

# Accounting for Family Background when Designing Optimal Income Taxes

*A Microeconomic Simulation  
Analysis*

Rolf Aaberge and Ugo Colombino

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# Previous literature on optimal taxation

- Mirrless (1971)
- Tuomala (1990, 2006, 2008),
- Bourgiognon and Spadaro (2005, 2006)
- Saez (2001, 2002)
- Blundell et al. (2006), Haan and Wrohlich (2007), Kleven et al. (2007)

# Optimal design requires

- Simulating the behavioral responses from tax changes
- Social evaluation of outcomes from the tax simulations

# The micromodel for labor supply

- simultaneous treatment of spouses' decisions
- exact representation of complex tax rules
- heterogeneity of choice sets
- quantity constraints on the choice sets

# Random utility model of labour supply

$\max U(C, h, j)$

s.t.

Budget constraint:  $C = f(wh, I)$

Choice opportunities:  $(h, w, j) \in B$

# Basic assumptions

- $U(C, h, j) = v(C, h) \varepsilon(h, w, j)$   
 $= v(f(\mathbf{w}h, \mathbf{I}), h) \varepsilon(h, w, j)$
- $v(f(\mathbf{w}h, \mathbf{I}), h)$  is the systematic component
- $\varepsilon(h, w, j)$  is the stochastic component
- $\text{Prob}(\varepsilon < u) = \exp(-1/u)$

# Choice probability

The probability (density) that a single individual chooses a job  $(h,w)$  is given by:

$$\varphi(h, w) \equiv \Pr \left[ U(f(wh, I), h) = \max_{(x,y) \in B} U(f(xy, I), y) \right] = \frac{v(h, w) p(h, w)}{\iint v(x, y) p(x, y) dx dy}$$

Dagsvik, *Econometrica*, 1994 and Aaberge, Colombino and Strøm, *J. of Applied Econometrics*, 1999

## Structural part of the utility functions for *couples*:

The systematic part of the utility function is specified as follows:

$$\ln v_i = [\alpha_2 + \alpha_3 N] \cdot \left( \frac{C^{\alpha_1} - 1}{\alpha_1} \right) + [a_5 + a_6 \ln A_M + a_7 (\ln A_M)^2] \cdot \left( \frac{L_M^{a_4} - 1}{a_4} \right)$$

(A.11)

$$+ [a_9 + a_{10} \ln A_F + a_{11} (\ln A_F)^2 + a_{12} CU6 + a_{13} CO6] \cdot \left( \frac{L_F^{a_8} - 1}{a_8} \right)$$



# Labour supply elasticities w.r.t. wage

## Married couples, Italy 1993

Family status	Type of elasticity	Decile of income distribution	Female elasticities		Male elasticities	
			Own wage elasticities	Cross elasticities	Own wage elasticities	Cross elasticities
Couples	Elasticity of the probability of participation	I	2.40	0.26	0.04	-0.02
		II	1.35	-0.19	0.05	-0.02
		III	0.54	-0.18	0.01	-0.01
		IV	0.16	-0.16	0.02	-0.01
		V	0.10	-0.15	0.02	0.00
	Elasticity of the conditional expectation of total supply of hours	I	1.60	0.55	0.28	0.08
		II	0.83	0.05	0.12	0.02
		III	0.18	-0.06	0.08	-0.02
		IV	0.04	-0.04	0.06	-0.02
		V	0.04	-0.02	0.04	-0.02
	Elasticity of the unconditional expectation of total supply of hours	I	4.44	0.82	0.32	0.06
		II	2.31	-0.15	0.17	0.00
		III	0.73	-0.24	0.10	-0.04
		IV	0.20	-0.20	0.08	-0.03
		V	0.13	-0.17	0.06	-0.02

Note: I = first decile; II = second decile; III = third to eighth deciles; IV = ninth decile; V = tenth decile.

# Simulating tax reforms

Given a **new tax function**  $t(\cdot)$  and using the estimated  $U(\cdot)$  and  $B$  the simulation consists of solving for each household

$$\max U(C, h, j)$$

s.t.

$$C = t(wh, I)$$

$$(h, w, j) \in B$$

to get new values of  $h$  and  $C$

# What is meant by an optimal tax system?

- *The social welfare function* = weighted sum of the equivalent incomes of the individuals
- *Optimal tax system* = the tax system that maximizes the social welfare function

# Social Welfare Functions (EO)

$$W = \int_0^1 p(t) F^{-1}(t) dt,$$

$$p_k(t) = \begin{cases} -\log t, & k=1 \\ \frac{k}{k-1} (1-t^{k-1}), & k=2,3,\dots \end{cases}$$

# Distributional weight profiles of four different social welfare functions

	$W_1$ (Bonferroni)	$W_2$ (Gini)	$W_3$	$W_\infty$ (Utilitarian)
p(.01)/p(.5)	6.64	1.98	1,33	1
p(.05)/p(.5)	4.32	1.90	1.33	1
p(.30)/p(.5)	1.74	1.40	1.21	1
p(.95)/p(.5)	0.07	0.10	0.13	1

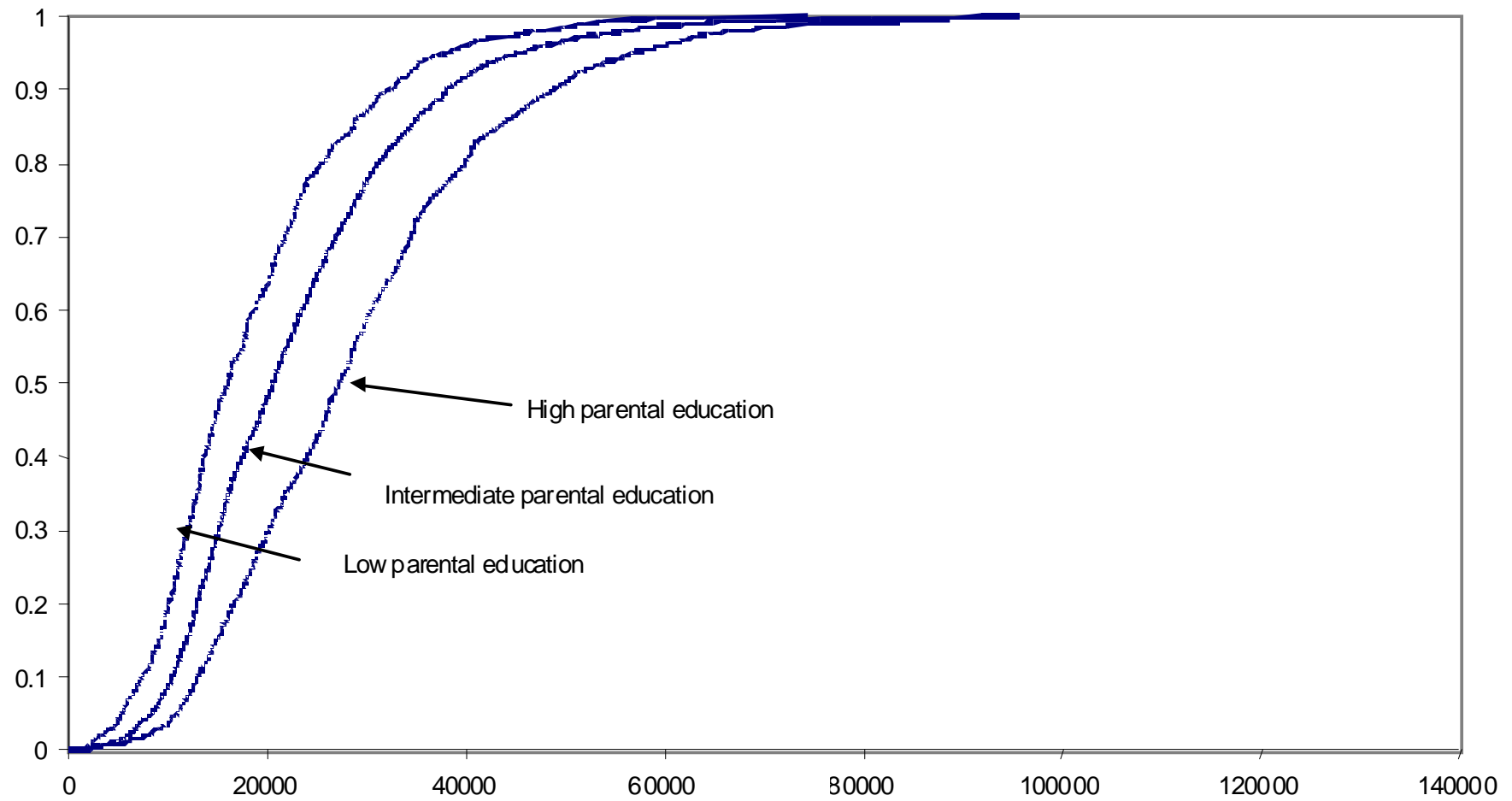
# Accounting for family background

- We classify the individuals into three types according to father's years of education:
- less than 5 years ( $F_1$ ),
- 5-8 years ( $F_2$ ),
- more than 8 years ( $F_3$ )

## ***EOp social welfare function***

$$\tilde{W}_k = \int_0^1 p_k(t) \min_j F_j^{-1}(t) dt, \quad k = 1, 2, \dots,$$

**Figure 1. Distributions of observed equivalent income by type. 1000 ITL**



# Optimal taxation

## Class of 3-parameter tax-transfer rule

$$x = \begin{cases} c + (1-t_1)y & \text{if } y \leq \bar{y} \\ c + (1-t_1)\bar{y} + (1-t_2)(y - \bar{y}) & \text{if } y > \bar{y} \end{cases}$$

where

$x$  = disposable income,

$y$  = gross income,

$\bar{y}$  = average individual gross income in Italy on the survey year (1993)



## *The tax reform simulations consist of five main steps:*

1. The tax rule is applied to individual earners' gross incomes in order to obtain disposable incomes. New labour supply responses in view of a new tax rule are taken into account by the household labour supply models for singles and couples described in the Appendix. Note that the utility functions (and choice sets) of the underlying micro-econometric model(s) are stochastic. Thus, we use stochastic simulation to find, for each individual/couple, the optimal choice given a tax-transfer rule. *The simulations are made under the conditions of **fixed total tax revenue** and non-negative disposable household incomes.*
2. To each decision making individual between 18 and 54 years old, an *equivalent income* is imputed, computed as total disposable household income divided by the square root of the number of household members.
3. We then build the individual equivalent income distributions  $F_1$ ,  $F_2$  and  $F_3$  for the types defined according to parental (actually father's) education: less than 5 years (type 1), 5-8 years (type 2) and more than 8 years (type 3).
4. Finally, we compute  $\tilde{W}_k$  for  $k = 1, 2, 3$  and  $\infty$ .

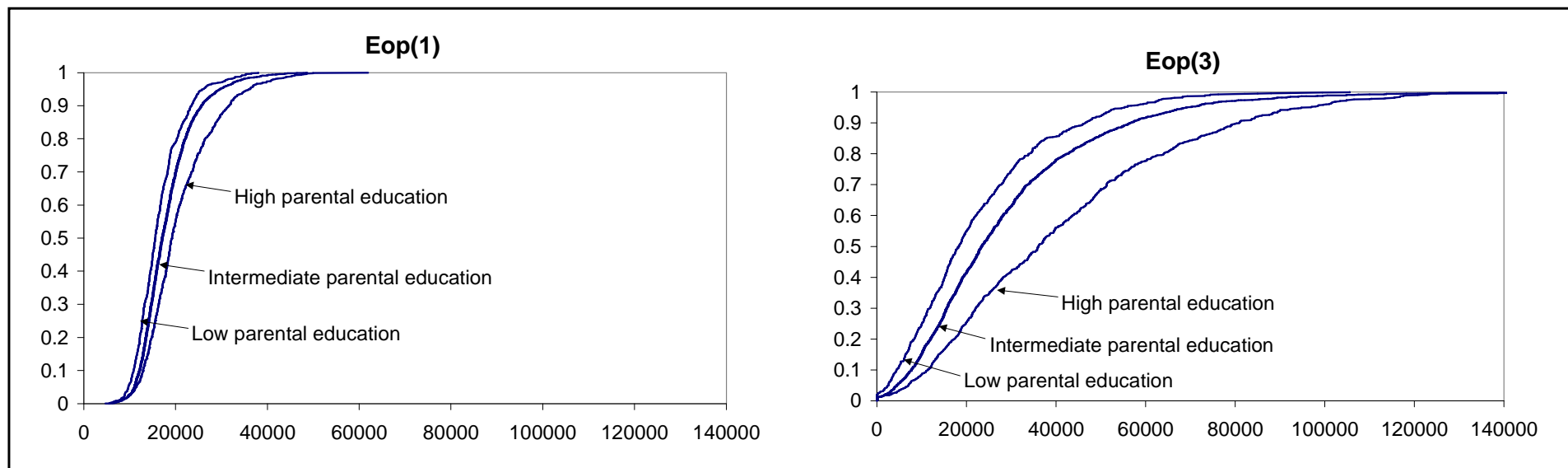
Optimization is performed by iterating the above steps, in order to find the tax rule that produces the highest value of  $\tilde{W}_k$  for each value of  $k$  *under the constraint of unchanged tax revenue*, provided that the tax rule is a member of certain sets of three-parameter tax rules

# EOp optimal three-parameter tax rules

**Table 6. Optimal three-parameter tax systems under various EOp social objective criteria ( $\tilde{W}_k$ )**

k	1	2	3	$\infty$
$t_1$	.856	.251	0	0
$t_2$	.776	.531	.168	0
c	12,500	3,500	-3,500	-5,790

**Figure 2. Distributions of individual equivalent income by type under the EOp(1) and EOp(3) tax systems. 1000 ITL**



**Table 7. Decomposition of EOp social welfare ( $\tilde{W}_k$ ) under various three-parameter tax systems**

Tax system	$\tilde{W}_\infty$	Measure of inequality		
		$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$
1993 tax system	18,323	.426	.302	.242
EOp3 (1) $\begin{pmatrix} t_1 = .856 \\ t_2 = .776 \\ c = 12,500 \end{pmatrix}$	15,393	.176	.116	.091
EOp3 (2) $\begin{pmatrix} t_1 = .251 \\ t_2 = .531 \\ c = 3,500 \end{pmatrix}$	18,508	.364	.253	.201
EOp3 (3) $\begin{pmatrix} t_1 = 0 \\ t_2 = .168 \\ c = -3,500 \end{pmatrix}$	21,156	.497	.355	.285
EOp3 ( $\infty$ ) $\begin{pmatrix} t_1 = t_2 = 0 \\ c = -5,790 \end{pmatrix}$	22,231	.553	.403	.326

# EO-optimal three-parameter tax rules

**Table 6. Optimal three-parameter tax systems under various EO social objective criteria ( $W_k$ )**

k	1	2	3	$\infty$
$t_1$	0	0	0	0
$t_2$	0	0	0	0
c	-5,790	-5,790	-5,790	-5,790

**Table 8. Decomposition of the EO social welfare ( $W_k$ ) with respect to mean and income inequality under different tax systems**

Tax system	Mean income	Measure of inequality		
		$C_1$	$C_2$	$C_3$
1993 tax system	23,540	.416	.295	.237
EOp3 (1) $\left( \begin{array}{l} t_1 = .856 \\ t_2 = .776 \\ c = 12,500 \end{array} \right)$	16,560	.193	.130	.104
EOp3 (2) $\left( \begin{array}{l} t_1 = .251 \\ t_2 = .531 \\ c = 3,500 \end{array} \right)$	21,477	.364	.255	.203
EOp3 (3) $\left( \begin{array}{l} t_1 = 0 \\ t_2 = .168 \\ c = -3,500 \end{array} \right)$	27,573	.499	.363	.294
EOp3 ( $\infty$ ) $\left( \begin{array}{l} t_1 = t_2 = 0 \\ c = -5,790 \end{array} \right)$	30,510	.544	.402	.327