Unambiguous Comparison of Intersecting Distribution Functions

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How do we compare intersecting distribution functions?

Important issue in both policy work, descriptive analysis and causal inference:

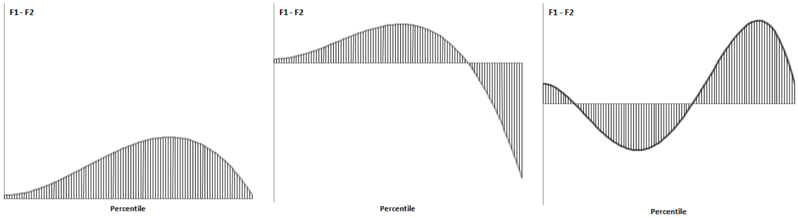
1. statistical offices and gov agencies compare distribution functions across countries, subgroups and time
2. descriptive research compares distributions of earnings, income, consumption and wealth to evaluate economic welfare
3. growing interest in econometrics in how to estimate the counterfactual outcome distribution
    - yet little attention has been devoted to how to compare counterfactual and actual outcome distributions

Suppose we want to rank the actual and counterfactual distributions, F_1 and F_2

- Straightforward with 1st or 2nd-degree dominance
  - but many empirical applications require weaker criteria

Theoretical literature: Offers higher order dominance criteria

Empirical literature: Tends to use parametric social welfare function



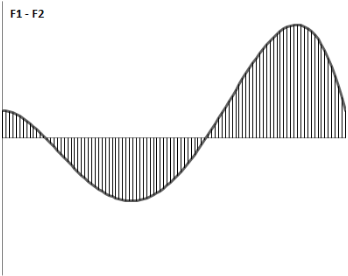
Concerns

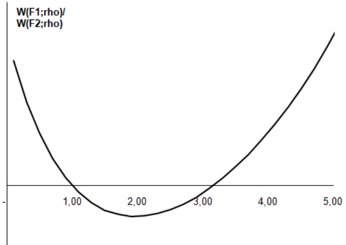
General dominance criteria: Hard to interpret and justify

- Rely on assumptions about third and higer order derivatives (see e.g Atkinson, 2003)

Parametric social welfare functions:

- Conclusion rests on more or less arbitrary parameter choice (and functional form)
- Ranking is non-monotonic in inequality aversion
  - An example: $W(F) = \int \frac{y^{1-\rho}}{1-\rho} dF(y), \quad \rho \in [0, \infty)$
    $\rho = 0$: ineq. neutral, $\rho = 1$: log, $\rho \to \infty$: mini-max





Outline

Aim: Proposes a general framework to *unambigously* compare *any set* of distributions functions in an *economically interpretable* way

1. Social welfare functions and 2nd-degree dominance
2. Social welfare functions and 3rd-degree upward and downward dominance
3. Social welfare functions and ith-degree upward and downward dominance
4. Parametric subfamilies
    - Upward: Gini family
    - Downward: Lorenz family
5. Asymptotic theory
6. Application

The general family of social welfare function

We will rely on the general family of rank-dependent measures of social welfare introduced by Yaari (1987,1988)

$$W_P(F) = \int_0^1 P'(t)F^{-1}(t)dt,$$

The weighting function P' is the derivative of a preference function that is a member of the following the set of preference functions:

$$\begin{aligned} \mathcal{P} &= \{P : P'(t) > 0 \textit{ and } P''(t) < 0 \\ &\quad \textit{for all } t \in (0,1),\ P(0) = P'(1) = 0,\ P(1) = 1\} \end{aligned}$$

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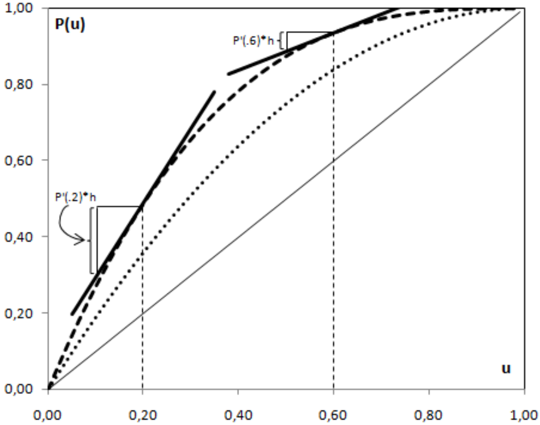
- W_P preserves 1st-degree dom, since $P'(t) > 0$, and
- W_P preserves 2nd-degree dom (and Pigou-Dalton), since $P''(t) < 0$
- $W_P \leq \mu_F$, and $W_P = \mu_F$ iff F is the egalitarian distribution

$P(t)$ reveals the inequality aversion profile of the social planner



Normative justification of the general family

The normative justification of W_P can be made in terms of a

(a) Theory for ranking distribution functions:

- With basic ordering and continuity assumptions, the dual independence axiom characterizes W_P (Yaari, 1988)

(b) Value judgement of the trade-off between the mean and (in)equality in the distributions (Ebert, 1987; Aaberge, 2001)

$$W_P = \mu_F[1 - J_P(F)]$$

where μ_F is the mean of F and
$J_P(F)$ is the family of rank-dependent measures of inequality aggregating the P'-weighted Lorenz curve of F

If we choose

$$P_{1k}(t) = 1 - (1-t)^{k-1},\ k > 2$$

then W_P is equal to the extended Gini family of social welfare functions (Donaldson and Weymark, 1980)

$$W_{G_k} \quad = \quad \mu\,[1 - G_k(F)] =, \qquad k > 2$$

where

- $G_k(F)$ is the extended Gini family of inequality measures
- $G_3(F)$ is the Gini coefficient and $W_{G_2} = \mu$
- Note that $\{\mu, W_{G_i}(F) :\ i = 3, 4, ...\}$ uniquely determines the distribution function F (Aaberge, 2000)

If we instead choose

$$P_{2k}(t) = \frac{(k-1)\,t - t^{k-1}}{k-2},\ k > 2$$

then W_P is the Lorenz family of social welfare functions (Aaberge, 2000)

$$W_{D_k} \quad = \quad \mu\left[1 - D_k(F)\right], \qquad k > 2$$

where

- $D_k(F)$ is the Lorenz family of inequality measures
- $D_3(F)$ is the Gini coefficient
- Note that $\{\mu, W_{D_i}(F) : i = 3, 4, ...\}$ uniquely determines the distribution function F (Aaberge, 2000)

Note that second degree inverse stochastic dominance is defined by

$$\Lambda_F^2(u) \equiv \int_0^u F^{-1}(t)dt, \quad u \in [0,1]$$

To define third degree upward inverse stochastic dominance, we use the notation

$$\Lambda_F^3(u) \equiv \int_0^u \Lambda_F^2(t)dt = \int_0^u (u-t)F^{-1}(t)dt, \quad u \in [0,1]$$

Definition

A distribution F_1 is said to *third* degree *upward inverse stochastic dominate* a distribution F_0 if and only if

$$\Lambda_{F_1}^3(u) \geq \Lambda_{F_0}^3(u) \text{ for all } u \in [0,1]$$

and the inequality holds strictly for some $u \in (0,1)$.

Proposition

Let F_1 and F_0 be members of F. Then the following statements are equivalent:

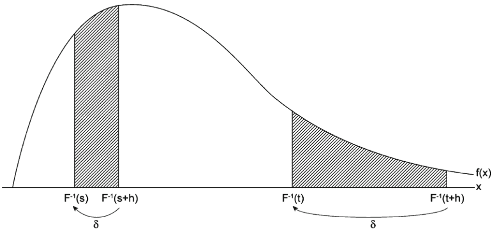
(i) F_1 third degree upward inverse stochastic dominates F_0

(ii) $\mu_{F_1}(u)\,(1 - G_3(u;\, F_1)) \geq \mu_{F_0}(u)\,(1 - G_3(u;\, F_0))$

for all $u \in [0, 1]$ *and the inequality holds strictly for some $u \in (0, 1)$.*

where:

- $\mu_F(u)$ is the quantile-specific lower tail mean
- $G_3(u;\, F)$ is the quantile-specific lower tail Gini coefficient
- $\mu_F(u)\,(1 - G_3(u;\, F))$ is the quantile-specific lower tail Gini social welfare function



$\Delta_s W_P(\delta, h)$: change in W_P of a fixed progressive transfer δ from an individual with rank $s+h$ to an individual with rank s.
$\Delta^1_{st} W_P(\delta, h) \equiv \Delta_s W_P(\delta, h) - \Delta_t W_P(\delta, h)$.

Definition

(Zoli, 1999; Aaberge, 2000, 2009) W_P satisfies the principle of first degree downside positional transfer sensitivity (DPTS) if and only if

$$\Delta^1_{st} W_P(\delta, h) > 0, \quad \text{when } s < t.$$

Let $\mathcal{P}_3$ be the family of preference functions defined by

$$\mathcal{P}_3 = \left\{ P \in \mathcal{P} : P^{'''}(t) > 0, \right\}$$

Theorem

Let F_1 and F_0 be members of F. Then the following statements are equivalent.
(i) F_1 third-degree upward inverse stochastic dominates F_0
(ii) $W_P(F_1) > W_P(F_0)$ for all $P \in \mathcal{P}_3$
(iii) $W_P(F_1) > W_P(F_0)$ for all $P \in \mathcal{P}$ where W_P satisfies first-degree DPTS

- ⇒ (i) and (ii): least-restrictive set of social welfare functions that unambiguously rank in accordance with 3-UID
- ⇒ (i) and (iii): normative justification for 3-UID

Upward vs. downward dominance

Upward dominance criteria justified through DPTS

- More sensitive to changes in the lower part of the distribution

Issues with upward dominance criteria:

- Prone to measurement error in the lower tail
- Changes in the upper part may be viewed as more important
  - Long vs. short transfers
  - Upper part is the focus: Test scores, top income, etc.

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We propose a complementary sequence of dominance criteria:

$\Rightarrow$ Downward inverse stochastic dominance

- More sensitive to changes in the upper part of the distribituion

Sequences coincide at 2nd-degree dom.$\Rightarrow$ both obey Pigou-Dalton

The criteria of 3rd-order downward inverse dominance aggregates $\Lambda_F^2(u)$ from above (rather than from below). Let

$$\tilde{\Lambda}_F^3(u) \equiv \int_u^1 \Lambda_F^2(t)dt = (1-u)\mu_F - \int_u^1 (t-u)F^{-1}(t)dt,\ u \in [0,1]$$

Definition

A distribution F_1 is said to *third* degree *downward inverse stochastic dominate* a distribution F_0 if and only if

$$\tilde{\Lambda}_{F_1}^3(u) \geq \tilde{\Lambda}_{F_0}^3(u) \text{ for all } u \in [0,1]$$

and the inequality holds strictly for some $u \in (0,1)$.

Proposition

Let F_1 and F_2 be members of F. Then the following statements are equivalent:

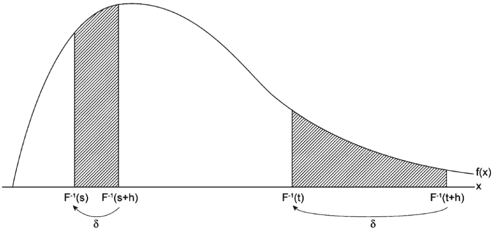
(i) F_1 third degree downward inverse stochastic dominates F_2

(ii) $\tilde{\mu}_{F_1}(u)\,(1 - D_3(u;\, F_1)) \geq \tilde{\mu}_{F_2}(u)\,(1 - D_3(u;\, F_2))$

for all $u \in [0, 1]$ *and the inequality holds strictly for some* $u \in (0, 1)$

where:

- $\tilde{\mu}_F(u)$ is the quantile-specific upper tail mean
- $D_3(u;\, F)$ is the quantile-specific upper tail Gini coefficient
- $\tilde{\mu}_F(u)\,(1 - D_3(u;\, F))$ is the quantile-specific upper tail Gini social welfare function



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$\Delta^1_{st} W_P(\delta, h) \equiv \Delta_s W_P(\delta, h) - \Delta_t W_P(\delta, h)$.

Definition

(Aaberge, 2009) W_P satisfies the principle of first degree upside positional transfer sensitivity (UPTS) if and only if

$$\Delta^1_{st} W_P(\delta, h) < 0, \quad \text{when } s < t.$$

Let $\widetilde{\mathcal{P}}_3$ be the family of preference functions defined by

$$\tilde{\mathcal{P}}_3 = \left\{ P \in \mathcal{P} : P^{'''}(t) < 0 \right\}.$$

Theorem

Let F_1 and F_0 be members of F. Then the following statements are equivalent.
(i) F_1 third-degree downward inverse stochastic dominates F_0
(ii) $W_P(F_1) > W_P(F_0)$ for all $P \in \tilde{\mathcal{P}}_3$
(iii) $W_P(F_1) > W_P(F_0)$ for all $P \in \mathcal{P}$ where W_P satisfies first-degree UPTS

- ⇒ (i) and (ii): least-restrictive set of social welfare functions that unambiguously rank in accordance with 3-DID
- ⇒ (i) and (iii): normative justification for 3-DID

To define upward inverse stochastic dominance of degree i, we use the notation

$$\begin{aligned} \Lambda_F^i(u) &= \int_0^u \Lambda_F^{i-1}(t)dt = \frac{1}{(i-3)!}\int_0^u (u-t)^{i-3}\Lambda_F^2(t)dt \\ &= \frac{1}{(i-2)!}\int_0^u (u-t)^{i-2}F^{-1}(t)dt,\ i>2 \end{aligned}$$

Definition

A distribution F_1 is said to ith degree upward inverse stochastic dominate F_0 for $i > 2$ if and only if

$$\Lambda_{F_1}^i(u) \geq \Lambda_{F_0}^i(u) \text{ for all } u \in [0,1]$$

and the inequality holds strictly for some $u \in (0,1)$.

Proposition

Let F_0 and F_1 be members of $\mathcal{F}$. Then for i = 3,4, ... the following statements are equivalent:

(i) F_1 i^{th} *degree upward inverse stochastic dominates* F_0
(ii) $\mu_{F_1}(u)\,(1 - G_i(u;\, F_1)) \geq \mu_{F_0}(u)\,(1 - G_i(u;\, F_0))$.

for all $u \in [0,1]$ *and the inequality holds strictly for some* $u \in (0,1)$.

where:

- $G_i(u;\, F)$ is the quantile-specific lower tail i^{th} member of the Gini family of inequality measures
- $\mu_F(u)\,(1 - G_i(u;\, F))$ is the quantile-specific lower tail i^{th} member of the Gini family of social welfare functions

The family of preference functions $\mathcal{P}_i$ is defined by

$$\mathcal{P}_i = \Big\{P \in \mathcal{P} : (-1)^{i-1}P^{(i)}(t) > 0 \text{ with } P^{(j)} \text{ continuous on } (0,1) \text{ and } (-1)^{i-1}P^{(j)}(1) \geq 0 \text{ for all } j = 3, 4, \dots, i-1\Big\}$$

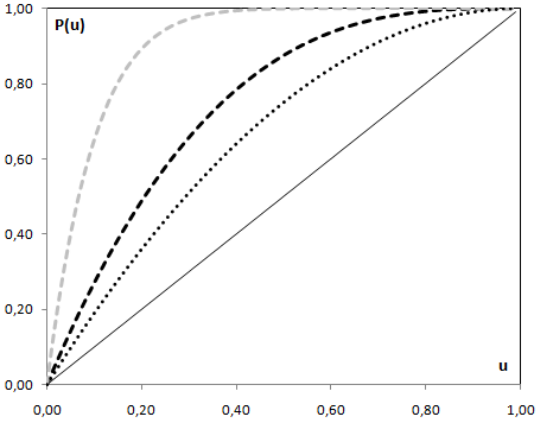
where $P^{(i)}$ denote the ith degree derivative of P.

Theorem

Let F_1 and F_0 be members of $\mathcal{F}$. Then for i = 3,4, ... the following statements are equivalent,
(i) F_1 ith degree upward inverse stochastic dominates F_0
(ii) $W_P(F_1) > W_P(F_0)$ for all $P \in \mathcal{P}_i$
(iii) $W_P(F_1) > W_P(F_0)$ for all $P \in \mathcal{P}$ where W_P satisfies DPTS of degree $i-2$

- ⇒ (i) and (ii): least-restrictive set of social welfare functions that unambiguously rank in accordance with ith-degree UID
- ⇒ (i) and (iii): normative justification for ith-degree UID

Upward dominance: The weighting function



To define downward inverse stochastic dominance of degree i, we use the notation

$$\tilde{\Lambda}^i_F(u) = \int_u^1 \tilde{\Lambda}^{i-1}_F(u) = \frac{1}{(i-3)!}\int_u^1 (t-u)^{i-3}\Lambda^2_F(t)dt = \frac{1}{(i-2)!}\left[(1-u)^{i-2}\mu_F - \int_u^1 (t-u)^{i-2}F^{-1}(t)dt\right]$$

$$i = 3, 4, \dots$$

Definition

A distribution F_1 is said to ith degree downward inverse stochastic dominate F_0 for $i > 2$ if and only if

$$\tilde{\Lambda}^i_{F_1}(u) \geq \tilde{\Lambda}^i_{F_0}(u) \text{ for all } u \in [0,1]$$

and the inequality holds strictly for some $u \in (0,1)$.

Proposition

Let F_0 and F_1 be members of $\mathcal{F}$. Then for $i = 3,4, \ldots$ the following statements are equivalent:

(i) F_1 *i^{th} degree downward inverse stochastic dominates* F_0

(ii) $\tilde{\mu}_{F_1}(u)\,(1 - D_i(u;\, F_1)) \geq \tilde{\mu}_{F_0}(u)\,(1 - D_i(u;\, F_0))$.

for all $u \in [0, 1]$ *and the inequality holds strictly for some* $u \in (0, 1)$.

where:

- $D_i(u;\, F)$ is the quantile-specific upper tail i^{th} member of the Lorenz family of inequality measures
- $\tilde{\mu}_F(u)\,(1 - D_i(u;\, F))$ is the quantile-specific upper tail i^{th} member of the Lorenz family of social welfare functions

The family of preference functions $\widetilde{\mathcal{P}}_i$ is defined by

$$\widetilde{\mathcal{P}}_i = \Big\{ P \in \mathcal{P} : P^{(i)}(t) < 0 \text{ with } P^{(j)} \text{ continuous on } (0,1) \quad (1)$$
$$\text{and } P^{(j)}(0) \leq 0 \text{ for all } j = 3, 4, \ldots, i-1 \Big\}$$

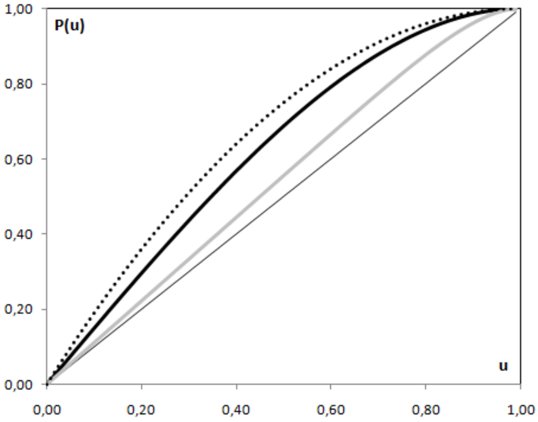
where $P^{(i)}$ denote the ith degree derivative of P.

Theorem

Let F_1 and F_0 be members of $\mathcal{F}$. Then for $i = 3, 4 \ldots$ the following statements are equivalent
(i) F_1 ith degree downward inverse stochastic dominates F_0
(ii) $W_P(F_1) > W_P(F_0)$ for all $P \in \widetilde{\mathcal{P}}_i$
(iii) $W_P(F_1) > W_P(F_0)$ for all $P \in \mathcal{P}$ where W_P satisfies UPTS of degree $i-2$

- ⇒ (i) and (ii): least-restrictive set of social welfare functions that unambiguously rank in accordance with ith-degree DID
- ⇒ (i) and (iii): normative justification for ith-degree DID

Downward dominance: The weighting function



As i goes to infinity, we get from the definitions of upward and downward dominance:

$$(i-1)!\Lambda^i(u) \rightarrow \begin{cases} 0, & 0 \leq u < 1 \\ F^{-1}(0+), & u = 1 \end{cases}$$

$$(i-2)!\tilde{\Lambda}(u) \rightarrow \begin{cases} \mu - F^{-1}(1-)dt, & u = 0 \\ 0, & 0 < u \leq 1 \end{cases}$$

where $F^{-1}(0+)$ and $F^{-1}(1-)$ denote the lowest and highest income in F

- Limit upward dominance: Social welfare function corresponding to the (Rawlsian) maximin criterion
- Limit downward dominance: Social welfare function approaches the utilitarian criterion

Proposition

Let F_1 and F_0 be members of F. Then for $i = 3, 4..$

(i) F_1 i^{th} degree upward inverse stochastic dominates F_0 implies

(ii) $W_{G_k}(F_1) > W_{G_k}(F_0)$ for $k > i$

Remark. The extended Gini family of social welfare functions has the following properties.

(i) W_{G_i} preserves upward inverse stochastic dominance of degree$< i$

(ii) W_{G_i} obeys the Pigou-Dalton principle of transfers

(iii) W_{G_i} obeys the principles of DPTS up to and including $(i - 2)$th-degree for $i = 3, 4, \ldots$.

(iv) The sequence $\{W_{G_i}\}$approaches μ_F as $i \to 2$

(v) The sequence $\{W_{G_i}\}$ approaches the Rawlsian maximin criterion as $i \to \infty$.

Proposition

Let F_1 and F_0 be members of F. Then $i = 3, 4..$

(i) F_1 ith degree downward inverse stochastic dominates F_0 implies

(ii) $W_{D_k}(F_1) > W_{D_k}(F_0)$ for $k > i$

Remark. The extended Lorenz family of social welfare functions has the following properties,
(i) W_{D_i} preserves downward inverse stochastic dominance of degree$< i$
(ii) W_{D_i} obeys the Pigou-Dalton principle of transfers.
(iii) W_{D_i} obeys the principles of UPTS up to and including $(i-2)$th-degree.
(iv) The sequence $\{W_{D_i}\}$ approaches μ_F as $i \to \infty$
(v) The sequence $\{i(W_{D_i} - \mu_F\}$ approaches $\mu_F - F^{-1}(1)dt$ as $i \to \infty$

Weights at quantiles relative to median

Quantile:	*0+*	*.05*	*.30*	*.70*	*.95*	*1−*
Panel (a): Gini social welfare function (upward)						
$i \to 2$	1.00	1.00	1.00	1.00	1.00	1.00
$i = 3$	2.00	1.90	1.40	0.60	0.10	0+
$i = 4$	4.00	3.61	1.96	0.36	0.01	0+
$i = 5$	8.00	6.86	2.74	0.22	0.00	0+
$i = 6$	16.00	13.03	3.84	0.13	0.00	0+
$i \to \infty$	∞	0	0	0	0	0
Panel (b): Lorenz social welfare function (downward)						
$i = 3$	2.00	1.90	1.40	0.60	0.10	0+
$i = 4$	1.33	1.33	1.21	0.68	0.13	0+
$i = 5$	1.14	1.14	1.11	0.75	0.16	0+
$i = 6$	1.07	1.07	1.06	0.81	0.20	0+
$i \to \infty$	1	1	1	1	1	0+

1) Since F_n is a consistent estimator of F

- $\Lambda^i_{F_n}(u)$ and $\tilde{\Lambda}^i_{F_n}(u)$ are consistent estimators of $\Lambda^i_F(u)$ and $\tilde{\Lambda}^i_F(u)$

2) The asymptotic properties of $\Lambda^i_{F_n}(u)$ and $\tilde{\Lambda}^i_{F_n}(u)$ can be obtained by

- considering the limiting distribution of the empirical processes

$$Y^i_n(u) = \sqrt{n}\left[\Lambda^i_{F_n}(u) - \Lambda^i_F(u)\right]$$

$$\tilde{Y}^i_n(u) = \sqrt{n}\left[\tilde{\Lambda}^i_{F_n}(u) - \tilde{\Lambda}^i_F(u)\right]$$

We can then show that $\tilde{Y}^i_n(u)$ and $Y^i_n(u)$

- converge to a Gaussian process and thus are asymptotically normally distributed

Theorem

Let $W_0(t)$ denote a Brownian bridge on $[0,1]$. Suppose that F has a continuous nonzero derivative f on $[a,b]$. Then $Y_n^i(u)$

converges in distribution to the processes

$$Y^i(u) = \frac{1}{(i-2)!}\int_0^u (u-t)^{i-2}\frac{W_0(t)}{f(F^{-1}(t))}dt$$

which has the same probability distribution as the Gaussian process $\sum_{j=1}^{\infty} h_j(u) Z_j$, where $h_j(u)$ is given by

$$h_j(u) = \frac{1}{(i-2)!}\left[\frac{\sqrt{2}}{j\pi}\int_0^u (u-t)^{i-2}\frac{\sin(j\pi t)}{f(F^{-1}(t))}dt\right]$$

and $Z_1, Z_2, \ldots$ are independent $N(0,1)$-variables.

Theorem

Let $W_0(t)$ denote a Brownian bridge on $[0,1]$. Suppose that F has a continuous nonzero derivative f on $[a,b]$. Then $\tilde{Y}_n^i(u)$ converges in distribution to the processes

$$\tilde{Y}^i(u) = \frac{1}{(i-2)!}\left[(1-u)^{i-2}\int_0^1 \frac{W_0(t)}{f(F^{-1}(t))}dt - \int_u^1 (t-u)^{i-2}\frac{W_0(t)}{f(F^{-1}(t))}dt\right]$$

which has the same probability distribution as the Gaussian process $\sum_{j=1}^{\infty}\tilde{h}_j(u)Z_j$, where $\tilde{h}_j(u)$ is given by

$$\tilde{h}_j(u) = \frac{1}{(i-2)!}\frac{\sqrt{2}}{j\pi}\left[(1-u)^{i-2}\int_0^1 \frac{\sin(j\pi t)}{f(F^{-1}(t))}dt - \int_u^1 (t-u)^{i-2}\frac{\sin(j\pi t)}{f(F^{-1}(t))}dt\right]$$

and $Z_1, Z_2, \ldots$ are independent $N(0,1)$-variables.

Application: Jobs First field experiment

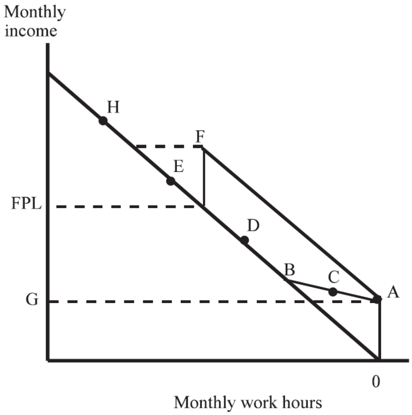
We apply our framework to the Jobs First program Apr 96–Dec 00, analyzed in Bitler et al. (2005, AER)

- Random assignment to Jobs First or AFDC
- Two counties in Connecticut: New Haven and Manchester
- Sample of about 4803 welfare recipients

Key features of Job First program:

- Expanded earnings disregard
- Introduced 21 month time limit

Application: Jobs First – Budget constraint



AB = AFDC AF = Jobs First

Application: Jobs First – Estimation

We use QTE-estimates from Bitler et al. (2008).

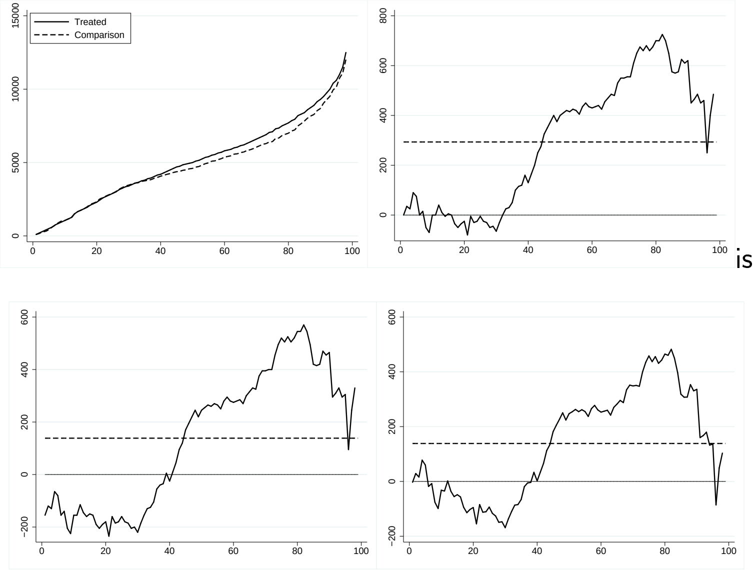
- Compares the quantiles of the treatment and control distribution: $\Delta_q = F_1^{-1}(q) - F_0^{-1}(q)$

Outcomes: Total income

Financial costs:

- Job First: Higher cash transfers, admin costs, and operating costs
  - Assess gains and losses with\without balanced budget

QTE: Averaged income q1–q16



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Dom. and soc welfare: Averaged income q1–q16

	Upward dominance			Downward dominance		
	No tax	*Lump sum*	*Prop. Tax*	*No tax*	*Lump sum*	*Prop. Tax*
Inverse stochastic dominance						
Dom.	4	4	231	3	3	3
Distr.	1	0	0	1	1	1
ΔW_p	8.8%	-6.2%	N/A	10.9%	0.6%	3.9%
$W(F_0)$	$341	$341	N/A	$742	$742	$742
Social welfare weights relative to median						
p(.05)	3.61	3.61	7E+63	1.90	1.90	1.90
p(.30)	1.96	1.96	3E+33	1.40	1.40	1.40
p(.70)	0.36	0.36	2E-51	0.60	0.60	0.60
p(.95)	0.01	0.01	1E-229	0.10	0.10	0.10

Conclusion

We characterize the relationship between dominance criteria and two nested subfamilies of least restrictive social welfare functions

- higher-order UID = stronger downside inequality aversion
- higher-order DID = stronger upside inequality aversion
    - Useful to unambiguously say whether F_1 is better than F_0

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We then derive parametric subfamilies of these social welfare functions that are easily implementable

- UID $\Rightarrow$ Gini family, W_{G_i}
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We illustrate the usefulness of the framework by applying to an experimental policy intervention