Inequality at the Top of the Distribution: Affluence in Income and Wealth

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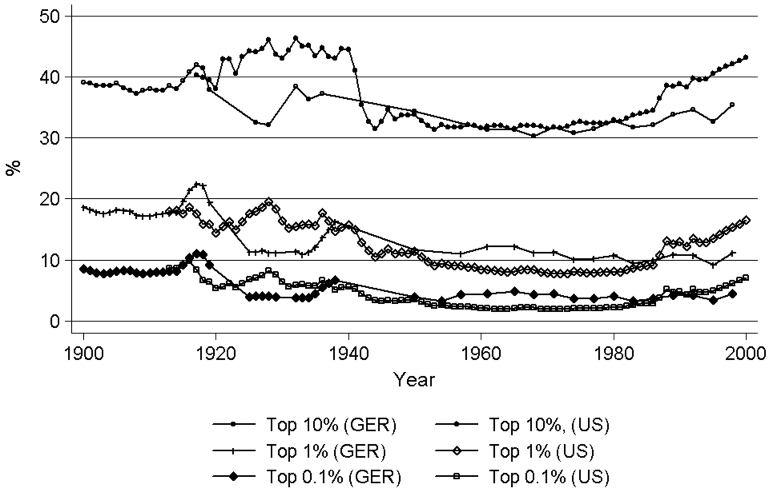

Figure: Alvaredo et al. (2011): “The World Top Incomes Database”

- Increasing inequality (and awareness of it) around the world
- Growing interest in top of income distribution:
  Piketty (2001/3/5); Piketty/Saez (2006); Atkinson/Piketty (2007,2010); **Atkinson/Piketty/Saez (2011)**; Aaberge/Atkinson (2010); Roine/Waldenström (2008); Jäntti et al. (2010); **Peichl/Schaefer/Scheicher (2010)**

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- design of public policies

Outline

2. Measuring Richness

- Outcome distribution $x = (x_1, x_2, \ldots, x_n) \in R^n_+$,
  π : poverty line (eg. 60% of median income),
  $p = \#\{i | x_i < \pi, \ i = 1, 2, \ldots, n\}$ number of poor people

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- Foster-Greer-Thorbecke (1984):

$$\varphi_{FGT}(x) = \frac{1}{n} \sum_{i=1}^{n} \left(\left(\frac{\pi - x_i}{\pi} \right)_+ \right)^{\alpha},$$

$(\alpha > 0$ und $y_+ := \max\{y, 0\}$.)

- ρ richness line, $r = \#\{i | x_i > \rho,\ i = 1, 2, \ldots, n\}$ number rich people.
- Headcount ratio (HCR):

$$R_{HC}(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{x_i > \rho} = \frac{r}{n}\,.$$

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- Income shares of the top $p\%$ (TIS) of the income distribution (Atkinson/Piketty/Saez):

$$IS_p(\mathbf{x}) = \frac{\sum_{i=1}^{n} x_i \mathbf{1}_{x_i > q_{1-p}}}{\sum_{i=1}^{n} x_i}$$

with q_p being the $(1-p)\%$ quantile.

- Advantage: simple descriptive stats, no normative choices
- Problems:
  - HCR only concerned with number of individuals above fixed cutoff level without taking income variation into account
  - TIS do not account for changes in the composition of the population nor changes in the distribution of income among the top

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- Problems:
  - HCR only concerned with number of individuals above fixed cutoff level without taking income variation into account
  - TIS do not account for changes in the composition of the population nor changes in the distribution of income among the top
- Solution 1: compute HCR using different richness lines and different TIS to capture some information about distribution

- Solution 2: simultaneously account for composition and distribution with same measure (cf. poverty measurement, e.g.: FGT).
- Medeiros (2006) defines (absolute) affluence gap by

$$R^{Med}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \rho)_+ = \frac{1}{n}\sum_{i=1}^{n}\max\{x_i - \rho, 0\}\,. \tag{1}$$

  - Advantage: increasing in income.
  - But: absolute measure that is proportional to income, i.e. transfer between two rich individuals will not change index.

- Peichl, Schaefer & Scheicher (2006,2010): class of richness measures that take into account the number of rich people as well as the intensity (distribution and amount) of richness:

$$R(\mathbf{x},\rho) = \frac{1}{n}\sum_{i=1}^{n} f\left(\frac{x_i}{\rho}\right) ,$$

where f is continuous, strictly increasing function measuring the individual contribution to overall richness

- This weighting function shall have some desirable properties which are derived following the literature on axioms for poverty indices
- Transfer axiom: concave or convex

3. Examples

	1	2	3	4	5	6	7	8	9	10
w	1	1	1	1	1	1	1	1	1	1
y1	5	5	5	5	5	5	5	5	5	55
x1	4	4	4	4	4	4	4	4	4	64
y2	5	5	5	5	5	5	5	5	11	49
y3	5	5	5	5	5	5	5	5	30	30

	1	2	3	4	5	6	7	8	9	10
w	1	1	1	1	1	1	1	1	1	1
y1	5	5	5	5	5	5	5	5	5	55
x1	4	4	4	4	4	4	4	4	4	64
y2	5	5	5	5	5	5	5	5	11	49
y3	5	5	5	5	5	5	5	5	30	30

	RL	HCR	Concave	Convex	Absolute	T10
y1	10	0.100	0.082	2.025	4.500	0.550
x1	10	0.100	0.084	2.916	5.400	0.640
y2	10	0.200	0.089	1.522	4.000	0.490
y3	10	0.200	0.133	0.800	4.000	0.300

	1	2	3	4	5	6	7	8	9	10	11
w	9.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
y1	5.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
x1	4.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0
x2	4.0	55.0	57.0	59.0	61.0	63.0	65.0	67.0	69.0	71.0	73.0
y4	5.0	46.0	48.0	50.0	52.0	54.0	56.0	58.0	60.0	62.0	64.0

	1	2	3	4	5	6	7	8	9	10	11
w	9.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
y1	5.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
x1	4.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0
x2	4.0	55.0	57.0	59.0	61.0	63.0	65.0	67.0	69.0	71.0	73.0
y4	5.0	46.0	48.0	50.0	52.0	54.0	56.0	58.0	60.0	62.0	64.0

	RL	HCR	Concave	Convex	Absolute	T10	T01
y1	10	0.100	0.082	2.025	4.500	0.550	0.055
x1	10	0.100	0.084	2.916	5.400	0.640	0.064
x2	10	0.100	0.084	2.949	5.400	0.640	0.073
y4	10	0.100	0.082	2.058	4.500	0.550	0.064

	1	2	3	4	5	6	7	8	9	10	11
w	9.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
y1	5.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
x1	4.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0
x2	4.0	55.0	57.0	59.0	61.0	63.0	65.0	67.0	69.0	71.0	73.0
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	1	2	3	4	5	6	7	8	9	10	11
w	9.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
y1	5.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
x1	4.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0	64.0
x2	4.0	55.0	57.0	59.0	61.0	63.0	65.0	67.0	69.0	71.0	73.0
y4	5.0	46.0	48.0	50.0	52.0	54.0	56.0	58.0	60.0	62.0	64.0

	RL	HCR	Concave	Convex	Absolute	T10	T01
y1	50	0.100	0.009	0.001	0.500	0.550	0.055
x1	50	0.100	0.022	0.008	1.400	0.640	0.064
x2	50	0.100	0.021	0.009	1.400	0.640	0.073
y4	50	0.070	0.009	0.002	0.560	0.550	0.064

4. Empirical Application

	Tax return data	Survey data
Samples	large	small
Representativeness	taxpayers	whole population (less for top 1%)
Income	taxable Y	gross & net
Socio-demographics	little	detailed
Problems	avoidance & evasion	measurement error
	varying definitions (income, tax unit)	survey / sampling methods

- Pareto distribution for income y:
  density: $f(y) = \alpha \frac{k^{\alpha}}{y^{(1+\alpha)}}$, $(k > 0, \alpha > 1)$
  α: Pareto parameter; k scale parameter
  $\beta = \frac{\alpha}{(\alpha-1)}$: inverted Pareto parameter;
  lower α (higher β): more inequality [fatter upper tail]

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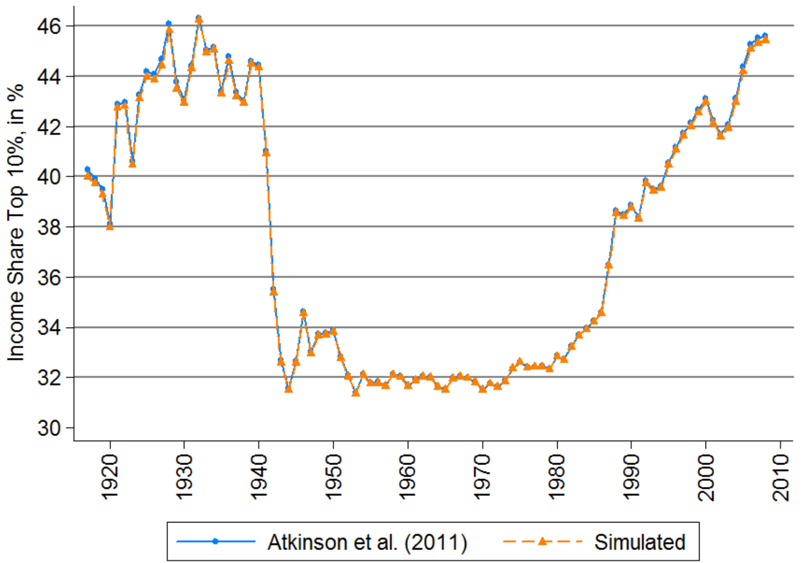
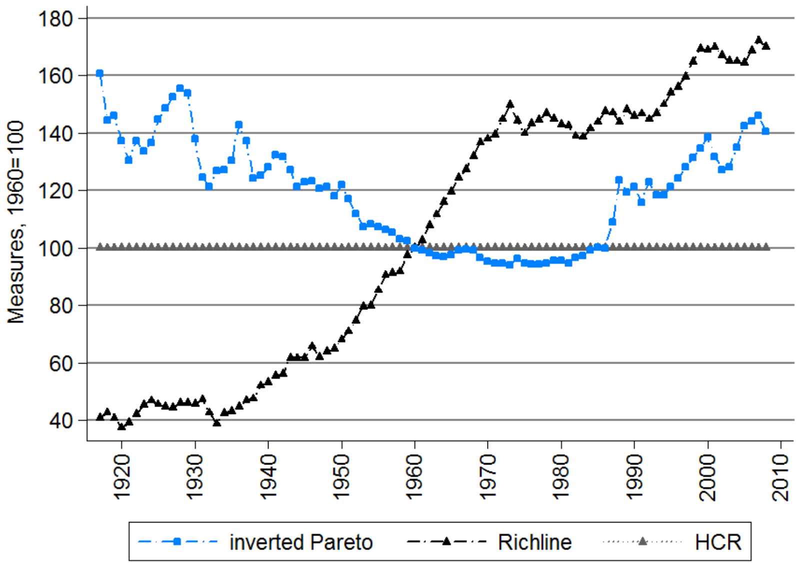
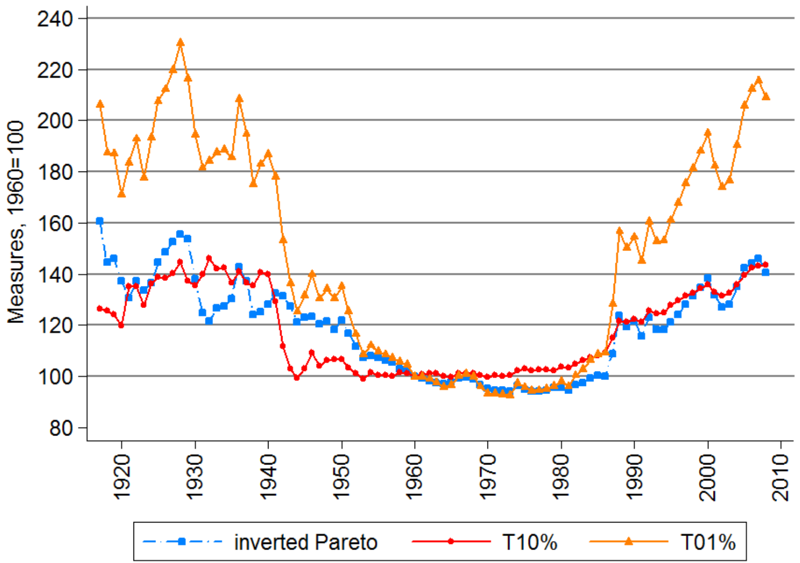
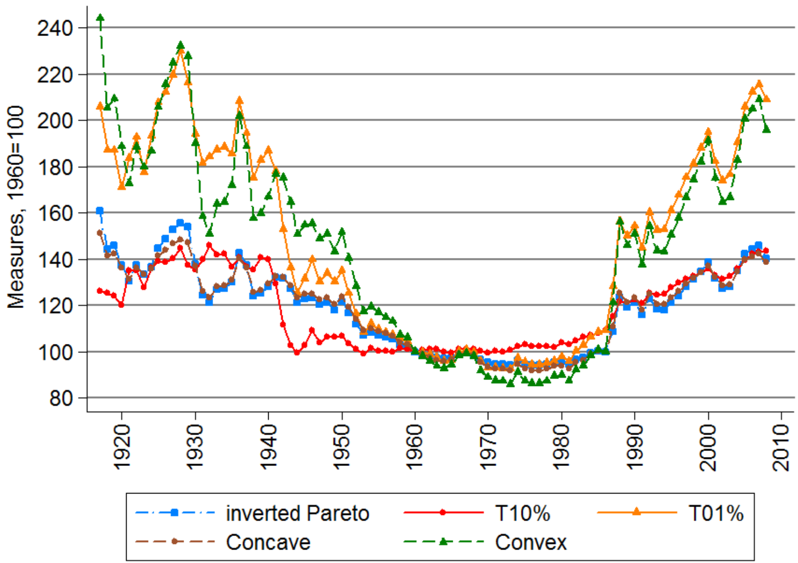
  lower α (higher β): more inequality [fatter upper tail]
- "The World Top Incomes Database": income shares and averages
- α / β and k can be computed from this data
- Assumption: upper tail follows Pareto law (Atkinson/Piketty/Saez)
- Simulate (top) income distribution for each country-year in database
- compute and compare various affluence measures / trends
  - richness line: P90 (T10) threshold ($HCR = 10\%$)


Income Share Top 10%, in %
46
44
42
40
38
36
34
32
30
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
Atkinson et al. (2011)
Simulated

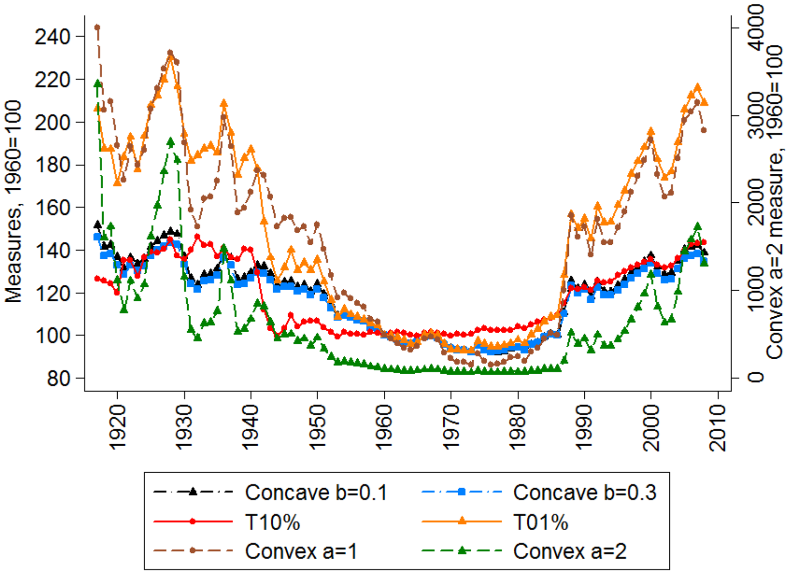


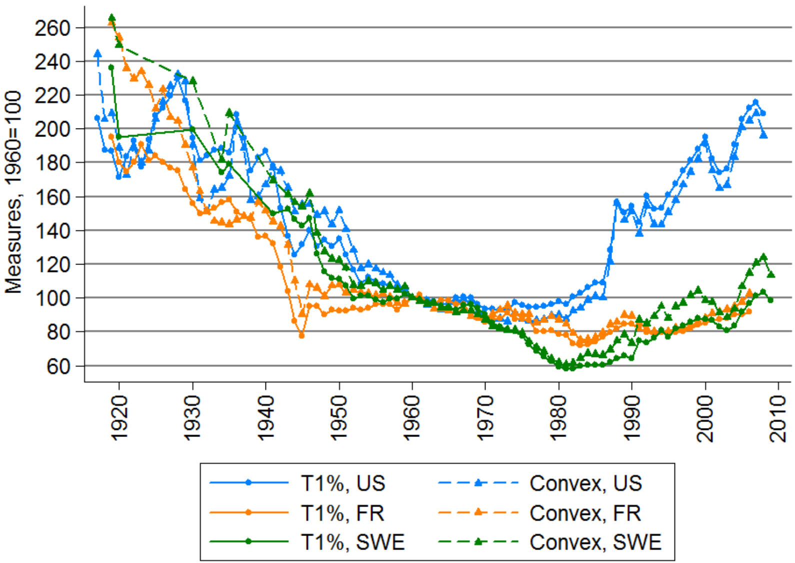
Measures, 1960=100
180
160
140
120
100
80
60
40
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
inverted Pareto
Richline
HCR


Measures, 1960=100
240
220
200
180
160
140
120
100
80
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
inverted Pareto
T10%
T01%



Measures, 1960=100
240
220
200
180
160
140
120
100
80
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
inverted Pareto
T10%
T01%
Concave
Convex


Measures, 1960=100
240
220
200
180
160
140
120
100
80
Convex a=2 measure, 1960=100
4000
3000
2000
1000
0
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
Concave b=0.1
Concave b=0.3
T10%
T01%
Convex a=1
Convex a=2


Measures, 1960=100
260
240
220
200
180
160
140
120
100
80
60
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
T1%, US
T1%, FR
T1%, SWE
Convex, US
Convex, FR
Convex, SWE

correlations all countries

	Concave1	Concave2	Convex1	Convex2	Absolute	T10	T01
Concave1	1.000						
Concave2	0.980	1.000					
Convex1	0.966	0.906	1.000				
Convex2	0.461	0.394	0.627	1.000			
Absolute	-0.020	-0.030	0.015	0.083	1.000		
T10	0.793	0.801	0.741	0.312	-0.101	1.000	
T01	0.948	0.904	0.950	0.506	-0.023	0.905	1.000

5. Multidimensional Affluence

Peichl, A. and N. Pestel (2011): Multidimensional Affluence: Theory and Applications to Germany and the US, IZA Discussion Paper No. 5926.

- Peichl / Pestel (2011): extend affluence measures (Peichl et al. 2010) to the multidimensional case following Alkire/Foster (2011)
- incorporate wealth as dimension of multidimensional affluence
- empirical application to Germany and the US

Dual cutoff method

- *so far:* affluence w.r.t. single dimensions separately (*1st cutoff*)
- *now:* individual (multidimensionally) affluent if affluence counts at least at certain threshold (*2nd cutoff*)

Measures:

- dimension adjusted "headcount ratio"
- dimension adjusted multidimensional richness measures

- German Socio-Economic Panel Study (SOEP)
- Survey of Consumer Finances 2007 (SCF)
- Income
  - market income from all sources and household members
  - substract asset income (*interest, dividends, gains etc.*)
- Wealth
  - household net worth (assets - debt)
- Cutoffs
  - distinguish affluent person from a non-poor but non-affluent
  - 80%-quantile of age group (*head aged* $<$*30, 30–59, 60*$+$)
- Adjustments
  - equivalence weighting $\rightarrow$ square root scale
  - currency $\rightarrow$ values expressed in 2007 PPP $US

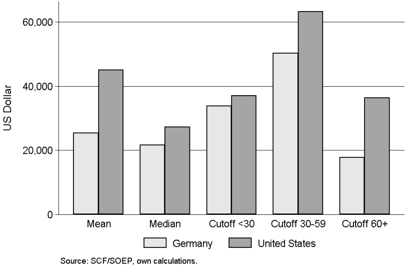


Figure: Income

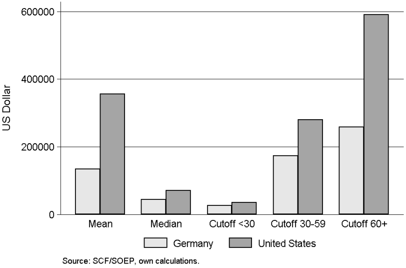
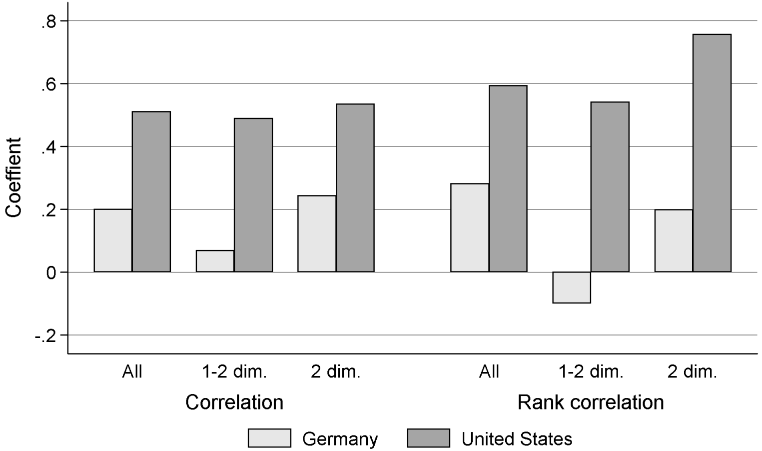


Figure: Wealth



Source: SCF/SOEP, own calculations.
All: affluent and non-affluent. 1-2 dim.: affluent in at least one dimension. 2 dim.: affluent in both dimensions.

Figure: Correlations between income and wealth

k	R^M_{HR}	$R^M_{\alpha=1}$	$R^M_{\alpha=2}$	$R^M_{\beta=1}$	$R^M_{\beta=3}$
		United States 2007			
1	0.199	0.133	9.143	0.020	0.030
2	0.111	0.103	8.446	0.012	0.016
		Germany 2007			
1	0.200	0.104	0.997	0.030	0.049
2	0.081	0.051	0.457	0.013	0.020

Note: k denotes the second cutoff threshold. Source: SCF/SOEP, own calculations.

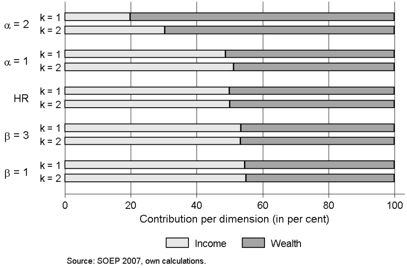


Figure: Germany

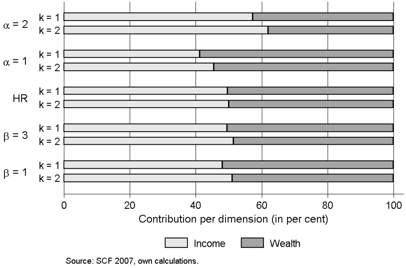


Figure: US

Robustness

- survey data vs. administrative tax data (for Germany)
- different cutoff thresholds (larger quantiles, % of median)

Discussion

- data requirements (availability of *all* dimensions)
- pension wealth and further dimensions

Summing up

- propose multidimensional affluence measures (convex and concave)
- conclusions from GE-US comparison depends on (normative) view
- importance of dimensions at the top different

6. Conclusion

- Increasing inequality at top since 1970s
- Top income shares: simple descriptive stats; but powerful
- Different (normative) measurement choices can lead to slightly different conclusions
- Correlation between measures relatively high
- Multidimensional measurement allows taking into account correlation between dimensions

Inequality at the top related to

- Macroeconomics: (big) recessions; financial crisis, inflation, war
- Roine / Vlachos / Waldenström (2009): e.g. financial development
- Executive remuneration: tournament / superstar theories, bargaining
- Progressive taxation: elasticity of income w.r.t. net-of-tax rate (Saez / Slemrod / Giertz, 2011): supply side, income shifting and bargaining
- Political Economy: partisanship?
- Globalization, (skill-biased) technol. change (how relevant at top?)

- Long-run elasticity of top incomes w.r.t net-of-tax rate appears to be relatively large, i.e. $e = e_1 + e_2 + e_3 \approx 0.5$
- optimal tax formula (Piketty/Saez/Stantcheva, 2011): $\tau^* = \frac{1-g+tae_2+ae_3}{1-g+a(e_1+e_2+e_3)}$
- Pareto coefficient $a = 1.5$; alternative tax rate $t = 20\%$
- Scenarios (current US top tax rate: 42.5%):
  - Pure labor supply ($e_1 = 0.5; e_2 = e_3 = 0$): $\tau^* = 57\%$
  - Tax avoidance ($e_1 = 0.2; e_2 = 0.3; e_3 = 0$): $\tau^* = 62\%$
  - TA after base broadening ($e_1 = 0.2; e_2 = 0.1; e_3 = 0$): $\tau^* = 71\%$
  - Compensation bargaining ($e_1 = 0.2; e_2 = 0; e_3 = 0.3$): $\tau^* = 83\%$

Thank you for your attention!

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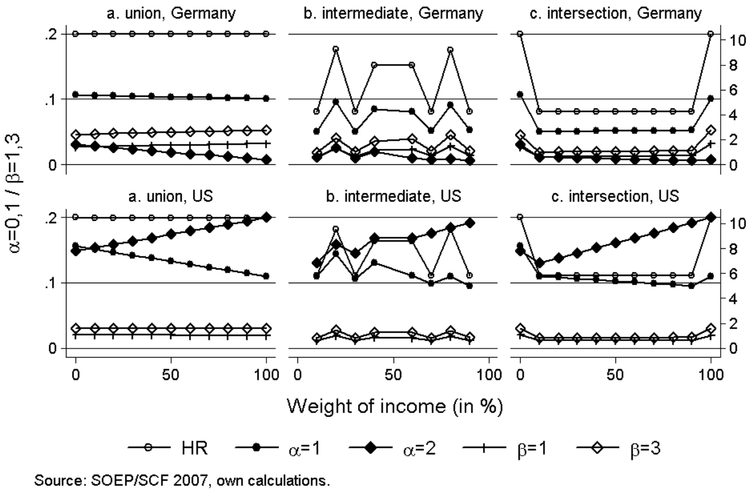
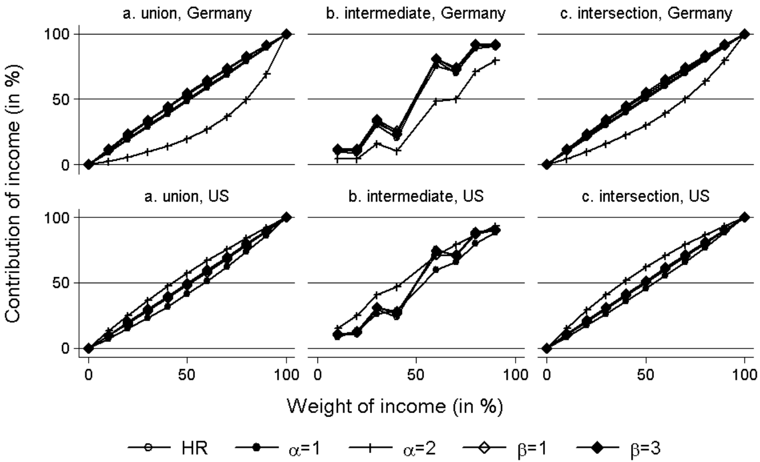


Figure: Multidimensional affluence for different weights



Source: SOEP/SCF 2007, own calculations.

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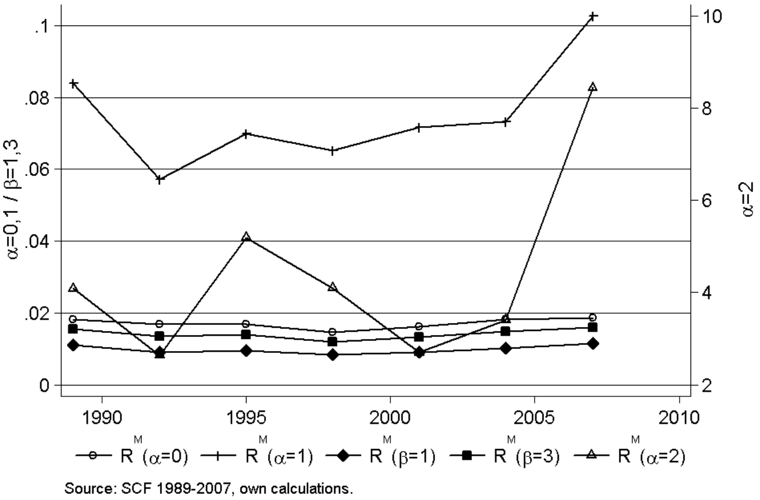


Figure: Multidimensional affluence (United States, 1989–2007)

- *Focus axiom:* a richness index shall be independent of the incomes of the non-rich.
- *Continuity axiom:* the index shall be a continuous function of incomes, i.e. small changes in the income structure shall not lead to discontinuously large changes in the richness index.
- *Monotonicity axiom:* a richness index shall increase if c.p. the income of a rich person increases.
- *Subgroup decomposability axiom:* the overall degree of richness may be decomposed into the (population) weighted sum of subgroup richness indices.

Transfer axiom in poverty: index shall decrease with rank-preserving progressive transfer from a poor person to someone who is poorer.

$\Rightarrow$ Translation to richness?:

- *Transfer axiom T1 (concave):* richness index shall **increase** with rank-preserving progressive transfer between two rich persons.
- *Transfer axiom T2 (convex):* richness index shall **decrease** with rank-preserving progressive transfer between two rich persons.

Question behind these two opposite axioms: shall richness index increase if

- (i) a billionaire gives an amount x to a millionaire,
- (ii) the millionaire gives the same amount x to the billionaire.

Concave:

- FGT index satisfying T1:

$$R_{\alpha}^{FGT,T1}(\mathbf{x},\rho) = \frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{x_i-\rho}{x_i}\right)_+\right)^{\alpha}, \quad \alpha \in (0,1).$$

- index analogous to the poverty index of Chakravarty (1983):

$$R_{\beta}^{Cha}(\mathbf{x},\rho) = \frac{1}{n}\sum_{i=1}^{n}\left(1-\left(\frac{\rho}{x_i}\right)^{\beta}\right)_+, \beta > 0.$$

Convex:

- FGT index satisfying T2:

$$R_{\alpha}^{FGT,T2}(\mathbf{x},\rho) = \frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{x_i-\rho}{\rho}\right)_+\right)^{\alpha}, \alpha > 1$$

Consider two populations with income distribution

$$\mathbf{x} = (5, 5, 5, 11, 11) \text{ and } \mathbf{y} = (5, 5, 5, 100, 100) .$$

Let $\rho_{\mathbf{x}}$, $\rho_{\mathbf{y}}$ be 200% of the median income. Then $\rho_{\mathbf{x}} = \rho_{\mathbf{y}} = 10$ and we obtain

$$R^{HC}(\mathbf{x},\rho = 10) = R^{HC}(\mathbf{y},\rho = 10) = 0.400 ,$$

and

$$R^{Cha}_{\beta=1}(\mathbf{x}) = 0.036 \quad \text{and} \quad R^{Cha}_{\beta=1}(\mathbf{y}) = 0.360,$$
$$R^{FGT,T2}_{\alpha=2}(\mathbf{x}) = 0.004 \quad \text{and} \quad R^{FGT,T2}_{\alpha=2}(\mathbf{y},) = 32.4.$$

$$\mathbf{x} = (5, 5, 5, 11, 9989) \text{ and } \mathbf{y} = (5, 5, 5, 1000, 9000)\,,$$

where **y** is obtained from **x** by a progressive transfer of 989 monetary units between the two rich persons. Again we obtain

$$R^{HC}(\mathbf{x}) = R^{HC}(\mathbf{y}) = 0.400\,,$$

but different results for the intensity measures:

$$R^{Cha}_{\beta=1}(\mathbf{x}) = 0.218 \quad \text{and} \quad R^{Cha}_{\beta=1}(\mathbf{y}) = 0.398,$$
$$R^{FGT,T2}_{\alpha=2}(\mathbf{x}) = 19,916,088 \quad \text{and} \quad R^{FGT,T2}_{\alpha=2}(\mathbf{y}) = 16,360,039.$$

Technical reasons:

- possibility to standardize the index (unit interval)
- use of survey data

Normative judgements:

- "equiprobability model for moral value judgments" (Harsanyi, 1977): a concave value function with diminishing marginal utility
- "polarization view", i.e. richness is increasing when the homogeneity of the top of the distribution increases
- people are rather envious of a rich dentist living next door but admire superstars gaining several millions
- progressive tax system where the (marginal) tax payment is a concave function of taxable income.

- n individuals, $d \geq 2$ dimensions and matrix $\mathbf{Y} = [\ y_{ij}\ ]_{n \times d}$
- for each dimension j some cutoff value γ_j

$$\theta_{ij}(y_{ij}; \gamma) = \begin{cases} 1 & \text{if } y_{ij} > \gamma_j, \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

- $0 - 1$ matrix of dimension-specific affluence:

$$\mathbf{\Theta^0} = [\ \theta_{ij}\ ]_{n \times d} \tag{3}$$

  - vector of affluence counts $\mathbf{c} = (c_1, \ldots, c_n)'$ with $c_i = \sum_j \theta_{ij}$

- matrix $\mathbf{\Theta^0}$ only provides binary information
- instead: evaluate *intensity* of affluence (Peichl et al. 2010):
  - **convex case**:

$$\mathbf{\Theta}^\alpha = \left[\left(\frac{y_{ij} - \gamma_j}{\gamma_j} \right)_+^\alpha \right]_{n \times d} \quad \text{for } \alpha \geq 1 \tag{4}$$

  - **concave case**:

$$\mathbf{\Theta}^\beta = \left[\left(1 - \left(\frac{\gamma_j}{y_{ij}} \right)^\beta \right)_+ \right]_{n \times d} \quad \text{for } \beta > 0 \tag{5}$$

- for *larger* (*smaller*) values of α (β) more weight on the “very” rich

Dual cutoff method

- **so far:** affluence w.r.t. single dimensions separately (*1st cutoff*)
- **now:** individual (multidimensionally) affluent if affluence counts at least at certain threshold (*2nd cutoff*)
  - see Alkire/Foster (2011)

- identification for integer $k \in \{1, \ldots, d\}$:

$$\phi_i^k(y_i, \gamma) = \begin{cases} 1 & \text{if } c_i \geq k, \\ 0 & \text{if } c_i < k \end{cases} \quad (6)$$

  - number of the affluent: $s = \mid \Phi^k \mid$

- replace affluence counts ($\mathbf{c}$) with zero when $\phi_i^k = 0$ (*focus axiom*):

$$c_i^k = \begin{cases} c_i & \text{if } c_i \geq k, \\ 0 & \text{if } c_i < k \end{cases} \quad (7)$$

  - vector of affluence counts $\mathbf{c^k} = (c_1^k, \ldots, c_n^k)'$ with $c_i^k = c_i \cdot \phi_i^k$

- **dimension adjusted "headcount ratio":**

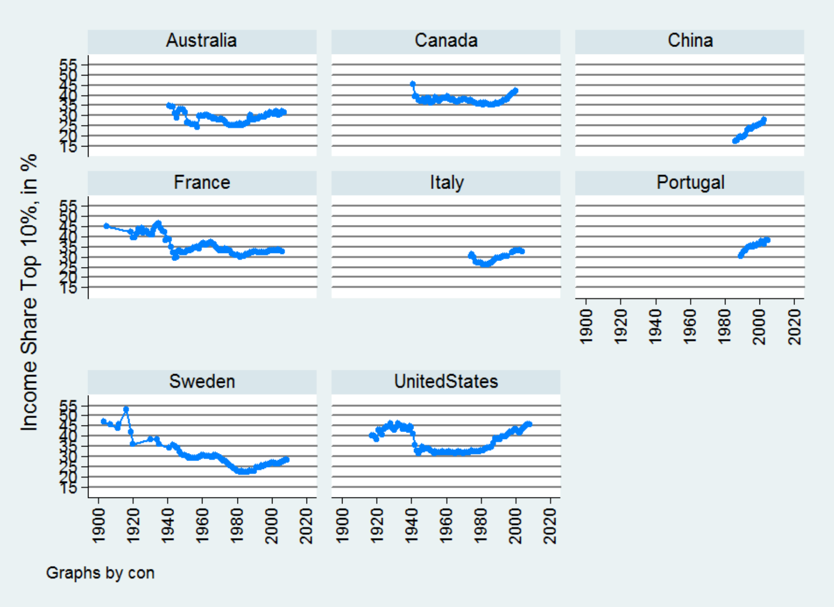
$$R^M_{HR} = \frac{|\mathbf{c^k}|}{n \cdot d} \tag{8}$$

  - satisfies *dimensional monotonicity*, but not *monotonicity*

- **dimension adjusted multidimensional richness measures:**

$$R^M_c = R^M_{HR} \cdot \frac{|\mathbf{\Theta^c(k)}|}{|\mathbf{c^k}|} = \frac{|\mathbf{\Theta^c(k)}|}{n \cdot d} \tag{9}$$

  - for $c = \alpha$ (convex case) and for $c = \beta$ (concave case)
  - measures satisfy *monotonicity*


Australia
Canada
China
France
Italy
Portugal
Sweden
UnitedStates
Income Share Top 10%, in %
15
20
25
30
35
40
45
50
55
1900
1920
1940
1960
1980
2000
2020
Graphs by con

correlations US

	Concave1	Concave2	Convex1	Convex2	Absolute	T10	T01
Concave1	1.000						
Concave2	0.999	1.000					
Convex1	0.992	0.986	1.000				
Convex2	0.849	0.828	0.906	1.000			
Absolute	0.212	0.210	0.203	0.124	1.000		
T10	0.830	0.824	0.829	0.697	0.322	1.000	
T01	0.955	0.948	0.960	0.844	0.274	0.952	1.000