Welfare & Poverty Comparisons

Sequential Procedures

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PRELIMINARIES

- Household types $i = 1, \ldots, n$ differentiated and ranked by needs, type $i = 1$ judged the neediest, see on.
- Household money income distribution functions $F(x)$ and $G(x)$.
- Type specific distributions: $F_i(x)$ and $G_i(x)$, densities $f_i(x)$ and $g_i(x)$, $1 \leq i \leq n$.
- Distribution functions for first *j* merged subpopulations: ${}^jF(x)$ and ${}^jG(x)$
- Proportion of households belonging to type *i* : $P_{i,F}$ in F and $P_{i,G}$ in G

MORE PRELIMINARIES

- **SWFs:** additively separable over money incomes, with differentiable utility functions, $U_i(x)$ for type i:
  $W_F = \sum_{i=1}^{n} P_{i,F} \int_o^z U_i(x) f_i(x) dx$ &
  $W_G = \sum_{i=1}^{n} P_{i,G} \int_o^z U_i(x) g_i(x) dx$

  in which z is the highest income present in either F or G, or an arbitrary income level exceeding this maximum.

  In Ok and Lambert (1999) additivity of the SWF across types is relaxed

- **The needs structure:** conditions relating the utility functions $U_j(x)$ and $U_{j+1}(x)$ of adjacent types, $1 \le j \le n-1$. See on

The starting point for analysis:

$$W_F - W_G = \sum_{i=1}^{n} U_i(x) \int_0^z [P_{i,F} f_i(x) - P_{i,G} g_i(x)] dx$$

from which everything follows using a mix of Abel's lemma, changing the order of summation and integration, and integration by parts.

ABEL'S LEMMA: $\sum_{i=1}^{n} v_i w_i = \sum_{j=1}^{n} d_j t_j$ where, $d_i = v_j - v_{j+1}$,

$1 \le j \le n-1$, $d_n = v_n$ & $t_j = \sum_{i=1}^{j} w_i$, $1 \le j \le n$

INTEGRATION BY PARTS: $\int_a^b v(x) w'(x) dx = [v(x)w(x)]_a^b - \int_a^b v'(x) w(x) dx$

Begin by integrating by parts, reverse the order of summation and integration, apply Abel's lemma, and reverse the order of summation and integration again....!

Utility information: $D_j(x) = U_j'(x) - U_{j+1}'(x), 1 \le j \le n-1 \,\&\, D_n(x) = U_n'(x)$

Systematic differences between the utilities of adjacent types embody social judgements about needs

$$W_F - W_G = \sum_{j=1}^{n} \int_0^z D_j(x) T_j'(x) dx$$

$$W_F - W_G = \sum_{j=1}^{n} \{D_j(z) T_j(z) - \int_0^z D_j'(x) T_j(x) dx\}$$

$$W_F - W_G = \sum_{j=1}^{n} \{D_j(z) T_j(z) - D_j'(z) \int_0^z T_j(x) dx + \int_0^z D_j''(x) [\int_0^x T_j(y) dy] dx\}$$

Distributional configuration: $T_j(x) = \sum_{i=1}^{j} \int_0^x [P_{i,G} G_i(y) - P_{i,F} F_i(y)] dy$

A familiar-looking construction in dominance analysis. See on

Theorem

(1) [Atkinson and Bourguignon (1987), Jenkins and Lambert (1993), Chambaz and Maurin (1998)]

$W_F \ge W_G \forall W \in \mathbf{W_1} \Leftrightarrow T_j'(x) \ge 0, \forall j, \forall x \in [0, z]$

(2) [Atkinson and Bourguignon (1987), Chambaz and Maurin (1998)]

$W_F \ge W_G \forall W \in \mathbf{W_2} \Leftrightarrow T_j(x) \ge 0, \forall j, \forall x \in [0, z]$

(3) $W_F \ge W_G \forall W \in \mathbf{W_3} \Leftrightarrow T_j(z) \ge 0$ *and* $\int_0^x T_j(y) dy \ge 0 \forall j, \forall x \in [0, z]$

where, for $\mathbf{W_1}$, $D_j(x) > 0 \, \forall x$

There is additional social merit in awarding an extra $1 to a household of type *j* with an income of *x*, over a household of the next-less-needy type *j+1* at the same income level (*j* < *n*). Adding $D_n(x) > 0 \, \forall x$, all utility functions are strictly increasing.

for $\mathbf{W_2}$, *add* $D_j'(x) < 0 \, \forall x$

For *j* < *n*, the extra social value in granting a new unit of resource to a needier household at each income level declines with increases in that income level. Adding $D_n'(x) < 0 \, \forall x$, all utility functions are strictly concave

for $\mathbf{W_3}$, *add* $D_j''(x) > 0 \, \forall x$

As between types *j* and *j +1*, it is judged *even better* to give the extra $1 to a household of type *j* at *a lower income level*, rather than at a higher one. Adding $D_n''(x) > 0 \, \forall x$, all utility functions have positive third derivatives, *i.e.* satisfy Kolm's (1976) Principle of Diminishing Transfers.

and in each case, the social decision-maker does not care about the family types of the super-rich.

REFINEMENT/SIMPLIFICATION FOR NO DEMOGRAPHIC DIFFERENCES:

(a) can drop that requirement "the social decision-maker does not care about the family types of the super-rich";

(b) $\Rightarrow P_{i,F} = P_{i,G} = P_i \forall i \Rightarrow T_j(x) = \left[\sum_{i=1}^{j} P_i\right] \int_0^x [{}^jG(y) - {}^jF(y)]dy$

rank dominance (Saposnik (1981)) and generalized Lorenz dominance (Shorrocks (1983)) between ${}^jF(x)$ and ${}^jG(x)$ correspond respectively to signed values of $T_j(x)$ and $T_j{}'(x)$;

(c) $\Rightarrow$ third degree stochastic dominance (Whitmore (1970)) of ${}^jF(x)$ over ${}^jG(x)$ if $T_j(z) \geq 0$ *and* $\int_0^x T_j(y)dy \geq 0 \, \forall j, \forall x \in [0, z]$,

adapted to scenarios where generalized Lorenz curves cross once by Dardanoni and Lambert (1987).

Corollary

In the absence of demographic differences between F and G,

(1) [Atkinson and Bourguignon, 1987] $W_F \geq W_G \forall\, W \in \mathbf{W_1}$ $\Leftrightarrow$for each set of the j most needy subgroups, $1 \leq j \leq n$, ${}^jF$ rank dominates ${}^jG$;

(2) [Atkinson and Bourguignon, 1987] $W_F \geq W_G \forall\, W \in \mathbf{W_2}$ $\Leftrightarrow$for each set of the j most needy subgroups, $1 \leq j \leq n$, ${}^jF$ generalized Lorenz dominates ${}^jG$;

(3) [Lambert and Ramos (2002)] If sequential generalized Lorenz dominance as in *(2)* is not satisfied, but at each stage j for which it fails, *(a)* the two generalized Lorenz curves cross once with ${}^jF$'s initially dominant, *(b)* the means are the same, and *(c)* ${}^jF$ has no higher a variance than ${}^jG$, then $W_F \geq W_G \forall\, W \in \mathbf{W_3}$.

All of these tests are easy to implement using household survey microdata.

Notes

1. Bourguignon's (1989) welfare class $\mathbf{W}_{1½}$: needs structure of $\mathbf{W}_1$ plus concavity (as in $\mathbf{W}_2$). Non-sequential dominance criterion "not easy to evaluate": a numerical algorithm is provided.

2. Equivalence scales. Deflators $m_1 > m_2 > ... > m_n$, type-specific utility functions $U_i(x) = m_i U(x/m_i)$ (Ebert (1997, 1999), Ebert and Moyes (2000))

→welfare classes $\boldsymbol{E}_k \subset \boldsymbol{W}_k$ → welfare results as in the theorem and corollary *for all equivalence scales* (with a tweak if there are demographic differences).

- *Try sequential methods using money income distributions before beginning the restrictive business of choosing an equivalence scale and resorting to mythical populations!*

POVERTY COMPARISONS: SEQUENTIAL PROCEDURES

Atkinson (1992) began this; Chambaz and Maurin (1998) the best source

- Money poverty lines $Z_1 \geq Z_2 \geq \cdots .. \geq Z_n$
- Generic poverty measure:

$$P(F|\underline{Z}) = \sum_{i=1}^{n} P_{i,F} \int_0^{Z_i} \theta_i(x|Z_i) f_i(x) dx$$

- Chambaz and Maurin's (1998) class $\mathbf{P}_{\mathbf{CM}}$ of poverty indices, in which:

$\theta_i(x|Z_i)$ twice differentiable

$\theta_i(x|Z_i) \geq 0 \ \forall x \in (0, Z_i], \theta_i(x|Z_i) = 0 \ \forall x > Z_i$

$\theta_1'(x|Z_1) \leq \theta_2'(x|Z_2) \leq ... \leq \theta_n'(x|Z_n) \leq 0 \forall x$ — At each fixed x, the potential of an extra $1 addition to reduce overall poverty is greatest if the recipient is of the neediest type, less for each successively less needy type and least of all for least needy type

$\theta_1''(x|Z_1) \geq \theta_2''(x|Z_2) \geq ... \geq \theta_n''(x|Z_n) \geq 0 \forall x$ — Benefit to overall poverty of giving $1 to someone having x who is of type i rather than type $i + 1$ is less at higher income levels than at lower ones

Sequential criteria obtained include this one, in particular:

Theorem (Chambaz and Maurin)

In the absence of demographic differences,

$$P(F|\underline{Z}) \leq P(G|\underline{Z}) \quad \forall P \in P_{CM} \;\&\; \forall \underline{Z}: Z_1 \geq Z_2 \geq \cdots .. \geq Z_n$$

$\Leftrightarrow$ *F* sequentially generalized Lorenz dominates *G*