# Social Choice with Risk and Time

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# Outline

Johnsen and Donaldson (1985): "The Structure of Intertemporal Preferences under Uncertainty and Time Consistent Plans", *Econometrica*

Political decisions typically have consequences for both present and future generations (e.g., public investments, tax policy), and the future is uncertain

- How should a benevolent and rational policy maker decide in an intertemporal and uncertain context ?
- What do actually decide policy makers?

### Classical (normative) Macro (Barro, Lucas-Stokey ...)

- Intertemporal economy: $\{c_t, x_t\}_{t\geq 0}$
- Objective of a unique and benevolent social planner: $\max E_0 V_0(c, x)$

### Alesina and Tabellini, 1990

- two policy makers, with different objectives, alternate in office.
- Political uncertainty

$\Rightarrow$ stock of public debt larger than it is socially optimal

## Framework

- Time: $1, \ldots, T$
- One (representative) individual appears at each period
- Each individual faces a risky future (unknown date of death)

## Textbook: Benevolent Social Planner

- Individual $t$'s utility in $t$: $V_t^t(c_t, \ldots, c_T)$
- Social Welfare: $W(V_1^1, \ldots, V_T^T)$

## Who decides?

- At time $t$, a set $N_t$ of individuals alive
- Utilities: $V_t^\tau(c_t^\tau, \ldots, c_T^\tau)$
- Social Welfare at time $t$: $W_t((V_t^\tau)_{\tau \in N_t})$

## Key issue

- As time goes, some people die, and some other are born
- Successive decision makers have different objectives, because they care about different populations

## Question

- Under what conditions can decisions made by a rational and benevolent social planner be implemented by successive social planners?
- Can we find $W$, $(W_t)_t$, $(V_t^\tau)_{t,\tau}$ such that:

$$W(V_1^1, \ldots, V_T^T) = \Phi(W_1((V_1^\tau)_{\tau \in N_1}), \ldots, W_T((V_T^\tau)_{\tau \in N_T}))?$$

# Outline

Two periods, $t \in \{1, 2\}$

One good: $K$ interval of $\mathbb{R}$

Two individuals

Individual $a$:

- born in period 1
- consumes $x$ in period 1
- has a probability $p$ to be alive in period 2
- if alive, consumes $y$ in period 2
- thus faces a prospect $(x, y_a, p) \in \mathscr{L}$ in period 1

Individual $b$ lives in period 2, and consumes $y_b$

## Assumption 1: Ex ante individual preferences

$a$'s preferences $\succsim_1$ in period 1, complete and continuous on $\mathscr{L}$.

(*i*) $x \geq x'$, $y \geq y'$ and $p \geq p' \Rightarrow (x, y, p) \succsim_1 (x', y', p')$ (resp., $>, \succ_1$)

(*ii*)

$$[(x' < x) \,\&\, (x', y', p) \sim_1 (x, y, p)]$$
$$\Rightarrow (x', y', p') \succ_1 (x, y, p'),\ \forall p' > p$$

## Consequence

- continuous function $u_1 : \mathscr{L} \to \mathbb{R}$ represents $\succsim_1$
- $u_1$ is strictly increasing in $p$ and $x$, and strictly increasing in $y$ whenever $p > 0$

## Assumption 2: Individual ex post preferences

- $a$ (if alive) and $b$'s preferences $\succsim_2$ in period 2 on $K$
- $y \succsim_2 y' \Leftrightarrow y \geq y'$

## Main assumptions

The social planner

- Only cares about people actually alive
- Is paretian with respect to *a*'s preferences

## Assumption 3 (Social Planner 1 preferences)

- Complete and continuous preferences $\tilde{\succcurlyeq}_1$ on $\mathscr{L}$
- $\tilde{\succcurlyeq}_1 = \succcurlyeq_1$

## Consequence

- $\tilde{\succcurlyeq}_1$ can be represented by a continuous function

$$V_1 : \mathscr{L} \to \mathbb{R}$$

- There exists $h$ cont. and strict. increasing: $V_1 = h \circ u_1$

## Main assumption

Only cares about individual who are actually alive: dead do not count

## Assumption 4 (Social Planner 2 preferences, one individual)

- $\succsim_2^1$ on $K$
- $y \succsim_2^1 y' \Leftrightarrow y \geq y'$

## Consequence

$\succsim_2^1$ represented by a continuous and strictly increasing function

$$V_2^1 : K \to \mathbb{R}$$

## Assumption 5 (Social Planner 2 preferences, 2 individuals)

- $\succsim_2^2$ continuous and complete on $K^3$
- $x \geq x'$, $y_a \geq y'_a$ and $y_b \geq y'_b \Rightarrow (x, y_a, y_b) \succsim_2^2 (x', y'_a, y'_b)$
- If, moreover, $y_a > y'_a$ or $y_b > y'_b$, then $(x, y_a, y_b) \succ_2^2 (x', y'_a, y'_b)$

## Consequence

There exists a continuous function

$$V_2^2 : K^3 \to \mathbb{R}$$

non decreasing in its first argument and strictly increasing in its two last arguments, that represents $\succsim_2^2$

# Outline

## Benevolent Social Planner

Preferences $\succcurlyeq^*$ complete and continuous over $\mathscr{L} \times K$

## Axiom (Non Paternalism)

For all $((x, y_a, p), y_b), ((x', y'_a, p'), y'_b) \in \mathscr{L} \times K$,

$$\left.\begin{array}{l}(x, y_a, p) \succcurlyeq_1 (x', y'_a, p') \\ y_b \geq y'_b\end{array}\right\} \Rightarrow ((x, y_a, p), y_b) \succcurlyeq^* ((x', y'_a, p'), y'_b).$$

If a LHS inequality strict: $((x, y_a, p), y_b) \succ^* ((x', y'_a, p'), y'_b)$.

## Consequence

$\succcurlyeq^*$ can be represented by a continuous and strictly increasing

$$W(u_1(x, y_a, p), y_b)$$

## Question

Can we find $(W, u_1, V_1, V_2^1, V_2^2)$ such that decisions made by a rational and benevolent social planner $(W)$ can be implemented by successive social planners $(V_1, V_2^1, V_2^2)$?

## Definition: Aggregated welfare

An aggregated welfare function is a continuous function

$$V(V_1(x, y_a, p), V_2^1(y_b), V_2^2(x, y_a, y_b), p)$$

strictly increasing in $V_1$, $V_2^1$ and $p$, strictly increasing in $V_2^2$ if $p > 0$, and constant in $V_2^2$ if $p = 0$.

## Proposition

Assume Assumptions 1 to 5 hold. Then $\succcurlyeq^*$ cannot simultaneously be non-paternalistic and be represented by an aggregated welfare function.