Identification of the Sharing Rule and Collective Demand Systems

M. Menon[1] F. Perali[2]

[1,2]University of Verona, Department of Economics and CHILD

Canazei - Winter School, 2011

Outline

1. Objectives
2. The Collective Model
3. Identification of the Sharing Rule
   - Specification of the Consumption Model
   - Specification of the Consumption Model and of the Sharing Rule
   - Summary of the identifying restrictions
4. Empirical Results
   - Estimation Methods
   - Estimation Results
5. The Collective Demand System
   - Individual Specific System with Common Sharing Rule
   - Hybrid - Household System with Individual Engel Curves

Objectives of the Paper

1. **Identification of the sharing rule.** We execute a controlled experiment to answer the following questions:
    1. Does there exist a correspondence between the structural and reduced form representation of the sharing rule?
    2. What is the specification so that estimating the structure or the reduced form gives same results?
    3. This question has important implications: collective demand systems can be estimated directly from the structure.
2. **Specification of complete collective demand systems.**
    1. Pure - a per member system with common sharing rule.
    2. Hybrid - a household system with individual Engel curves.

Information Requirement to Identify the Sharing Rule

1. **Information set:** $\Omega_g = \{exclusive\ goods\}$
   1. At least one exclusive good for each member, that is individual consumption that can be observed also within the household - Chiappori's collective approach (Browning et al. JPE 1994, Chiappori, Fortin, and Lacroix JPE 2002, Lise and Seitz 2009, Vermuelen 2007)
2. **Information set:** $\Omega_p = \{singles;\ household\ technologies\}$
   1. Consumption of female and male individuals living alone .AND. the hh technology (Browning, Chiappori, and Lewbel 2010)

Experiment Design

1. **Theory**:
   1. Show that there should be a unique correspondence between a given structure and the associated reduced form.
2. **Application** (using the same consumption data):
   1. Estimates the structural model (S) *a la* Browning, Bourguignon, Chiappori, Lechene (1994).
   2. Estimate the reduced form (RF) model *a la* Chiappori, Fortin, Lacroix (2002).
3. Do S and RF give the same estimates? Do we find the correspondence? Under which conditions?

The Consumption Collective Model:General Assumptions

Consider a two-adult family. Assume that:

1. Each spouse $i = 1,2$ privately consumes an assignable good c^i and an ordinary good c_o^i ,
2. There are no public goods or externalities within the family,
3. The family faces a linear and convex budget constraint,
4. Household outcomes are Pareto efficient.

Individual Preferences

Preferences of each household member are represented by an egoistic quasi-concave utility function, twice differentiable and strictly increasing in its arguments:

$$u^i = U^i\left(c^i, c_o^i\right).$$

The Centralized Program

Household welfare can be represented by the following weighted Bergsonian welfare function

$$W^h = \mu U^1\left(c^1, c_o^1\right) + (1-\mu)U^2\left(c^2, c_o^2\right),$$

Pareto weight

The Pareto index $\mu \in [0,1]$ is a function of prices p, income y and distribution factors z: $\mu = \mu^*(p, y, s)$.

Distribution factors

Distribution factors s are exogenous variables affecting behaviour only through the decision process μ. Distribution factors simplify the theoretical identification of the sharing rule. Examples are divorce ratio and sex ratio, wealth at marriage.

The Decentralized Program

Given the previous assumptions, the centralized household program can be decentralized using the Second Fundamental Welfare Theorem for a given sharing rule $\phi(p,x,s)$:

$$Max_{c^i,c_o^i}\left\{U^i\left(c^i,c_o^i\right)|\,p_ic^i+p_oc_o^i=\phi_i(p,x,s)\right\},$$

For $i=1$, $\phi_1=\phi(p,x,s)$ is the part of total income x allocated to member 1. For $i=2$, $\phi_2=x-\phi(p,x,s)$ and $\phi_1+\phi_2=x$.

Observability of the Sharing Rule

Researchers are not able to observe the rule governing the resource allocation process; so $\phi(p,x,s)$ must be inferred by the observation of private consumption or labor supply. Note that μ and ϕ are linked by a relationship derivable by equating the individual demands of the centralized and decentralized program.

The Decentralized Marshallian Demand Functions

The individual demand functions in reduced and structural form

Reduced form		Structural form
$c^i(p_1, p_2, x, s)$	$=$	$C^i(p_i, \phi_i(p_1, p_2, x, s)),$
$c^i_o(p_1, p_2, x, s)$	$=$	$C^i_o(p_i, \phi_i(p_1, p_2, x, s)).$

Identification of the Sharing Rule

Proposition 1: Identification of the sharing rule

If for any continuously differentiable reduced and structural specifications of collective demands the elements of the Jacobian of the reduced form establishes a one-to-one correspondence with the elements of the Jacobian of the structural form, then the partial effects of the sharing rule $\{\phi_{p_1}, \phi_{p_2}, \phi_x, \phi_s\}$ can be identified up to an additive constant.

Identification of the Sharing Rule

Proof - Part A

Member 1		*Member 2*	
Reduced	*Structural*	*Reduced*	*Structural*
$c^1_{p_1}$	$C^1_{p_1} + C^1_\phi \phi_{p_1}$,	$c^2_{p_1}$	$-C^2_\phi \phi_{p_1}$,
$c^1_{p_2}$	$C^1_\phi \phi_{p_2}$,	$c^2_{p_2}$	$C^2_{p_2} - C^2_\phi \phi_{p_2}$,
c^1_x	$C^1_\phi \phi_x$,	c^2_x	$C^2_\phi (1 - \phi_x)$,
c^1_s	$C^1_\phi \phi_s$,	c^2_s	$-C^2_\phi \phi_s$,

Identification of the Sharing Rule

General Results: Proof - Part B

Define:

$$A = \frac{c^1_{p_2}}{c^1_x}\ ,\ B = \frac{c^2_{p_1}}{c^2_x}\ ,\ C = \frac{c^1_s}{c^1_x} \text{ and } D = \frac{c^2_s}{c^2_x}.$$

Then we can solve for the partial effects $\{\phi_{p_1}, \phi_{p_2}, \phi_x, \phi_s\}$:

ϕ_{p_1}	ϕ_{p_2}	ϕ_x	ϕ_s
$\frac{BC}{D-C}$	$\frac{AD}{D-C}$	$\frac{D}{D-C}$	$\frac{CD}{D-C}$

The sharing rule can be recovered up to an additive constant because all the partial effects of the sharing rule can be deduced from the observed partial effects of the reduced form demands.

The Decentralized Slustky Equation/1

In equilibrium the dual identitiy is

$$C^i_H\left(p_i, \bar{U}_i\right) \equiv C^i\left(p_i, \phi^*_i\left(p_i, \bar{U}_i\right)\right).$$

Differentiating with respect to p_i and using the envelope theorem, we obtain the symmetric Slutsky equation

$$C^i_{H_{p_i}}\mid_{d\bar{U}_i=0} = C^i_{p_i} + c^i C^i_{\phi_i},$$

where $C^i_{p_i}$ is the substitution effect and $C^i_{\phi_i}$ the income effect. Both these effects are unobservable.

The Decentralized Slustky Equation/2

After some manipulations we obtain the Slutsky equation in terms of all observable variables for member 1

$$C^1_{H_{p_1}} \mid_{d\bar{U}_1=0} = c^1_{p_1} - \frac{c^1_x}{\phi_x}\left(\phi_{p_1} - c^1\right) \leq 0,$$

and for member 2

$$C^2_{H_{p_2}} \mid_{d\bar{U}_2=0} = c^2_{p_2} - \frac{c^2_x}{1-\phi_x}\left(\phi_{p_2} - c^2\right) \leq 0$$

which is symmetric. Note that the Centralized Slutsky equation is not symmetric because the household welfare function depends on the Pareto weight $\mu = \mu^*(p, x, s)$ that depends on p.

Identification of the Sharing Rule

Experiment Design

1. Specification of the demand function
   1. identification issues
2. Specification of the demand function and of the sharing rule
   1. identification issues

Is this exercise going to unveal new “hidden” properties of the collective model?

Outline

1. Objectives
2. The Collective Model
3. Identification of the Sharing Rule
   - Specification of the Consumption Model
   - Specification of the Consumption Model and of the Sharing Rule
   - Summary of the identifying restrictions
4. Empirical Results
   - Estimation Methods
   - Estimation Results
5. The Collective Demand System
   - Individual Specific System with Common Sharing Rule
   - Hybrid - Household System with Individual Engel Curves

Specification of the Consumption Model

Linear Specification of the Demand Functions

The assignable good $c^i(p_1, p_2, x, s)$ has the following structural and reduced continuously differentiable functional forms

Structural Form | **Reduced Form**

Member 1

$$\ln c_S^1 = \alpha_0 + \sum_{j=1}^{D} \alpha_j \ln d_j + \alpha_{p_1} \ln p_1 + \alpha_x \ln \phi(p_1, p_2, x, s), \quad (1.S)$$

$$\ln c_R^1 = a_0 + \sum_{j=1}^{D} a_j \ln d_j + \sum_{i=1}^{2} a_{p_i} \ln p_i + a_x \ln x + \sum_{r=1}^{R} a_{s_r} \ln s_r, \quad (1.R)$$

Member 2

$$\ln c_S^2 = \beta_0 + \sum_{j=1}^{D} \beta_j \ln d_j + \beta_{p_2} \ln p_2 + \beta_x \ln(x - \phi(p_1, p_2, x, s)), \quad (2.S)$$

$$\ln c_R^2 = b_0 + \sum_{j=1}^{D} b_j \ln d_j + \sum_{i=1}^{2} b_{p_i} \ln p_i + b_x \ln x + \sum_{r=1}^{R} b_{s_r} \ln s_r. \quad (2.R)$$

Specification of the Consumption Model

Identification given a linear form of demand /1

Using Proposition 1, from the structural and reduced equations of member 1, the Jacobian matrixes are

Structural Form	Reduced Form
$\frac{\partial \ln c_S^1}{\partial \ln p_1} = \alpha_{p_1} + \alpha_x \phi_{p_1}$	$\frac{\partial \ln c_R^1}{\partial \ln p_1} = a_{p_1},$
$\frac{\partial \ln c_S^1}{\partial \ln p_2} = \alpha_x \phi_{p_2}$	$\frac{\partial \ln c_R^1}{\partial \ln p_2} = a_{p_2},$
$\frac{\partial \ln c_S^1}{\partial \ln x} = \alpha_x \phi_x$	$\frac{\partial \ln c_R^1}{\partial \ln x} = a_x,$
$\frac{\partial \ln c_S^1}{\partial \ln s_r} = \alpha_x \phi_{s_r}$	$\frac{\partial \ln c_R^1}{\partial \ln s_r} = a_{s_r}.$

Specification of the Consumption Model

Identification of the Sharing Rule/2

Similarly for member 2

Structural Form	Reduced Form
$\frac{\partial \ln c_S^2}{\partial \ln p_1} = -\beta_x \phi_{p_1}$	$\frac{\partial \ln c_R^2}{\partial \ln p_1} = b_{p_1},$
$\frac{\partial \ln c_S^2}{\partial \ln p_2} = \beta_{p_2} - \beta_x \phi_{p_2}$	$\frac{\partial \ln c_R^2}{\partial \ln p_2} = b_{p_2},$
$\frac{\partial \ln c_S^2}{\partial \ln x} = \beta_x (1 - \phi_x)$	$\frac{\partial \ln c_R^2}{\partial \ln x} = b_x,$
$\frac{\partial \ln c_S^2}{\partial \ln s_r} = -\beta_x \phi_{s_r}$	$\frac{\partial \ln c_R^2}{\partial \ln s_r} = b_{s_r}.$

Specification of the Consumption Model

Identification of the Sharing Rule - Proportionality Condition (Bourguignon, Browning, and Chiappori 2009)

Property 1: Proportionality Condition

(Bourguignon, Browning, and Chiappori 2009) With several distribution factors, the correspondence identifying condition implies that the ratio of the parameters associated with a distribution factor across individuals is the same for all distribution factors:

$$\frac{a_{s_1}}{b_{s_1}} = \frac{a_{s_r}}{b_{s_r}},$$

for any point such that $b_1, b_{s_r} \neq 0$ and for all $r = 2, ..., R$.

Specification of the Consumption Model

Identification of the Sharing Rule - Extension of the Proportionality Condition

Proposition 2: Extended Proportionality

If there exists at least one distribution factor, then the ratio of the parameters of the reduced demand equations (1.R) and (2.R) associated with the distribution factor must be proportional to the (negative) ratio of the income parameters of the structural demand equations (1.S) and (2.S)

$$\frac{a_{s_r}}{b_{s_r}} = -\frac{\alpha_x}{\beta_x},$$

for any point such that $\beta_x, b_{s_r} \neq 0$. With several distribution factors, given Property 1, the income proportionality condition extends to

$$\frac{a_{s_1}}{b_{s_1}} = \frac{a_{s_r}}{b_{s_r}} = -\frac{\alpha_x}{\beta_x}, \quad \forall\, r = 2, \ldots, R.$$

Specification of the Consumption Model

Identification of the Sharing Rule - Recovering the Partial Effects

The resulting partial effects of the sharing rule $\{\phi_x, \phi_{p_1}, \phi_{p_2}, \phi_s\}$ are:

$$\phi_x = \frac{a_x b_{s_1}}{a_x b_{s_1} - a_{s_1} b_x},$$

$$\phi_{p_1} = \frac{b_{p_1} a_{s_r}}{a_x b_{s_r} - a_{s_r} b_x}$$

$$\phi_{p_2} = \frac{a_{p_2} b_{s_r}}{a_x b_{s_r} - a_{s_r} b_x}$$

$$\phi_{s_r} = \frac{a_{s_r} b_{s_r}}{a_x b_{s_r} - a_{s_r} b_x} = \frac{a_{s_r} b_{s_r}}{a_x - \frac{a_{s_r}}{b_{s_r}} b_x}.$$

Specification of the Consumption Model

Identification of the Sharing Rule - Recovering the Sharing Rule Function

Implicit Sharing Rule Functional Form

The sharing rule underlying the consumption equations is then recovered up to an additive constant solving the system of differential equations of the partial effects $(\phi_x, \phi_{p_1}, \phi_{p_2}, \phi_{s_r})$

$$\ln\phi = \phi_x \ln x + \phi_{p_1} \ln p_1 + \phi_{p_2} \ln p_2 + \phi_{s_1} \ln s_1 + \phi_{s_2} \ln s_2,$$

with two distribution factors $r = 1, 2$. Note that this equation can be aggregated as

$$\ln\phi = \phi_x \ln x + \ln m(p_1, p_2, s_1, s_2),$$

where $\ln m(\cdot) = \phi_{p_1} \ln p_1 + \phi_{p_2} \ln p_2 + \phi_{s_1} \ln s_1 + \phi_{s_2} \ln s_2$.

Specification of the Consumption Model

Identification of the Sharing Rule - Recovering the Structural Demand Parameters

The parameters of the structural demand equations (1.S) and (2.S) $\alpha_{p_1}, \alpha_x, \beta_{p_2}$ and β_x are recovered as follows, for member 1

$$\alpha_x = a_x - b_x \frac{a_{s_r}}{b_{s_r}},$$

$$\alpha_{p_1} = a_{p_1} - a_{s_r} \frac{b_{p_1}}{b_{s_r}},$$

and for member 2

$$\begin{aligned} \beta_x &= b_x - a_x \frac{b_{s_r}}{a_{s_r}}, \\ \beta_{p_2} &= b_{p_2} - a_{p_2} \frac{b_{s_r}}{a_{s_r}}. \end{aligned}$$

What is the source of the identification problem?

$$\ln c_S^1 = \alpha_0 + \sum_{j=1}^{D} \alpha_j \ln d_j + \alpha_{p_1} \ln p_1 + \alpha_x \ln \phi \left(p_1, p_2, x, s\right)$$

where, zooming on the last term:

$$\alpha_x \ln \phi \left(p_1, p_2, x, s\right) = \alpha_x \left(\phi_x \ln x + \phi_{p_1} \ln p_1 + \phi_{p_2} \ln p_2 + \phi_{s_1} \ln s_1 + \phi_{s_2} \ln s_2\right)$$

we observe the source of the identification because, e.g.
$A_x = \alpha_x \phi_x$, thus making it impossible to estimate the model directly.

Summarizing

1. Up to now, we have shown
   1. Identification of the sharing rule given a linear functional form of the demand model
   2. We learned about a novel property: the income proportionality condition
   3. a mixed property in the sense that the ratio of reduced form parameters of distribution factors must be proportional to the ratio of the income parameters of the structure
2. Next, we re-do the same exercise, but now we adopt the same functional form of the sharing rule shown above. This functional form of the sharing rule has been also used by Browning et al. (1994), Lewbel and Pendakur (2008), and Lise and Seiz (2010).

Outline

1. Objectives
2. The Collective Model
3. Identification of the Sharing Rule
   - Specification of the Consumption Model
   - Specification of the Consumption Model and of the Sharing Rule
   - Summary of the identifying restrictions
4. Empirical Results
   - Estimation Methods
   - Estimation Results
5. The Collective Demand System
   - Individual Specific System with Common Sharing Rule
   - Hybrid - Household System with Individual Engel Curves

Specification of the Sharing Rule

Aim and Procedure

1. The aim is now to uncover the identifying conditions underlying the structural specification of the demand equations and of the sharing rule. We then compare the identifying conditions.
2. The procedure is:
    1. Specification of a functional form for the sharing rule.
    2. Using Proposition 1, we derive the Jacobian matrices of the structural and reduced demand equations as done before in order to unveil the partial effects of the sharing rule $\{\phi_x, \phi_p, \phi_s\}$ and the structural parameters $\{\alpha, \beta\}$.

Sharing Rule: Functional Form Specification/1

Sharing Rule Functional Form

The amount of family resources allocated to member *1* is modelled by the following functional form

$$\phi_1 (p_1, p_2, x, s) = xm (p_1, p_2, s),$$

and household resources allocated to member 2 are obtained as $\phi_2 = x - \phi_1 (p_1, p_2, x, s) = x (1 - m (p_1, p_2, s))$, such that $\phi_1 (p_1, p_2, x, s) + \phi_2 (p_1, p_2, x, s) = x$.

Sharing Rule: Functional Form Specification/2

Note that given the structural definition of the sharing rule of equation, distribution factors are not a key information for the identification of the structure underlying collective models.

The Income Scaling Function

The income scaling function $m(p_1, p_2, s)$ takes the Cobb-Douglas form

$$m(p_1, p_2, s) = p_1^{\phi_{p1}} p_2^{\phi_{p2}} s_1^{\phi_{s_1}} s_2^{\phi_{s_2}},$$

where we limit the analysis to two distribution factors s_1 and s_2. It is worth remarking that the scaling function $m(p_1, p_2, s)$ depends on individual prices p_1, p_2 and distribution factors s only, but not on total expenditure.

Sharing Rule: Functional Form Specification/3

Property of the Scaling Function

For $\phi_1(p_1, p_2, x, s)$ be the amount of family resources allocated to member 1, the scaling function $m(p_1, p_2, s)$ must satisfy the condition that $m(p_1, p_2, s) \in (0, 1)$. In consumption models, the extreme values 0 and 1 are not admissible because when $m(p_1, p_2, s) = 1$, then $\phi_2 = 0$ and member 2 has no monetary resources to support his consumption. When $m(p_1, p_2, s) = 0$, then $\phi_1 = 0$ and similarly for member 1 will be equal to zero. In consumption models both solutions are meaningless.

Remark: Is there any plausible restriction keeping $m(p_1, p_2, s) \in (0, 1)$?

Structural Demand Functions

Substituting the sharing rule into the structural demand equations (1.S) and (2.S) we obtain

$$\ln c_S^1 = \alpha_0 + \sum_{i=1}^{D} \alpha_{d_i} d_{1i} + \alpha_{p_1} \ln p_1 + \alpha_x \ln \left(xm \left(p_1, p_2, s \right) \right),$$

$$\ln c_S^2 = \beta_0 + \sum_{i=1}^{D} \beta_{d_i} d_{2i} + \beta_{p_1} \ln p_2 + \beta_x \ln \left(x \left(1 - m \left(p_1, p_2, s \right) \right) \right),$$

where $\ln(1 - m(\cdot))$ can be linearly approximated to $-m(\cdot)$ for sufficiently small values of m, and equation is simplified to

$$\ln c_S^2 = \beta_0 + \sum_{i=1}^{D} \beta_{d_i} d_{2i} + \beta_{p_2} \ln p_2 + \beta_x \ln x - \beta_x \ln m \left(p_1, p_2, s \right).$$

Income Proportionality Property

Income Proportionality Property

Given the parametric specification of the collective consumption model, and the sharing rule, if there exists at least one distribution factor then the ratio of the reduced parameters of the distribution factor must be proportional to the (negative) ratio of the reduced income parameters

$$\frac{a_{s_r}}{b_{s_r}} = -\frac{a_x}{b_x} = -\frac{\alpha_x}{\beta_x},$$

because $\alpha_x = a_x$ and $\beta_x = b_x$.

Remark: Recall that $\phi_x = \frac{a_x}{a_x - b_x \frac{a_{sr}}{b_{sr}}}$. Therefore, given the income proprotionality property, then

$$\phi_x = \frac{1}{\cdot}.$$

Recovering the partials of the Sharing Rule

Using Proposition 1 again, the partial effects of the sharing rule become $\{\phi_{p_1}, \phi_{p_2}, \phi_s\}$.

$$\phi_{p_1} = -\frac{b_{p_1}}{b_x},$$

$$\phi_{p_2} = \frac{a_{p_2}}{a_x},$$

and

$$\phi_{s_r} = \frac{a_{s_r}}{a_x}.$$

Note that, conditional upon the specification of the sharing rule that has an explicit income term, the partial effect ϕ_x is not needed. Note also the simpler form of the restrictions.

Recovering the Structural Demand Parameters

The structural demand parameters for member 1 become

$$\alpha_x = a_x,$$

$$\alpha_{p_1} = a_{p_1} + b_{p_1}\frac{a_x}{b_x},$$

and for member 2

$$\beta_x = b_x,$$

$$\beta_{p_2} = b_{p_2} + a_{p_2}\frac{b_x}{a_x}.$$

Household or Individual Total Expenditure?
A Novel Issue

Individual Total Expenditure

We argue that total expenditure at the individual level (rather than at the household level) has to be used to identify the sharing rule.

Therefore the structural sharing rule becomes

$$\phi_1^*\left(p_1, p_2, x, s\right) = x_1 m^*\left(p_1, p_2, s\right),$$

with $\phi_2^*\left(p_1, p_2, x, s\right) = x - \phi_1^*\left(p_1, p_2, x, s\right)$. The value of the scaling function $m^*\left(p_1, p_2, s\right)$ must be bounded between $0 < m^*\left(p_1, p_2, s\right) < \frac{x}{x_i}$.

Outline

1. Objectives
2. The Collective Model
3. Identification of the Sharing Rule
   - Specification of the Consumption Model
   - Specification of the Consumption Model and of the Sharing Rule
   - Summary of the identifying restrictions
4. Empirical Results
   - Estimation Methods
   - Estimation Results
5. The Collective Demand System
   - Individual Specific System with Common Sharing Rule
   - Hybrid - Household System with Individual Engel Curves

Summary of the identifying restrictions

Table 1. Identification Conditions

		Restrictions		
		$\frac{a_{s_1}}{b_{s_1}} = \frac{a_{s_2}}{b_{s_2}}$		$\frac{a_{s_1}}{b_{s_1}} = \frac{a_{s_2}}{b_{s_2}} = -\frac{a_x}{b_x}$
ϕ_x	$=$	$\frac{a_x}{a_x - b_x \frac{a_{s_1}}{b_{s_1}}}$	$=$	$\frac{1}{2}$
ϕ_{p_1}	$=$	$\frac{b_{p_1}}{a_x \frac{b_{s_1}}{a_{s_1}} - b_x}$	$=$	$-\frac{b_{p_1}}{b_x}$
ϕ_{p_2}	$=$	$\frac{a_{p_2}}{a_x - b_x \frac{a_{s_1}}{b_{s_1}}}$	$=$	$\frac{a_{p_2}}{a_x}$
ϕ_{s_1}	$=$	$\frac{a_{s_1}}{a_x - b_x \frac{a_{s_1}}{b_{s_1}}}$	$=$	$\frac{a_{s_1}}{a_x}$
ϕ_{s_2}	$=$	$\frac{a_{s_2}}{a_x - b_x \frac{a_{s2}}{b_{s_2}}}$	$=$	$\frac{a_{s_2}}{a_x}$

Data

Data: A Sample of Italian Households

Variable	Definition	Mean	Std. Dev.
$\ln q_p$	Log of parents' clothing demand	5.752	0.492
$\ln q_c$	Log of child's clothing demand	5.941	0.716
$\ln p_p$	Log of parent clothing price	-0.831	0.773
$\ln p_c$	Log of child clothing price	-1.708	1.019
$\ln x_p$	Log of parent individual expenditure	5.016	0.326
$\ln x_c$	Log of child individual expenditure	2.512	0.157
$\ln x$	Log of household total expenditure	7.527	0.479
North	$= 1$ if family located in North regions	0.257	
South	$= 1$ if family located in South or Islands	0.337	
December	$= 1$ if family interviewed in December	0.087	
DE	$= 1$ if double-earner family	0.588	
Parent age	Parents' age in classes	7.505	1.427
Child age	Child's age in classes	1.684	0.706
Child 1517	$= 1$ if dependent child 15-17 years old	0.141	
Ageratio	Age ratio: age wife/(age wife + age husband)	0.478	0.034
Eduratio	Education ratio: edu. wife/(edu. wife + edu. husband)	0.488	0.084

Outline

1. Objectives
2. The Collective Model
3. Identification of the Sharing Rule
   - Specification of the Consumption Model
   - Specification of the Consumption Model and of the Sharing Rule
   - Summary of the identifying restrictions
4. Empirical Results
   - Estimation Methods
   - Estimation Results
5. The Collective Demand System
   - Individual Specific System with Common Sharing Rule
   - Hybrid - Household System with Individual Engel Curves

Estimation Methods

We use a 2 equation system (husband and wife) of the demand for clothing and estimate it using both a:

1. Maximum Likelihood (Direct estimation of the structure)
2. Minimum Distance (Indirect estimation of the reduced form)
    1. I stage: Estimate the reduced form
    2. II stage: Impose restrictions by min distance between structural and reduced form parameters

Structural Estimation (Maximum Likelihood)

Outline

1. Objectives
2. The Collective Model
3. Identification of the Sharing Rule
   - Specification of the Consumption Model
   - Specification of the Consumption Model and of the Sharing Rule
   - Summary of the identifying restrictions
4. Empirical Results
   - Estimation Methods
   - Estimation Results
5. The Collective Demand System
   - Individual Specific System with Common Sharing Rule
   - Hybrid - Household System with Individual Engel Curves

	Reduced Method	*Structural Method*	
		x	x_i
ϕ_{p_1}	0.075(0.026)	0.074(0.025)	0.049(0.017)
ϕ_{p_2}	0.082(0.016)	0.082(0.013)	0.109(0.017)
ϕ_x	0.500(0.010)		
ϕ_{s_1}	−0.057(0.078)	−0.057(0.074)	−0.072(0.053)
ϕ_{s_2}	0.026(0.023)	0.027(0.021)	0.028(0.021)

Note: Standard errors are in parenthesis.

	Reduced Method	*Structural Method*	
		x	x_i
α_{p_1}	$-1.138(0.057)$	$-1.138(0.057)$	$-1.089(0.039)$
β_{p_2}	$-0.713(0.028)$	$-0.713(0.028)$	$-0.632(0.039)$
α_x	$1.287(0.055)$	$1.287(0.057)$	$0.964(0.042)$
β_x	$0.991(0.062)$	$0.989(0.063)$	$1.487(0.094)$

Note: Standard errors are in parenthesis.

Outline

1. Objectives
2. The Collective Model
3. Identification of the Sharing Rule
   - Specification of the Consumption Model
   - Specification of the Consumption Model and of the Sharing Rule
   - Summary of the identifying restrictions
4. Empirical Results
   - Estimation Methods
   - Estimation Results
5. The Collective Demand System
   - Individual Specific System with Common Sharing Rule
   - Hybrid - Household System with Individual Engel Curves

Individual Specific Collective Demand System/1

- We assume individual preferences of Gorman polar form linear in individual full income Y_i and demographically transformed using the translating technique (Pollak and Wales 1971). Note: this is an extension of Apps-Rees model within the collective framework. Very data demanding.
- Thus the associated AIDS indirect utility function for individual $i = 1, 2$ is

$$V^i(P_{ik}, Y_i; d_i, d_f) = \frac{\ln \phi_i(\xi, Y_i) - \ln \wp_i^T(d_i, P_{ik}) - \ln A_i(P_{ik})}{B_i(P_{ik})},$$

where $P_{ik} = \{p_i, p_z, w_i\}$ is the set of prices differentiated by individual i and $\ln \wp_i^T = \sum_k t_{ik}(d_i) \ln(P_{ik})$ is the individual specific fixed cost component associated with the demographic characteristics.

Individual Specific Collective Demand System/2

- The function $\phi_i(\xi, Y_i) = Y_i m(\xi)$ is the sharing rule where ξ is a set of variables explaining the decision process within the family.
- The price indexes are $\ln A_i(P_{ik}) = \alpha_{i0} + \sum_{k=1}^{K} \alpha_{ik} \ln P_{ik} + \frac{1}{2} \sum_{r=1}^{R} \sum_{k=1}^{K} \nu_{ikr} \ln P_{ir} \ln P_{ik}$, and $B_i(P_{ik}) = \beta_{i0} \prod_{k=1}^{K} P_{ik}^{\beta_{ik}}$.
- Roy's identity yields the following system of modified share equations

$$s_{ik} = \alpha_{ik} + t_{ik}(d_i) + \sum_{r=1}^{R} \nu_{ikr} \ln P_{ir} + \beta_i \ln \left(\frac{\phi_i^*(\xi, Y_i, d_i)}{A_i(P_{ik})} \right) + \varepsilon_{ik},$$

where $s_{ik} = P_{ik} k^i / Y_i$, $\phi_i^* = \phi_i(\xi, Y_i) - \sum_k t_{ik}(d_i) \ln(P_{ik})$, and ε_{ik} is the error term assumed to be independent and identically distributed.

Empirical Evidences

prenidle da caumi

Outline

1. Objectives
2. The Collective Model
3. Identification of the Sharing Rule
   - Specification of the Consumption Model
   - Specification of the Consumption Model and of the Sharing Rule
   - Summary of the identifying restrictions
4. Empirical Results
   - Estimation Methods
   - Estimation Results
5. The Collective Demand System
   - Individual Specific System with Common Sharing Rule
   - Hybrid - Household System with Individual Engel Curves

Hybrid Collective Demand System/1

- Consider the following collective PIGLOG form, without demographic modifications, separable in the utilities of the household members

$$y(p,u) = A(p)\,B(p)^{u_1}\,B(p)^{u_2} = A(p)\,B(p)^{(u_1+u_2)},$$

where u_i is the welfare level of individual i, and the terms $A(p)$ and $B(p)$ are price aggregators.
- In the logarithms the hybrid cost function is $\ln y(p,u) = \ln A(p) + u_1 \ln B(p) + u_2 \ln B(p)$.

Hybrid Collective Demand System/2

- Then differentiation the cost function with respect to prices gives the AIDS budget shares as a function of prices and utility

$$w_i = \frac{\partial \ln y(p,u)}{\partial \ln p_i} = \alpha_i + \sum \gamma_{ij}^{*} \ln p_j + \beta_i u_1 ln \left(\beta_0 \prod_i p_i^{\beta_i} \right) + \beta_i u_2 ln \left(\beta_0 \prod_i p_i^{\beta_i} \right)$$

- Substitution of the individual utiity $u_i = \frac{\ln y_i - \ln A(p)}{\ln B(p)}$ $i{=}1{,}2$ gives the collective hybrid budget shares as a function of prices and individual incomes as

$$w_i = \alpha_i + 0.5 \sum_j \gamma_{ij} \ln p_j + \beta_{1i} \left[\ln y - \ln A(p) \right] + \beta_{2i} \left[\ln y - \ln A(p) \right].$$

Individual Income elasticities					
	food	alcohol	clothing	edu.rec.	other
Husband	0.775 (0.291)	1.467 (0.182)	0.89 (0.105)	0.914 (0.132)	2.067 (0.030)
Wife	0.689 (0.024)	0.512 (0.124)	1.188 (0.158)	1.975 (0.111)	0.927 (0.026)
Compensated price elasticities					
	food	alcohol	clothing	edu.rec.	other
food	-0.931 (0.054)	0.045 (0.028)	0.558 (0.053)	0.424 (0.050)	0.838 (0.047)
alcohol	0.669 (0.173)	-1.153 (0.226)	0.100 (0.270)	0.243 (0.208)	1.050 (0.188)
clothing	0.747 (0.133)	0.009 (0.081)	-1.272 (0.403)	0.521 (0.124)	0.899 (0.152)
edu.rec.	1.475 (0.118)	-0.050 (0.065)	0.286 (0.158)	-2.805 (0.225)	1.905 (0.130)
other	0.862 (0.061)	0.045 (0.031)	0.500 (0.056)	0.401 (0.055)	-0.892 (0.082)

Note 1: Standard deviations in parentheses.

Conclusions

- There exists a correspondence between a reduced and structural estimations of the sharing rule
- The set of restrictions traditionally used in the reduced method is incomplete.
- Because the reduced and structural estimates of the sharing rule are statistically close, the structural estimation of the sharing rule is a feasible estimation strategy BUT using the best estimate of individual incomes . . .
- The estimation is also feasible at a system level both with a individual specific or a hybrid demand system.
- Next:
    - Extend the experiment to labor supply as in CFL using individual non labor incomes
    - Combine information from exclusive consumption with information coming from the consumption pattern of singles and household technologies.