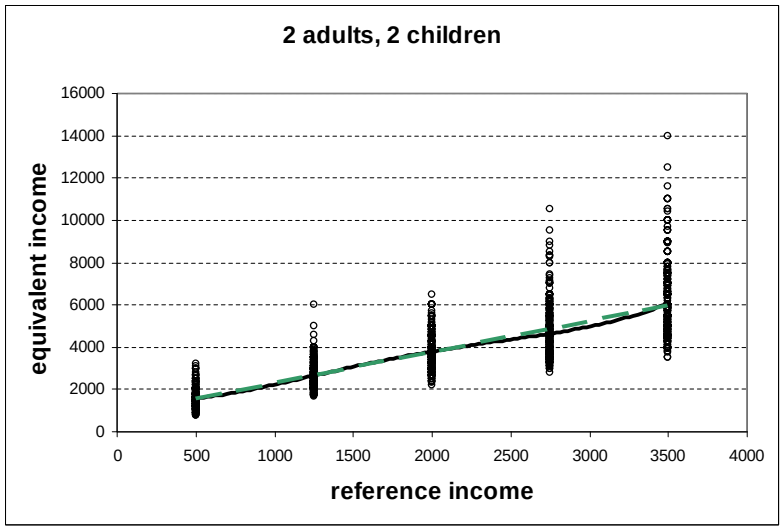


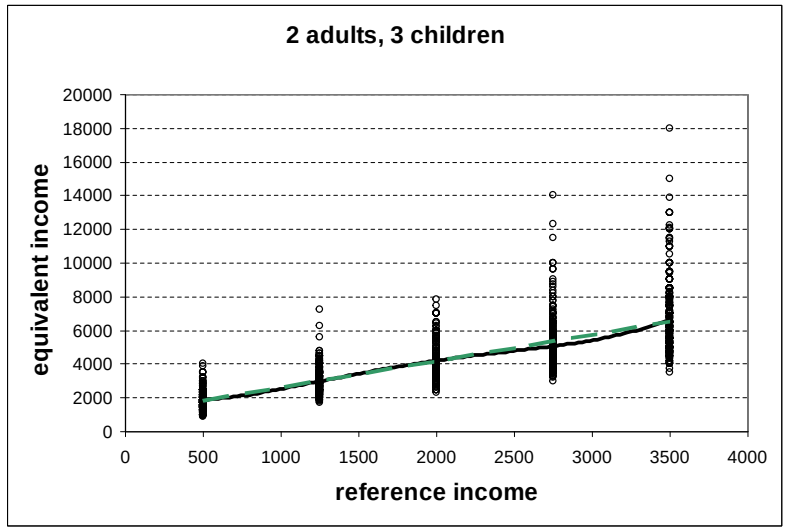



# Raw responses



# Regression Analysis

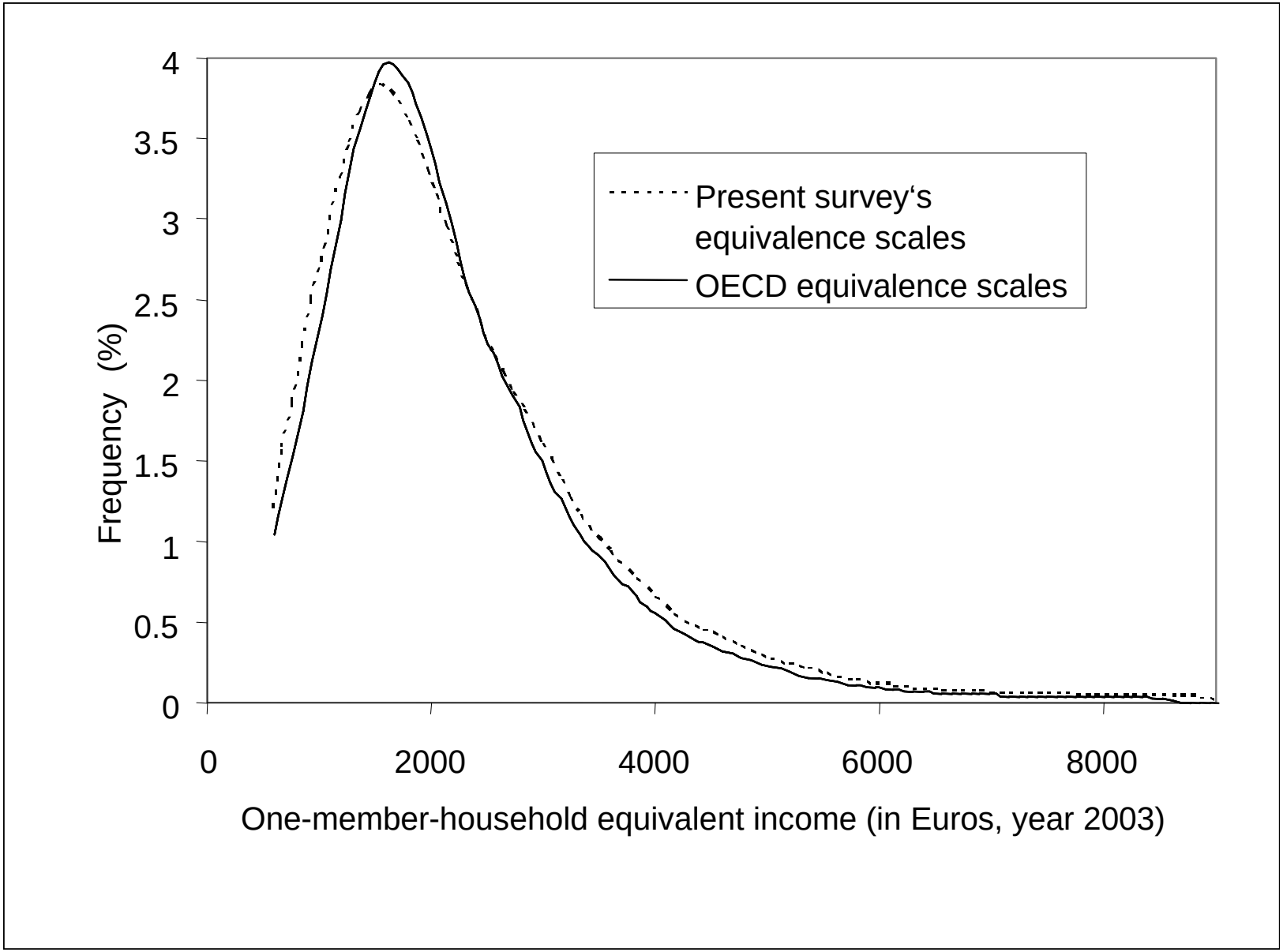
## Endogenous variable: equivalence scale ($EI_h / EI_{OMH}$)

| | Household type | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 adult, 1 child | 1 adult, 2 children | 1 adult, 3 children | 2 adults, 0 children | 2 adults, 1 child | 2 adults, 2 children | 2 adults, 3 children |
| Constant | 1.06*** | 1.12*** | 1.20*** | 1.42*** | 1.44*** | 1.53*** | 1.61*** |
| Reciprocal of reference income | 269.74*** | 498.34*** | 728.85*** | 329.38*** | 592.99*** | 839.25*** | 1,079.86*** |
| Dummy reference income equals 1,250 Euros | 0.00 | -0.00 | -0.02 | 0.03 | 0.00 | -0.02 | -0.04 |
| Dummy reference income equals 2,000 Euros | 0.02* | 0.02 | 0.02 | 0.00 | -0.00 | -0.00 | -0.02 |
| Dummy reference income equals 2,750 Euros | -0.02* | -0.04** | -0.07** | -0.05* | -0.08** | -0.11*** | -0.13*** |
| Normalized Likert-scale evaluation | -0.04*** | -0.07*** | -0.10*** | -0.05*** | -0.07*** | -0.09*** | -0.13*** |
| Same family type of respondent | 0.04 | -0.01 | -0.14* | 0.02 | 0.02 | 0.01 | 0.01 |
| Same living standard of respondent | -0.01 | -0.03 | -0.03 | -0.04 | -0.00 | -0.03 | -0.05 |
| Same family type and living standard of respondent | -0.06 | 0.13 | -0.03 | 0.05 | -0.16* | -0.02 | -0.04 |
| Adjusted $R^2$ | 0.46 | 0.53 | 0.54 | 0.30 | 0.46 | 0.52 | 0.54 |
| F test statistic for exclusion of all reference-income dummy variables | 2.36 [0.07] | 3.07* [0.03] | 3.29* [0.02] | 3.60* [0.01] | 3.37* [0.02] | 3.45* [0.02] | 3.51* [0.01] |

# Findings

- Repondents mean what they say
- They can evaluate household types and living standards different from their own very well
- Personal characteristics do not influence assessments (only education, but slightly and only for families with children)
- Evidence is rather in favor of RC!!!

# Germany: disposable incomes



# Momentary utility of the German RC in 2003

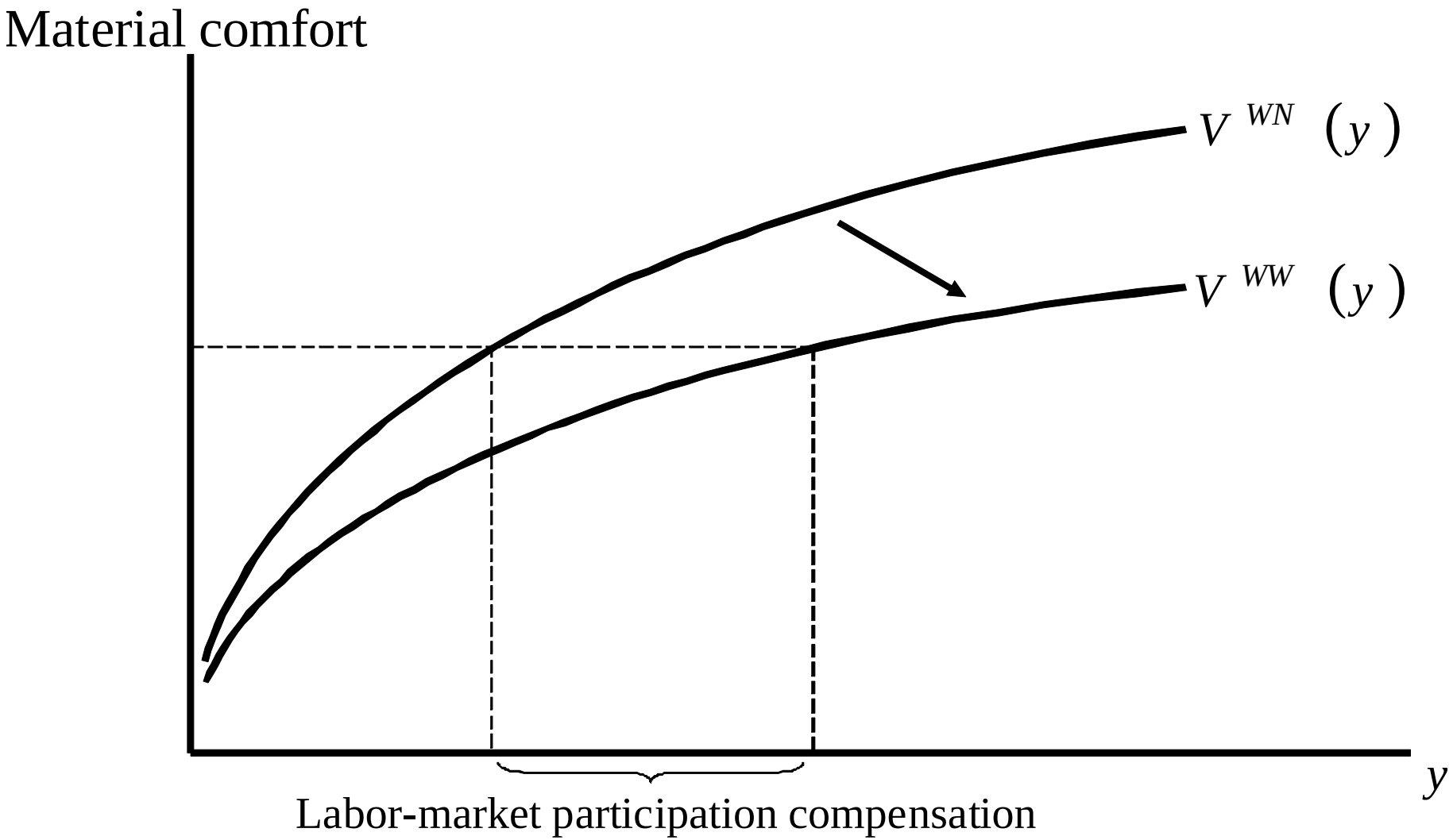
- If we go with the convention that RC exists, there are two free parameters:

$$u^{RC}\left(c, t = 2003\right) = \frac{\left(c + \frac{1.24}{\alpha} \cdot \beta_{OMH} - EUR\ 3,281\right)^{1-\frac{1}{\alpha}} - 1}{1 - \frac{1}{\alpha}}$$

# Extension to Household-Time Endowments

- It can give us information about home production
- Found in Koulovatianos, Schröder and Schmidt (JBES 2009)

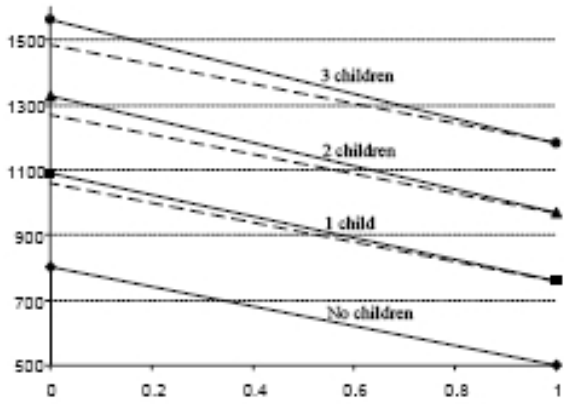
# Compensations for Time-Endowment loss



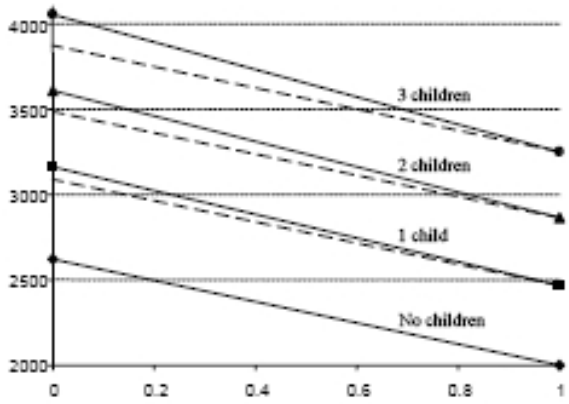
# The Questionnaire

| | 1 adult, nonworking | 1 adult, working (full time) | 2 adults, both nonworking | 2 adults, 1 nonworking 1 working (full time) | 2 adults, both working (full time) |
|---|---|---|---|---|---|
| 0 children | **Reference income** {EUR500, 2000, 3500} | | | | |
| 1 child | | | | | |
| 2 children | | | | | |
| 3 children | | | | | |

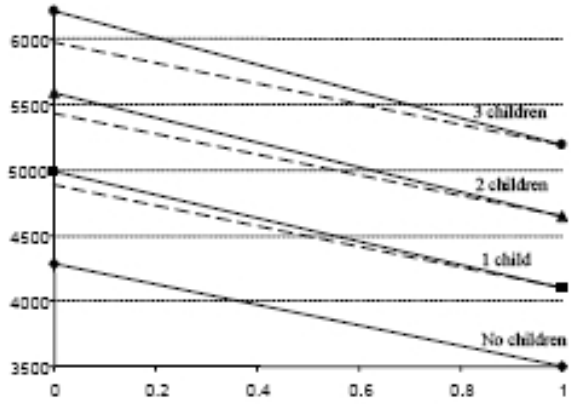
# The Results (pilot study in Belgium)



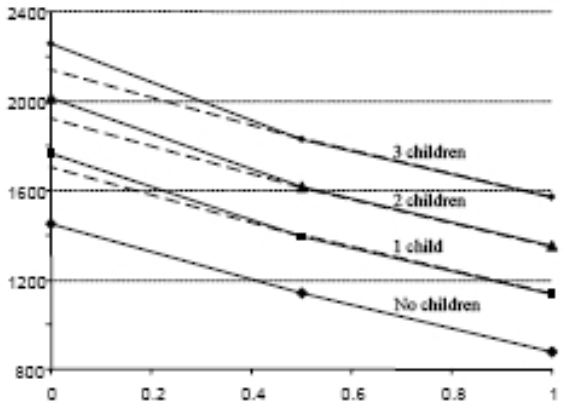

Singles, poor (EUR 500)



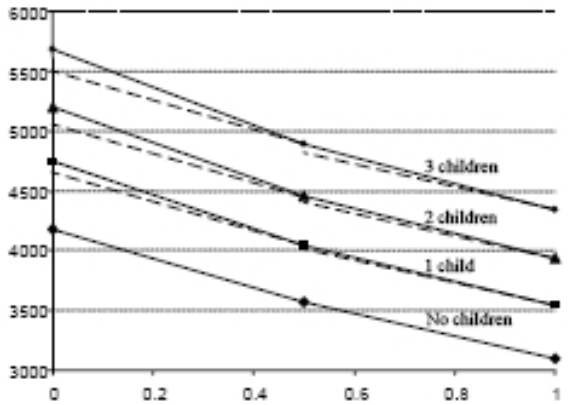

Singles, middle (EUR 2000)



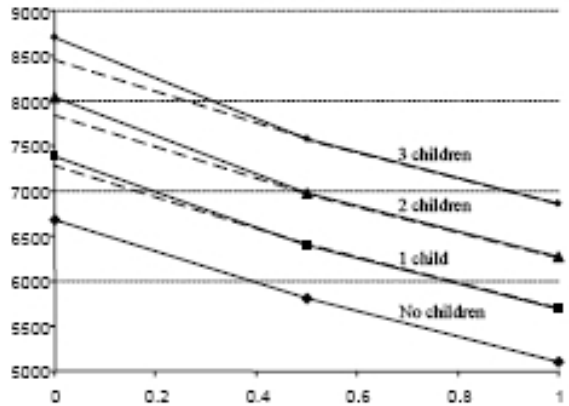

Singles, rich (EUR 3500)




Couples, poor (EUR 500)

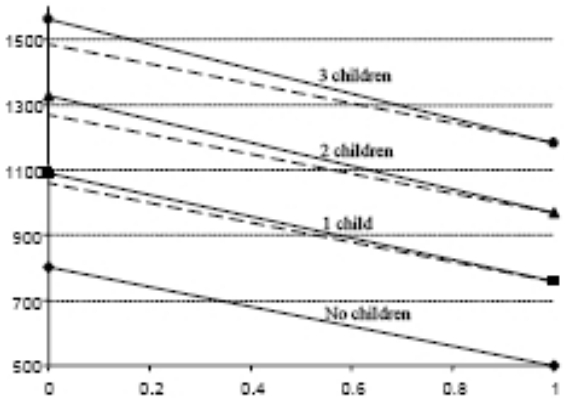



Couples, middle (EUR 2000)

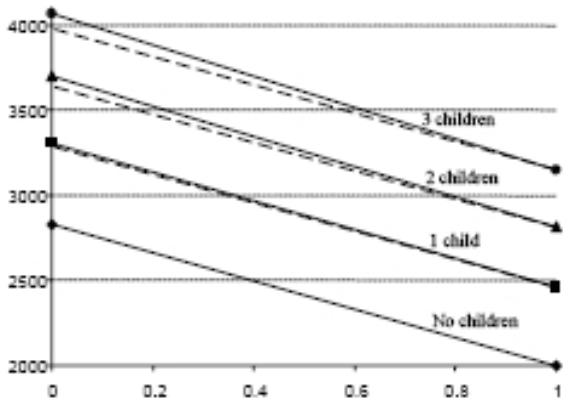



Couples, rich (EUR 3500)

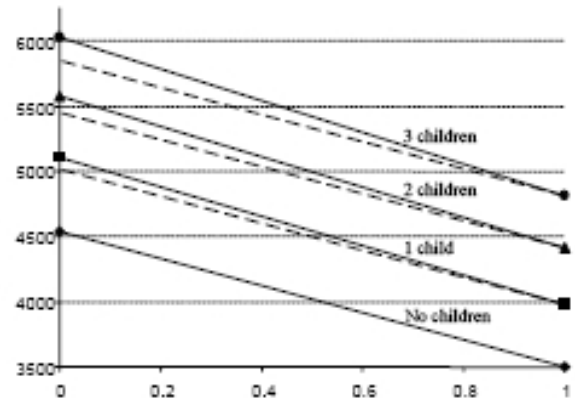
# Pilot study in Germany



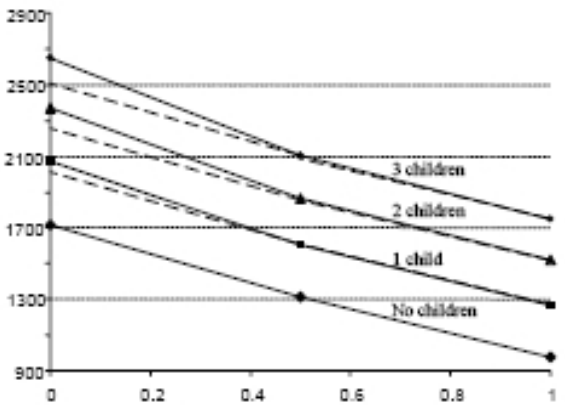

Singles, poor (EUR 500)



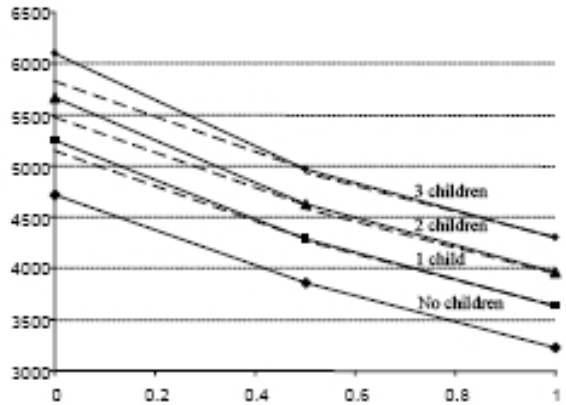

Singles, middle (EUR 2000)



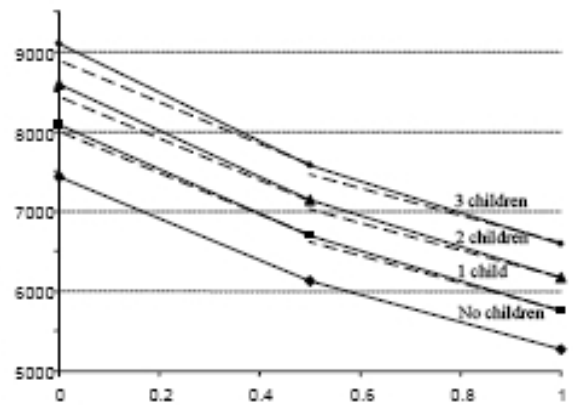

Singles, rich (EUR 3500)




Couples, poor (EUR 500)




Couples, middle (EUR 2000)




Couples, rich (EUR 3500)

# Child-Cost estimates

- Children are more costly for the poor

Table 5. Child costs relative to an adult in *WN* vs. *WW* households

| $y_r$ | Spec. 4 | | | | Apps & Rees (2001) | |
|---|---|---|---|---|---|---|
| | Belgium | | Germany | | | |
| | *WN* | *WW* | *WN* | *WW* | *WN* (average income) | *WW* (average income) |
| Poor (500) | 0.59 | 0.79 | 0.58 | 0.76 | 0.24-0.40[a]<br>0.82-0.98[b] | 0.53-0.69[a]<br>0.78-0.91[b] |
| Middle (2000) | 0.36 | 0.60 | 0.30 | 0.50 | | |
| rich (3500) | 0.33 | 0.55 | 0.37 | 0.49 | | |

NOTE: $y_r$ denotes the reference-income level in Euros.
[a] denotes a model specification without considering household production and parental child care.
[b] denotes a model specification considering household production and parental child care.

- Potential for fruitful combination of survey methods with econometric approaches

# Conclusions

- We do not disprove RC
- We do not prove RC's existence either!
- We offer aggregation theorems
- We offer (and have tested) a reliable survey instrument for estimating household-size economies
- We also provide numbers for equivalence scales for 6 countries (distinguishing poorer from richer households)
- It seems we must pay more attention to subsistence consumption