Revealed preferences and intra-household allocation

Sixth Winter School on Inequality and Social Welfare Theory

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Introduction

- The topic of this Sixth Winter School is “Inequality and the family”
- An important issue in this respect is that one usually does not observe how the household’s resources are distributed among the household members
- Researchers often make use of equivalence scales to transform the household’s resources to individual resources
- These equivalence scales can be rather a-theoretic (e.g. OECD scale) or embedded in a structural consumption model (e.g. Barten scale)
- Both approaches depend on assumptions about the intra-household distribution of resources and the importance of economies of scale

Introduction

- Most studies tackle the issue by assuming a **unitary model**
  - Standard textbook model: see e.g. Samuelson's (1947) *Foundations of Economic Analysis* or Deaton and Muellbauer's (1980) *Economics and Consumer Behavior*
  - Households behave like single rational decision makers: utility function is maximized subject to a budget constraint
  - Generates the testable implications of adding-up, homogeneity, negativity and symmetry
  - A unique preference ordering obtains if the theoretical restrictions are satisfied
  - This allows to use the model to construct traditional equivalence scales

Introduction

- Some theoretical and empirical issues
  - The unitary model ignores the intra-household distribution of resources
  - The estimation of equivalence scales is faced with a fundamental identification problem: demand data only identify the shape and the ranking of indifference curves but not the utility level attached to each of these curves; this utility level is in general needed to calculate equivalence scales
  - Theoretical restrictions usually rejected when applied to multi-person households (but not when applied to singles)

Introduction

- In this lecture, we propose a different approach to go from household resources to individual resources
- The approach is based on the **collective model**
  - Chiappori (Ecma 1988, JPE 1992); Apps and Rees (JPubE 1988)
  - Multi-person households consist of different individuals with own rational preferences
  - Intra-household allocations are assumed to be Pareto-efficient
  - Generates testable implications which fit the data better than those of the unitary model
  - Individual preferences and the sharing rule (which governs how the household's resources are distributed among the household members) can be identified under some assumptions
  - Model allows welfare analyses at the individual level (specific application: Browning, Chiappori and Lewbel's (WP 2010) indifference scales)

Introduction

- The standard modelling approach (both for the unitary and the collective model) is to use a **parametric structure** for the preferences and the intra-household bargaining process
  - Fully characterized by Chiappori (Ecma 1988), Browning and Chiappori (Ecma 1998), Chiappori and Ekeland (JET 2006, Ecma 2009)
  - Differentiable approach: assumes a demand function of which the value is known (usually after estimation) for a continuous range of price - total expenditure combinations
  - Results are influenced by the chosen functional specification

Introduction

- Alternative modelling approach (both for the unitary and the collective model) is to opt for a **nonparametric "revealed preference" approach**
  - Samuelson (Econ 1938), Houthakker (Econ 1950), Afriat (IER 1967), Varian (Ecma 1982)
  - Revealed preference axioms (WARP, SARP, GARP)
  - This lecture is in the tradition of Afriat (IER 1967): finite set of quantity and price data observed
  - Analyzes choice behaviour without imposing any parametric structure on preferences or demand
  - Global approach rather than local differentiable approach

Introduction

- Aim of this lecture
  - Introduce you to the revealed preference (RP) approach to consumption behaviour
  - Discuss testable implications of different models
  - Discuss how one can identify information about the intra-household allocation of the household's resources

Overview

- RP characterization of the unitary model
- RP characterizations of collective models
- Some empirical results
- Conclusion

Overview

- RP characterization of the unitary model

RP characterization of the unitary model

- We observe a finite set of price-quantity data: $S = \{(\mathbf{p}_t ; \mathbf{q}_t) ; t = 1, ..., T\}$
- A unitary rationalization of the data set S implies that the household acts as a single decision maker

Definition (Unitary rationalization)

Let $S = \{(\mathbf{p}_t ; \mathbf{q}_t) ; t = 1, ..., T\}$ be a set of observations. A utility function U provides a *unitary rationalization* of S if for each observation t we have $U(\mathbf{q}_t) \geq U(\mathbf{z})$ for all $\mathbf{z}$ with $\mathbf{p}_t' \mathbf{z} \leq \mathbf{p}_t' \mathbf{q}_t$

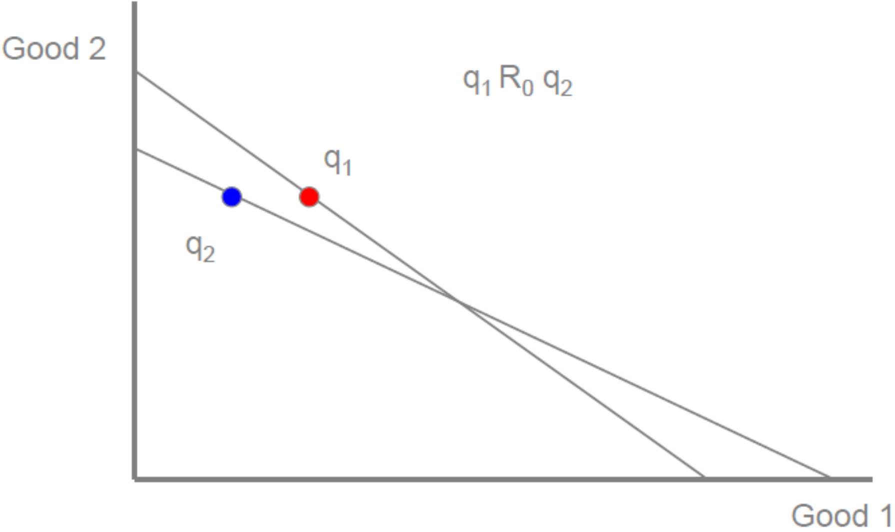
RP characterization of the unitary model

- Varian (Ecma 1982): a locally non-satiated utility function exists that provides a unitary rationalization of S if and only if the data satisfy the *Generalized Axiom of Revealed Preference* (*GARP*)

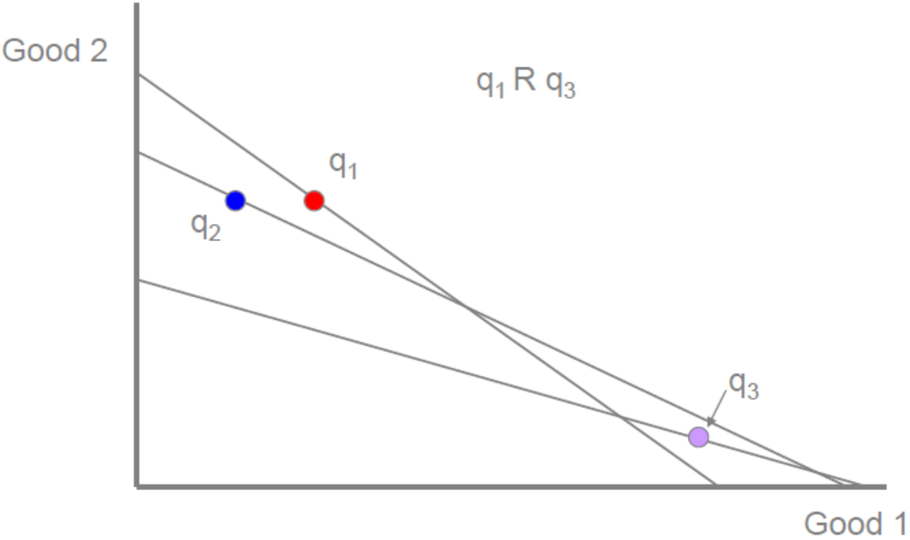
Definition (GARP)

The set $S = \{(\mathbf{p}_t; \mathbf{q}_t); t = 1, ..., T\}$ satisfies GARP if there exist relations R_0, R that meet:
(i) if $\mathbf{p}_s'\mathbf{q}_s \geq \mathbf{p}_s'\mathbf{q}_t$ then $\mathbf{q}_s\ R_0\ \mathbf{q}_t$;
(ii) if $\mathbf{q}_s\ R_0\ \mathbf{q}_u$, $\mathbf{q}_u\ R_0\ \mathbf{q}_v$, ..., $\mathbf{q}_w\ R_0\ \mathbf{q}_t$ for some (possibly empty) sequence $(u, v, ..., w)$ then $\mathbf{q}_s\ R\ \mathbf{q}_t$;
(iii) if $\mathbf{q}_s\ R\ \mathbf{q}_t$ then $\mathbf{p}_t'\mathbf{q}_t \leq \mathbf{p}_t'\mathbf{q}_s$

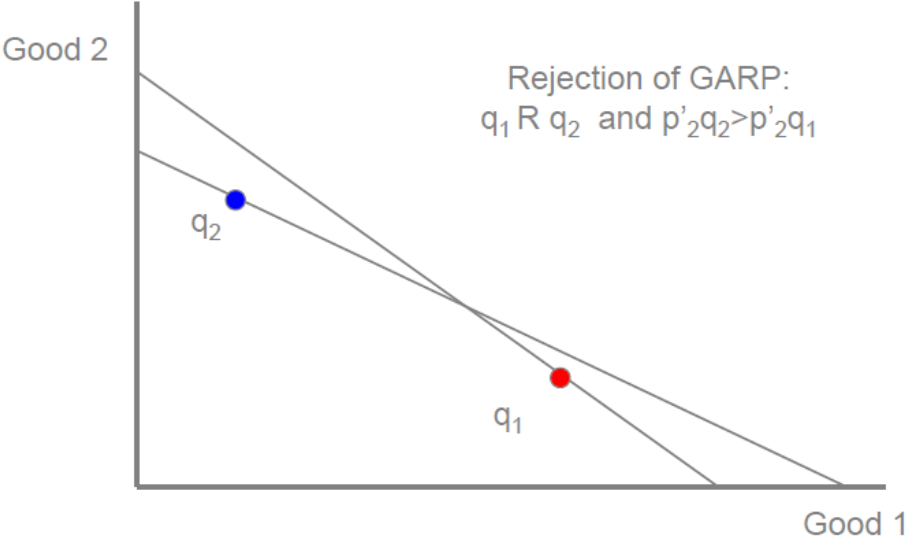
RP characterization of the unitary model



RP characterization of the unitary model



RP characterization of the unitary model



Overview

- RP characterization of the unitary model
- RP characterizations of collective models

RP characterizations of collective models

- We start with an RP characterization of collective models with private and public consumption; Cherchye, De Rock and Vermeulen (REStud 2011)
- Households consist of 2 household members (easily generalized for M members)
- Researcher knows which goods are privately consumed and which goods are publicly consumed; this is similar to Chiappori and Ekeland (Ecma 2009)
- Only aggregate quantities observed
- We observe a finite set of price-quantity data: $S = \{(\mathbf{p}_t, \mathbf{P}_t; \mathbf{q}_t, \mathbf{Q}_t); t = 1, ..., T\}$
- Household member m $(m = 1, 2)$ has preferences represented by the utility function $U^m(\mathbf{q}_t^m, \mathbf{Q}_t)$

RP characterizations of collective models

- Special cases of this model:
  - Egoistic model: $U^m\left(\mathbf{q}_t^m\right)$
  - Model with only public goods: $U^m\left(\mathbf{Q}_t\right)$
- Since the individual consumption of the private goods is not observed, we consider *feasible personalized quantities*

Definition (Feasible personalized quantities)

Let S be a set of observations. For each observation t, *feasible personalized quantities* $\mathfrak{q}_t^m \in \mathbb{R}_+^N$, $m = 1, \ldots, M$, satisfy $\mathfrak{q}_t^1 + \mathfrak{q}_t^2 = \mathbf{q}_t$

RP characterizations of collective models

- A collective rationalization (CR) of the data set S requires that the observed household consumption can be represented as a Pareto efficient outcome of some bargaining process

Definition (Collective rationalization)

Let S be a set of observations. A combination of utility functions U^1 and U^2 provides a collective rationalization of S if for each observation t there exist feasible personalized quantities $\mathfrak{q}_t^m$ and Pareto weights $\mu_t^m > 0$, $m = 1, 2$, such that

$$\mu_t^1 U^1\left(\mathfrak{q}_t^1, \mathbf{Q}_t\right) + \mu_t^2 U^2\left(\mathfrak{q}_t^2, \mathbf{Q}_t\right) \geq \mu_t^1 U^1\left(\mathfrak{z}^1, \mathbf{Z}\right) + \mu_t^2 U^2\left(\mathfrak{z}^2, \mathbf{Z}\right)$$

for all $\mathfrak{z}^m \in \mathbb{R}_+^N$ and $\mathbf{Z} \in \mathbb{R}_+^K$ such that $\mathbf{p}_t'\left(\mathfrak{z}^1 + \mathfrak{z}^2\right) + \mathbf{P}_t'\mathbf{Z} \leq \mathbf{p}_t'\mathbf{q}_t + \mathbf{P}_t'\mathbf{Q}_t$

RP characterizations of collective models

- To come to an RP characterization, we need to define *feasible personalized prices* (Lindahl prices) for the public goods; they capture the fraction of the market price that is borne by the household members

Definition (Feasible personalized prices)

Let S be a set of observations. For each observation t, *feasible personalized prices* $\mathfrak{P}_t^m \in \mathbb{R}_+^K$, $m = 1, 2$, satisfy $\mathfrak{P}_t^1 + \mathfrak{P}_t^2 = \mathbf{P}_t$

- Cherchye, De Rock and Vermeulen (REStud 2011) show that a CR is possible if and only if there exist feasible personalized prices and quantities such that GARP holds for both household member specific sets $(m = 1, 2)$

$$S^m = \{(\mathbf{p}_t, \mathfrak{P}_t^m; \mathfrak{z}^m, \mathbf{Q}_t); t = 1, ..., T\}$$

RP characterizations of collective models

- Setting allows a decentralized interpretation of collective rationality (see Chiappori (Ecma 1988, JPE 1992)):
  - Sharing rule distributes aggregate group income over household members
  - Each household member maximizes her/his utility subject to the given income share while accounting for personalized prices
- We are interested in the recovery of *feasible income shares*

Definition (Feasible income shares)

Consider feasible personalized prices and quantities for a set of observations S such that each set $\{(\mathbf{p}_t, \mathfrak{P}_t^m; \mathfrak{q}_t^m; \mathbf{Q}_t)\,;\, t = 1, ..., T\}$, $m = 1, 2$ satisfies *GARP*. For $y_t = \mathbf{p}_t'\mathbf{q}_t + \mathbf{P}_t'\mathbf{Q}_t$ the group income at observation t, the *feasible income share* for each member m at prices $\mathbf{p}_t$ and $\mathbf{P}_t$ is $\mathfrak{y}_t^m = \mathbf{p}_t'\mathfrak{q}_t^m + \mathfrak{P}_t^{m\prime}\mathbf{Q}_t$

RP characterizations of collective models

- The above characterization is not directly useful
- Observed prices and quantities define infinitely many specifications of feasible prices and quantities; each specification entails different revealed preference relations
- We therefore provide an equivalent *mixed integer linear programming* (MILP) characterization of collective rationality; allows using solution algorithms tailored for such problems
- The MILP formulation uses the binary variables $x_{st}^{m} \in \{0, 1\}$ where the variable equals 1 if household member m prefers the personalized quantity bundle in situation s to that in situation t for given personalized prices

RP characterizations of collective models

Proposition

Let S be a set of observations. There exists a combination of concave and continuous utility functions U^1 and U^2 that provide a collective rationalization of S if and only if there exist $\mathfrak{P}_t^m \in \mathbb{R}_+^K$, $\mathfrak{q}_t^m \in \mathbb{R}_+^N$, $\mathfrak{y}_t^m \in \mathbb{R}_+$ and $x_{st}^m \in \{0,1\}$, $m = 1,2$, that satisfy

(i) $\mathfrak{P}_t^1 + \mathfrak{P}_t^2 = \mathbf{P}_t$ *(i.e. personalized prices)*;

(ii) $\mathfrak{q}_t^1 + \mathfrak{q}_t^2 = \mathbf{q}_t$ *(i.e. personalized quantities)*;

(iii) $\mathfrak{y}_t^m = \mathbf{p}_t'\mathfrak{q}_t^m + \mathfrak{P}_t^{m\prime}\mathbf{Q}_t$ *(i.e. personal share)*;

(iv) $\mathfrak{y}_s^m - \mathbf{p}_s'\mathfrak{q}_t^m - \mathfrak{P}_s^{m\prime}\mathbf{Q}_t < y_s x_{st}^m$ *(i.e. if $\mathbf{p}_s'\mathfrak{q}_s^m + \mathfrak{P}_s^{m\prime}\mathbf{Q}_s \geq \mathbf{p}_s'\mathfrak{q}_t^m + \mathfrak{P}_s^{m\prime}\mathbf{Q}_t$ then $x_{st}^m = 1$)*;

(v) $x_{su}^m + x_{ut}^m \leq 1 + x_{st}^m$ *(i.e. transitivity)*;

(vi) $\mathfrak{y}_t^m - \mathbf{p}_t'\mathfrak{q}_s^m - \mathfrak{P}_t^{m\prime}\mathbf{Q}_s \leq y_t\left(1 - x_{st}^m\right)$ *(i.e. if $x_{st}^m = 1$ then $\mathbf{p}_t'\mathfrak{q}_t^m + \mathfrak{P}_t^{m\prime}\mathbf{Q}_t \leq \mathbf{p}_t'\mathfrak{q}_s^m + \mathfrak{P}_t^{m\prime}\mathbf{Q}_s$)*

RP characterizations of collective models

- *Testing consistency* with this model:
  - *Necessary* and *sufficient* RP test for any number of observations (also data sets with only a few observations)
  - Data can be collectively rationalized if the above MILP problem has a solution
  - First step of an empirical analysis
- *Recovery* and *forecasting*
  - Bounds on member specific consumption bundles and income shares
  - Add objective function to the MILP formulation (e.g. maximize η_t^1 or minimize η_t^1)
  - Second step of an empirical analysis: generates input for welfare analyses at the individual level

RP characterizations of collective models

- We now discuss the RP characterization of a more general collective model *à la* Browning and Chiappori (Ecma 1998); Cherchye, De Rock and Vermeulen (Ecma 2007, JET 2010)
- Households consist of 2 household members (easily generalized for M members)
- The model allows publicly consumed goods and externalities with respect to the privately consumed goods
- Researcher does not know what part of the consumption is privately consumed and what part of the consumption is publicly consumed, nor which consumption generates externalities
- Only aggregate quantities observed
- We observe a finite set of price-quantity data: $S = \{(\mathbf{p}_t; \mathbf{q}_t); t = 1, ..., T\}$
- Household member m $(m = 1, 2)$ has preferences represented by the utility function $U^m\left(\mathbf{q}_t^1, \mathbf{q}_t^2, \mathbf{q}_t^h\right)$, where $\mathbf{q}_t = \mathbf{q}_t^1 + \mathbf{q}_t^2 + \mathbf{q}_t^h$

RP characterizations of collective models

- A collective rationalization for this general model of the data set S requires again that the observed household consumption can be represented as a Pareto efficient outcome of some bargaining process

Definition (Collective rationalization general model)

Let S be a set of observations. A combination of utility functions U^1 and U^2 provides a collective rationalization of S if for each observation t there exist feasible personalized quantities $\widehat{\mathfrak{q}}_t = \left(\mathfrak{q}_t^1, \mathfrak{q}_t^2, \mathfrak{q}_t^h\right)$ and Pareto weights $\mu_t^m > 0$, $m = 1, 2$, such that

$$\mu_t^1 U^1\left(\mathfrak{q}_t^1, \mathfrak{q}_t^2, \mathfrak{q}_t^h\right) + \mu_t^2 U^2\left(\mathfrak{q}_t^1, \mathfrak{q}_t^2, \mathfrak{q}_t^h\right) \geq \mu_t^1 U^1\left(\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h\right) + \mu_t^2 U^2\left(\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h\right)$$

for all $\mathfrak{z}^1, \mathfrak{z}^2, \mathfrak{z}^h \in \mathbb{R}_+^N$ such that $\mathbf{p}_t'\left(\mathfrak{z}^1 + \mathfrak{z}^2 + \mathfrak{z}^h\right) \leq \mathbf{p}_t'\mathbf{q}_t$

RP characterizations of collective models

- We again need personalized (Lindahl) prices for the consumption bundles $\widehat{\mathfrak{q}}_t$: $\widehat{\mathfrak{p}}_t^1 = \left(\mathfrak{p}_t^1, \mathfrak{p}_t^2, \mathfrak{p}_t^h\right)$ and $\widehat{\mathfrak{p}}_t^2 = \left(\mathbf{p}_t - \mathfrak{p}_t^1, \mathbf{p}_t - \mathfrak{p}_t^2, \mathbf{p}_t - \mathfrak{p}_t^h\right)$
- Cherchye, De Rock and Vermeulen (Ecma 2007) show that a CR for this general model is possible if and only if there exist feasible personalized prices and quantities such that GARP holds for both member-specific sets $(m = 1, 2)$

$$S^m = \left\{\left(\widehat{\mathfrak{p}}_t^m; \widehat{\mathfrak{q}}_t\right); t = 1, ..., T\right\}$$

- Compare this with the GARP condition we had before: the condition for the general model turns out to be nonlinear in feasible prices and quantities, which makes it difficult to test the condition in practice
- Still, a necessity test can be derived that is formulated in terms of observable information

RP characterizations of collective models

- The idea is to make use of hypothetical member-specific preference relations (denoted by H_0^m and H^m)
- These hypothetical relations represent feasible specifications of the true individual preference relations in terms of observed prices and quantities
- A *necessary* RP condition then requires that there must exist at least one specification of the hypothetical member-specific preference relations that simultaneously meet a set of CR conditions

RP characterizations of collective models

Proposition

Suppose that there exists a pair of utility functions U^1 *and* U^2 *that provide a collective rationalization of the set of observations* $S = \{(\mathbf{p}_t; \mathbf{q}_t); t = 1, ..., T\}$. *Then there exist hypothetical relations* H_0^m, H^m *for each member* $m \in \{1, 2\}$ *such that:*

(i) if $\mathbf{p}_s'\mathbf{q}_s \geq \mathbf{p}_s'\mathbf{q}_t$, *then* $\mathbf{q}_s H_0^1 \mathbf{q}_t$ *or* $\mathbf{q}_s H_0^2 \mathbf{q}_t$;

(ii) if $\mathbf{q}_s H_0^m \mathbf{q}_k, \mathbf{q}_k H_0^m \mathbf{q}_l, ..., \mathbf{q}_z H_0^m \mathbf{q}_t$ *for some (possibly empty) sequence* $(k, l, ..., z)$, *then* $\mathbf{q}_s H^m \mathbf{q}_t$;

(iii) if $\mathbf{p}_s'\mathbf{q}_s \geq \mathbf{p}_s'\mathbf{q}_t$ *and* $\mathbf{q}_t H^m \mathbf{q}_s$, *then* $\mathbf{q}_s H_0^l \mathbf{q}_t$ *(with* $l \neq m$*);*

(iv) if $\mathbf{p}_s'\mathbf{q}_s \geq \mathbf{p}_s'(\mathbf{q}_{t_1} + \mathbf{q}_{t_2})$ *and* $\mathbf{q}_{t_1} H^m \mathbf{q}_s$, *then* $\mathbf{q}_s H_0^l \mathbf{q}_{t_2}$ *(with* $l \neq m$*);*

(v) $\begin{cases} \text{a) if } \mathbf{q}_s H^1 \mathbf{q}_t \text{ and } \mathbf{q}_s H^2 \mathbf{q}_t, \text{ then } \mathbf{p}_t'\mathbf{q}_t \leq \mathbf{p}_t'\mathbf{q}_s \\ \text{b) if } \mathbf{q}_{s_1} H^1 \mathbf{q}_t \text{ and } \mathbf{q}_{s_2} H^2 \mathbf{q}_t, \text{ then } \mathbf{p}_t'\mathbf{q}_t \leq \mathbf{p}_t'(\mathbf{q}_{s_1} + \mathbf{q}_{s_2}) \end{cases}$

RP characterizations of collective models

- In Cherchye, De Rock and Vermeulen (Ecma 2007, JPE 2009), some *sufficient* RP conditions for a CR are proposed (which differ from those where the nature of the goods is known a priori)
- One example is a *situation-dependent dictatorship*
  - There exists a partitioning of the observed set S into subsets $S^1 \subseteq S$ and $S^2 = S \backslash S^1$
  - Both subsets satisfy GARP
  - In S^1 (S^2), individual 1 (2) is the situation-dependent dictator

Overview

- RP characterization of the unitary model
- RP characterizations of collective models
- Some empirical results

Some empirical results

- Data from the Russia Longitudinal Monitoring Survey covering the period from 1994 to 2003
- 148 pure couples with both spouses employed which are 8 times observed
- 108 pure singles who are employed and who are 8 times observed
- Each household separately analyzed (no homogeneity across households assumed)

Some empirical results

- Test results from Cherchye, De Rock and Vermeulen (JPE 2009)

TABLE 1
UNITARY TEST RESULTS

	Frequency	Percentage
Singles:		
GARP rejected	0	.00
GARP not rejected	108	100.00
Couples:		
GARP rejected	31	20.95
GARP not rejected	117	79.05

Some empirical results

TABLE 2
NECESSITY TEST RESULTS

	Frequency	Percentage
	A. Necessity Test	
Collective rationality rejected	0	.00
Collective rationality not rejected	148	100.00
	B. Filtering Procedure	
Number of uninformative observations:		
0	0	.00
1	0	.00
2	0	.00
3	1	.68
4	1	.68
5	8	5.41
6	21	14.19
7	0	.00
8	117	79.05
	C. Subset Tests	
Number of subsets (of informative observations):		
0	117	79.05
1	30	20.27
2	1	.68

Some empirical results

TABLE 3
SUFFICIENCY TEST RESULTS

Model	Number of Rejections	Power 1	Power 2
α = .5	31	100.0	12.63
α = .495	19	100.0	11.74
α = .49	16	100.0	10.17
α = .47	5	100.0	5.89
α = .45	1	99.9	4.05
α = .4	0	96.3	2.15
α = .3	0	68.8	.77
α = .2	0	38.3	.32
α = .01	0	7.8	.06
α = 0*	0	7.5	.05

* Situation-dependent dictatorship.

Some empirical results

- Some test and recovery results from Cherchye, De Rock and Vermeulen (REStud 2011)
- Same 148 RLMS-couples as before
- All consumption assumed to be public: pass rate = 100%
- All consumption assumed to be private: pass rate = 100%
- Intermediate case: 3 public goods and 18 private goods with varying (assumed) assignability
  - 100% assignability: pass rate = 92.6%
  - 60% assignability: pass rate = 100%

Some empirical results

	$\theta = 1.00$ (137 households)				$\theta = 0.90$ (6 households)			
Observation	Lower bound		Upper bound		Lower bound		Upper bound	
	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
1	0.445	0.127	0.593	0.109	0.379	0.108	0.581	0.097
2	0.407	0.130	0.597	0.123	0.378	0.081	0.671	0.048
3	0.396	0.147	0.609	0.153	0.382	0.170	0.715	0.120
4	0.396	0.117	0.601	0.133	0.406	0.109	0.647	0.034
5	0.410	0.116	0.597	0.103	0.400	0.033	0.648	0.072
6	0.395	0.127	0.601	0.123	0.313	0.071	0.661	0.090
7	0.395	0.123	0.613	0.119	0.406	0.098	0.635	0.054
8	0.385	0.116	0.618	0.117	0.360	0.084	0.682	0.079

Overview

- RP characterization of the unitary model
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Conclusion

- *Testing*
  - We presented a series of RP tests for a variety of collective models that do not depend on any functional specification for demand, preferences or the intra-household bargaining process
  - Tests work for any number of observations (including small data sets, though the larger the data sets the more powerful the results)
- *Recovery*
  - Member-specific consumption bundles, personalized prices and income shares
  - The larger the data set and the information available, the sharper the lower and upper bounds on the unobservables