The disaggregation approach to welfare, poverty and inequality measurement

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Abstract

"It is quite difficult to understand household behavior".

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- Consequences:
  - Bias in inequality assessment (Anand and Sen 1994)
  - Underestimation of inequality (Haddad and Kanbur 1991)
  - Difficult evaluation of the effectiveness of redistributive policies

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- Most importantly, inequality decomposition requires a precise estimation of the intra-household **sharing rule.**

- Our research program focuses on welfare quasi-orders. We try to identify the minimal assumptions on the intra-household sharing rule required to **preserve** welfare**,** inequality and poverty orderings after the **disaggregation** of the income distribution.

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- Disaggregation: Only the between group term is known, (the within group term is private information)

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- Links with Mathematics (Schur-concavity and HLP Theorem) , statistics (the Lorenz Curve) and risk literature (Rothschild and Stiglitz 70)

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- **1) a common** judgement on two distributions of all risk averse expected utility maximizers
- **2) a statistical test** in terms of inequality between integrated distribution functions,
- **3)** How to get any dominated distributions from a dominant one: (Mean Preserving Spread)

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- **4)** The **Generalized Lorenz test** (Shorrocks 1983) (see Appendix)
- **5)** A **poverty ordering** pointed out by Shorrocks and Foster (1987).
- **6)** The Pigou-Dalton **transfer principle** is the suitable operation in the space of income

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- Population of couples, in every couple a **dominant** individual (husband or wife) gets a larger share of expenditure than under equal distribution (**dominated** individuals receive less).
- Intra-household inequality could neutralize the egalitarian effect among individuals of redistributive policies from rich to poor households.
- Intuition: a **progressive** transfer among households,can imply a **regressive** transfer from the dominated individual of the rich household to the dominant individual of the poor household. Overall outcome? Ambiguous.

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- The disaggregation issue: the “sharing function”
- The main result: Preservation of GL dominance and RL dominance
- Poverty orderings: the lost axiom
- Stochastic dominance of order higher than 2

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- F = functions satisfying Assumption 1. We also denote $f_r(y) = y - f_p(y)$.

Disaggregation

- Applying f_p and f_r to the elements of $\mathbf{y}$ leads to disaggregate

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- $\mathbf{r}(\mathbf{y}) = (r_1, ..., r_j, ..., r_n)$ (strong individuals).
- We get a (rearranged) vector of "individual" incomes $\mathbf{x}(\mathbf{y}) \in \mathbb{R}^{2n}_+$.

Preservation

- Let $\mathbf{y}$ and $\mathbf{y}'$ be two distributions of $\mathbb{R}^n_+$, and $\mathbf{x}(\mathbf{y}), \mathbf{x}(\mathbf{y}') \in \mathbb{R}^{2n}_+$ the corresponding (and **unknown**) individual distributions

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- Let us suppose that $\mathbf{y}$ and $\mathbf{y}'$ are ordered according to some dominance relation
- Solving the problem of preserving a dominance relation through disaggregation means:
- To identify the largest class of f_p that guarantees that the same dominance relation also holds at the individual level, that is between $\mathbf{x}(\mathbf{y})$ and $\mathbf{x}(\mathbf{y}')$

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Main results

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Main results

- Peluso and Trannoy: preservation under disaggregation of inequality, welfare (JET 2007) and poverty (JID 2009) dominance .
- In Couprie et Al. (JPub 2010) we also account for public consumption and implement an empirical application.
- Peluso and Trannoy (2009) extend the results to stochastic dominance of order higher than 2.
- The crucial condition on f_p for the preservation of SSD is that the weak individual receives less and less **at the margin**, when y increases.

The main result

Lemma

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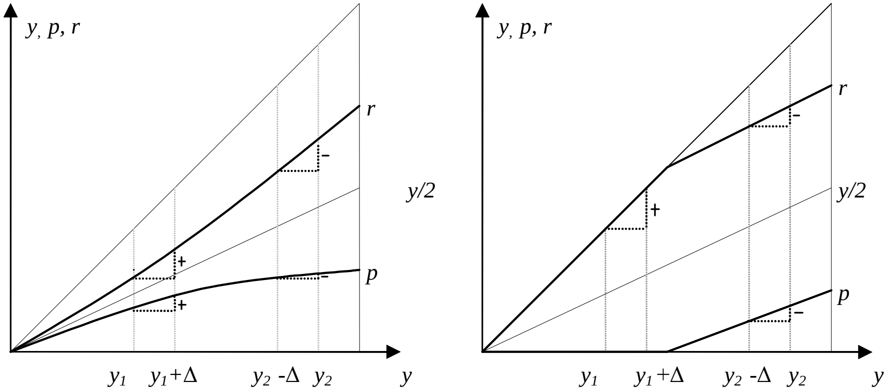
Theorem

Let f_p non decreasing and continuous. Then:

$$f_p \in \mathcal{C} \iff \left[\forall\ \mathbf{y}, \mathbf{y}' \in Y_n,\ \mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \Rightarrow \mathbf{x} \succcurlyeq_{GL} \mathbf{x}' \ \right].$$

The double dividend



a- Concave sharing function **b-** Convex sharing function

Inequality

Corollary (RL dominance)

Let $f_p \in F$ *and* $\beta \in [0, \frac{1}{2}]$
$f_p = \beta y \iff [\forall\ y, y' \in Y_n, \ , y \succcurlyeq_{RL} y' \implies x(y) \succcurlyeq_{RL} x(y')]$

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Corollary

Let $f_p \in F$ *and* $\mu_{\mathbf{y}} \leq \mu_{\mathbf{y}'}$
$f_p \in C \iff [\forall\ y, y' \in Y_n, \ , y \succcurlyeq_{RL} y' \implies x(y) \succcurlyeq_{RL} x(y')].$

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Corollary

If the sharing function is concave, a progressive taxation schedule implies a lower inequality at the individual level

An easy extension

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$f_p \in C$ *and* $f_g \in C^g \iff$
$[\mathbf{y} \succcurlyeq_{GL} \mathbf{y}' \implies \mathbf{x}(\mathbf{y}) \succcurlyeq_{GL} \mathbf{x}(\mathbf{y}'), \ \forall \ \mathbf{y}, \mathbf{y}' \in \mathbb{Y}_n]$.

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- "chain condition": concavity of all the group sharing functions is necessary and sufficient to get the preservation of the GL test.

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Theorem

(Foster-Shorrocks 1988)
Let Y and Y' be two income vectors of $\mathbb{R}^n_+$ ordered in the increasing way. The two following statements are equivalent:

i) Y dominates Y' according to stochastic dominance of order $h = 1, 2, 3$

ii) $P_{h-1}(Y) \leq P_{h-1}(Y')$, for any poverty line z (Poverty orderings $\succsim_{P_0}$, $\succsim_{P_1}, \succsim_{P_2}$)

Truncated poverty Orderings

It can be reasonable to require poverty dominance only below a fixed poverty line $z^{\max}$

Definition

$Y \succsim_{P_1 z^{\max}} Y' \Longleftrightarrow P_1(Y, z) \leq P_1(Y', z)$, *for all* $z \leq z^{\max}$.

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Lemma

For any $z_c > 0$ *and any* $z_s \leq f_p(z_c)$
$Y \succsim_{P_1 zc} Y' \Rightarrow X(Y) \succsim_{P_1 zs} X(Y')$ *iff* f_p *is increasing and concave*

The lost axiom

- To preserve the poverty order a the individual level at $z_{c/2}$, we need information about non-poor households: the "focus axiom" does not hold. We need to consider households with incomes below $f_p^{-1}(z_{c/2})$.

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Corollary

For any $z_c > 0$

$Y \succsim_{P_1 f_p^{-1}(z_{c/2})} Y' \Rightarrow X(Y) \succsim_{P_1 c/2} X(Y')$ *iff* f_p *is increasing and concave*

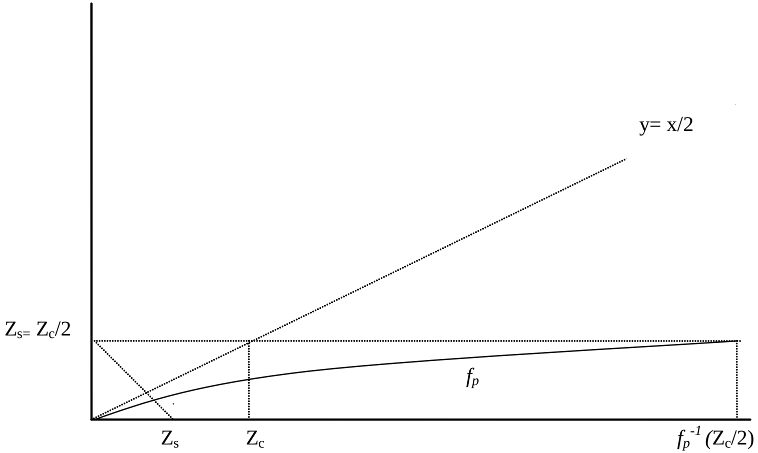

Figure 1 : Illustration of Proposition 6 (ii)

Implementation

Inequality or poverty Test	Econometrics
• Lorenz test, Poverty orderings	➢ Concavity test
• Z-Poverty orderings	➢ Concavity test, local estimation of the sharing rule
• Poverty and inequality indices	➢ Complete estimation of the sharing rule

A last extension: initial endowments.

- Let $\mathcal{F}$ be the set of sharing functions f_p, with $f_p(0) = 0$, $f_p'(y) \leq 1$ for all y and $f_p(y) + k_p \leq f_r(y) + k_r$ for all nonnegative y.

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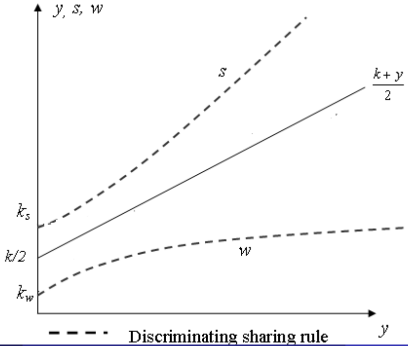
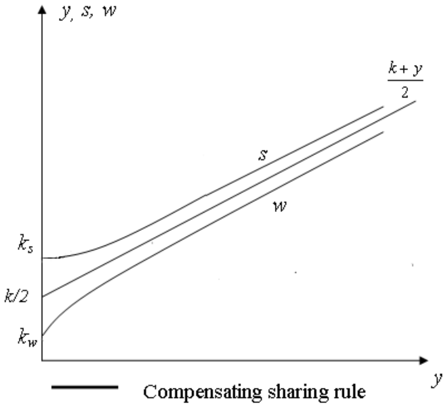
- **Discriminating** case $0 \leq f_p'(y) \leq f_r'(y) \leq 1$, the weak receive always less than the strong at the margin.



- **Compensating** case $0 \leq f_r'(y) \leq f_p'(y) \leq 1$, the dominated receive always more than the strong at the margin



Stochastic dominance of order > 2

- Let $\mathcal{F}_c$ be the set of continuous sharing functions f_p, with $f_p(0) = 0$, $\frac{1}{2} \leq f_p'(y) \leq 1$ for all y and $f_p(y) + k_w \leq f_r(y) + k_s$ for all nonnegative y.

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Theorem

For any $m \geq 3$,

$$f_p \in \mathcal{F}_c \cap \mathbb{U}_m \iff \left[\mathbf{y} \succsim_{\mathbf{m}} \mathbf{y}' \implies \mathbf{x}(\mathbf{y};\mathbf{k}) \succsim_m \mathbf{x}(\mathbf{y}';\mathbf{k}),\ \forall\ \mathbf{y},\mathbf{y}' \in \mathbb{D}\right]$$

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If f and g are real functions differentiable m times, then:

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Theorem

If f and g are real functions differentiable m times, then:

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Lemma

If h and g belong to $\mathbb{U}_m$*, then* $g \circ h$ *belongs to* $\mathbb{U}_m$

- And the theorem is proved thanks to a further result:

Lemma

Let $u \in U_m$ *and* $f_p \in \mathcal{F}_c \cap \mathbb{U}_m$*,*
Then the function $V(y) = u(f_p(x) + k_w) + u(f_r(x) + k_s)$ *belongs to* $\mathbb{U}_m$

Further extensions

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- Households with different size: the SGL test
- Empirical evidence: on Thursday (Hélène Couprie presentation)!