Winter School on Inequality and Social Welfare Theory
Alba di Canazei, 13-16 January 2014

Equilibrium policy simulation with discrete models of labour supply*

Ugo Colombino

Department of Economics and Statistics, Turin, Italy

*Colombino, U. (2013) A new equilibrium simulation procedure with discrete choice models, *International Journal of Microsimulation*, 6(3), 25-49

Outline

- Discrete models of labour supply
- The “dummies refinement”
- Standard policy simulation procedure
- Inconsistency with equilibrium conditions
- A structural interpretation of the “dummies refinement”
- A procedure consistent with equilibrium
- An empirical example

Discrete Model of Labour Supply

- The agent chooses among a set of alternatives or "job" types j = 0,…, M.

- $U_i(j,T_{ij}) = V_i(j,T_{ij}) + \varepsilon_{ij}$ = utility attained if a job of type *j* is chosen by individual *i* under tax-transfer regime *T*

- T_{ij} = net income of individual *i* when choosing job of type *j* under tax-transfer regime *T*

- ε_{ij} is i.i.d Type I extreme value

- Given the above assumption on ε_{ij}, the familiar Conditional Logit Model is obtained:

- $P_i(j,T_{ij}) = \dfrac{\exp\left\{V_i(j,T_{ij})\right\}}{\sum_{h=0}^{M}\exp\left\{V_i(h,T_{ij})\right\}}$ = probability that a job type j is chosen

- The model, as specified above, may not fit the data very well, e.g.

  - Part-timers are over-predicted
  - Non participants are under-predicted

The "Dummies Refinement"

Certain types of jobs might differ according to

a) Availability (density)
b) Fixed costs
c) Search costs
d) Systematic utility components not measured by V

For instance, we might define three dichotomous variables (not exhaustive):

$$D_{0j} = I\left[j \text{ is a market job} \right],$$

$$D_{1j} = I\left[j \text{ is a part-time job} \right],$$

$$D_{2j} = I\left[j \text{ is a full-time job} \right].$$

In general, they might also be specific of individual i.

Then we write the choice probability as follows:

$$P_i(j,T_{ij}) = \frac{\exp\left\{V_i(j,T_{ij}) + \gamma_0 D_{0j} + \gamma_1 D_{1j} + \gamma_3 D_{3j}\right\}}{\sum_{h=0}^{M} \exp\left\{V_i(h,T_{ij}) + \gamma_0 D_{0h} + \gamma_1 D_{1h} + \gamma_3 D_{3h}\right\}}$$

The estimated γ-s will account (somehow) for factors (a) – (d) above.

Many papers – with different interpretations – adopt the above procedure e.g. Van Soest (1995), Aaberge, Dagsvik and Strøm (1995), Aaberge, Colombino and Strøm (1999), Kalb (2000), Kornstad and Thoresen (2003), Aaberge and Colombino (2013).

All the papers mention availability (density, numerosity) of jobs.

Standard tax simulation procedure

Let *R* be a new tax-transfer regime (a "reform").

The basic step is simulating behavioural and welfare effects consists of computing the new choices or the new choice probabilities.

The standard method proceeds by replacing *T* with *R* **leaving unchanged the γ-s:**

$$P_i(j,R_{ij}) = \frac{\exp\left\{V_i(j,R_{ij}) + \gamma_0 D_{0j} + \gamma_1 D_{1j} + \gamma_3 D_{3j}\right\}}{\sum_{h=0}^{M} \exp\left\{V_i(h,R_{ij}) + \gamma_0 D_{0h} + \gamma_1 D_{1h} + \gamma_3 D_{3h}\right\}}$$

Inconsistency of the standard tax simulation procedure

The simulation of tax-transfer reforms is typically interpreted a comparative statics exercise, i.e. we compare different *equilibria*.

However, the standard procedure is not consistent with the comparative statics interpretation.

- The γ-s reflect (also) the n. of available jobs.
- Equilibrium requires:
  n. of available jobs = n. of workers
- The reform *R* in general entails a change in the n. of workers.
- It follows that also the γ-s must change.
- In order to specify how they should change, we need a structural interpretation of the γ-s.

A structural interpretation of the “dummies refinement”

- In a series of papers (e.g. Aaberge et al. 1995, Aaberge et al. 1999) a version of a matching model (Dagsvik 1994, 2000) is adopted, where the γ-s have a precise interpretation in terms of the density of jobs and jobs types.

- By allowing for different density or number of jobs of different types, the probability that individual *i* is matched to a job of type j turns out to be:

$$P_i(j,T_{ij}) = \frac{\exp\left\{V_i(j,T_{ij})g_j\right\}}{\sum_{h=0}^{M}\exp\left\{V_i(h,T_{ij})g_h\right\}}$$

where g_x is the number of available jobs of type x.

- A convenient specification of g_h*, for h > 0*:

$$\frac{g_h}{J} = \begin{cases} \alpha e^{\gamma_1} \text{ if type } h \text{ is a part-time job} \\ \alpha e^{\gamma_2} \text{ if type } h \text{ is a full-time job} \\ \alpha \text{ otherwise} \end{cases}$$

where $J = \sum_{h>0} g_h = \text{total number of available market jobs.}$

In general, both J and g_h might be specific of individual i.

Given the above specification, we can get again the “dummies refinement” choice probability:

$$P_i(j;T) = \frac{\exp\left\{V_i(j;T) + \gamma_0 D_{0j} + \gamma_1 D_{1j} + \gamma_3 D_{3j}\right\}}{\sum_{h=0}^{M} \exp\left\{V_i(h;T) + \gamma_0 D_{0h} + \gamma_1 D_{1h} + \gamma_3 D_{3h}\right\}}$$

with the following interpretation of the coefficients γ_ℓ:

$$\gamma_0 = \ln\left(\frac{\alpha J}{g_0}\right), \gamma_1 = \ln\left(\frac{J_1}{n_1 \alpha J}\right), \gamma_2 = \ln\left(\frac{J_2}{n_2 \alpha J}\right)$$

where

J_1 = number of available part-time jobs

J_2 = number of available full-time jobs

n_1 = number of job-types classified as part-time

n_2 = number of job-types classified as full-time

g_0 = number of available non-market “jobs” (non-participation alternatives).

- More generally, both α and g_0 can be thought of *as* normalization constants. If – besides reflecting the density of jobs – we think that γ also reflects other factors such as fixed costs of participation, search costs etc., then we should think of g_0 and α as a constants absorbing the effect of those other factors.

- The structural interpretation leads us to the definition of a simulation procedure that is consistent with equilibrium conditions.

- We assume here the simplest concept of equilibrium,

  n. of available jobs = n. of people willing to work

  and the simplest equilibrium mechanism:

  wage rate adjustment.

Implementing the equilibrium-consistent simulation: an example

- In order to further simplify the illustration we consider a special case of the above model, where we have just one dummy, D_{0j}:

$$P_i(j;T) = \exp\left\{V_i(j,T_{ij}) + \gamma_0 D_{0j}\right\} \Big/ \sum_{h=0}^{M} \exp\left\{V_i(h,T_{ij}) + \gamma_0 D_{0h}\right\}$$

where $\gamma_0 = \ln\left(\alpha J/g_0\right)$. Notice that given the estimated γ_0 and the observed J, we can compute α/g_0.

- The idea is to specify a relationship between the total number of market jobs J and the mean μ of the wage distribution. Therefore we get a relationship between μ and γ_0.
- When simulating a reform R, we must adjust μ, and therefore γ_0, so that the number of available market jobs is equal to the number of individual choosing a market job.

1) Define the jobs-supply (or labour demand) function:

$$J = K\mu^{-\eta}$$

(constant elasticity labour demand). Notice that, given the observed values of J and μ, and a (estimated or imputed) value of η, we can compute K.

Now use the definition of γ_0 together with definition of labour demand to write γ_0 as a function of μ:

$$\gamma_0(\mu) = \ln\left(\frac{\alpha}{g_0} K\mu^{-\eta}\right)$$

2) *Simulation under equilibrium*

We now write the probability that individual *i* is matched to a job of type j under tax-transfer regime *R*, given mean wage μ, as:

$$P_i\left(j,R_{ij}(\mu_R)\right)=\frac{\exp\left\{V_i\left(j,R_{ij}(\mu_R)\right)+\ln\left(\frac{\alpha}{g_0}K\mu_R^{-\eta}\right)D_{0j}\right\}}{\sum_{h=0}^{M}\exp\left\{V_i\left(h,R_{ih}(\mu_R)\right)+\ln\left(\frac{\alpha}{g_0}K\mu_R^{-\eta}\right)D_{0h}\right\}}$$

where $R_{ij}(\mu_R)$ shows that individual *i*'s net income on job j under regime R depends (also) on the mean wage is μ_R.

The probability that individual i is matched to a market job is equal to $\sum_{j>0} P_i\left(j, R_{ij}(\mu_R)\right)$.

The equilibrium simulation of a reform R is performed by iteratively computing the probabilities above and the equilibrium μ_R until the following condition is satisfied,

$$\sum_i \sum_{j>0} P_i\left(j, R_{ij}(\mu_R)\right) = K \mu_R^{-\eta},$$

where the left-hand side is the expected number of individuals holding a market job.

Extensions

- The procedure can be generalized to couples (as in the empirical example illustrated hereafter) and to multiple “dummies”.

- We might adopt more general specifications of the job-supply function.

- We might make J to depend on more moments of the wage distribution.

- The job-supply function might be jointly estimated together with the utility function (better with repeated cross-sections or longitudinal data).

- More sophisticated concepts of equilibrium might be used (e.g. matching: Dagsvik, 2000).

An empirical example

- We illustrate the procedure with a model of labour supply of Italian couples and singles estimated with a EUROMOD dataset.
- We simulate the effects of hypothetical reforms whereby the current income support policies are replaced various version of universal policies.
- The marginal tax rates are iteratively calibrated so that the total net fiscal revenue remains unchanged.
- The complete exercise is illustrated in Colombino (2011)

We present the results obtained with five different procedures:

a) Standard simulation (no equilibrium)
b) Equilibrium simulation with $\eta = 0$
c) Equilibrium simulation with $\eta = 0.5$
d) Equilibrium simulation with $\eta = 1$
e) Equilibrium simulation with $\eta \to \infty$

Computations

The exercise is computer-intensive, since it requires the simulation of the choice probabilities given the reforms, subject to the constraints:

- Constant total net tax revenue
- Equilibrium conditions

We used *Amoeba*, a global optimization routine written for STATA.

Reforms

UBI = Universal Basic Income: Every household receives a transfer alternatively equal to 50%, 75% or 100% of the poverty line

GMI = Guaranteed Minimum Income: Every household receives a transfer that brings household income up to, alternatively, 50%, 75% or 100% of the poverty line – provided household income is below that threshold.

Social Welfare evaluation

Comparable money-metric individual welfare, King (1983).

1) Compute

$$y_{iR} = \ln \sum_{h=0}^{M} \exp\left\{V_i\left(h, R_{ih}(\mu_R)\right) + \gamma_o(\mu_R) D_{0h}\right\}$$ = expected maximum utility attainable by individual *i* under regime *R*.

2) Compute m_{iR} such that

$$\ln \sum_{h=0}^{M} \exp\left\{V_K\left(h, m_{iR}\right) + \gamma_o(\mu_R) D_{0h}\right\} = y_{iR}$$

where K denotes the index of the “reference individual”.

Gini Social Welfare function, Aaberge (2007):

(Average m_R) × (1 – Gini index of the distribution of m_R).

Alternative treatment of equilibrium conditions

Social Welfare Rankings

	N.E.	$\eta=0$	$\eta=0.5$	$\eta=1$	$\eta=\infty$
Pre-Reform	7	7	7	7	3
GMI-50%	4	4	4	4	2
GMI-75%	6	2	5	5	5
GMI-100%	5	1	6	6	7
UBI-50%	2	3	1	1	1
UBI-75%	1	5	2	2	4
UBI-100%	3	6	3	3	6

Alternative treatment of equilibrium conditions

Monthly household net income

	N.E.	η=0	η=0.5	η=1	η=∞
Pre-Reform	2234	2228	2231	2231	2214
GMI-50%	2185	2366	2198	2200	2163
GMI-75%	2176	2421	2189	2192	2134
GMI-100%	2168	2476	2169	2173	2091
UBI-50%	2185	2334	2195	2199	2162
UBI-75%	2173	2244	2180	2186	2127
UBI-100%	2158	1986	2164	2170	2087

Alternative treatment of equilibrium conditions

Top Marginal Tax Rate (%)

	N.E.	η=0	η=0.5	η=1	η=∞
Pre-Reform	43.7	43.7	43.7	43.7	43.7
GMI-50%	45.9	44.9	45.8	45.7	45.9
GMI-75%	48.2	45.7	47.3	47.2	47.7
GMI-100%	51.3	48.2	50.9	51.1	52.1
UBI-50%	50.9	50.0	50.9	50.8	50.9
UBI-75%	55.4	54.7	55.3	55.2	55.5
UBI-100%	59.9	62.0	59.8	59.7	60.3

Alternative treatment of equilibrium conditions

Poverty Rate (%)

	N.E.	η=0	η=0.5	η=1	η=∞
Pre-Reform	4.33	4.33	4.33	4.33	4.42
GMI-50%	2.26	2.95	2.48	2.48	2.44
GMI-75%	0.87	1.32	0.81	0.81	0.72
GMI-100%	0.01	0.26	0.00	0.01	0.00
UBI-50%	0.52	0.62	0.52	0.52	0.46
UBI-75%	0.04	0.06	0.04	0.04	0.02
UBI-100%	0.00	0.00	0.00	0.00	0.00

Comments

- A first point emerges from comparing the no-equilibrium and the ($\eta=\infty$) simulation.

- One might be tempted to interpret the common practice of ignoring equilibrium constraints, while leaving the wage rates unchanged, as being consistent with a perfectly elastic demand scenario.

- The results confirm that this interpretation in general is not appropriate: the simulation performed under the correctly specified scenario with perfectly elastic demand produces a ranking of policies that is very different from the one produced by the no-equilibrium simulation.

- The perfectly inelastic scenario leads to choosing a generous means-tested mechanism (GMI-100) as the best reform, while the elastic and perfectly elastic scenarios favours a less generous unconditional mechanism (UBI-50).

- In general, scenarios with more elastic labour demand seem to favour UBI mechanisms over GMI mechanisms.

- This is so because a more elastic demand allows less constrained choices and more variability in the number of jobs (and workers).

- As an implication, the perverse effects of the poverty trap on labour supply and income, present with GMI but not with UBI, have more space to manifest themselves.

- Moreover, the unconditional transfers of UBI have a negative effect on labour supply and income bur they are much more effective than GMI in reducing the Poverty Ratio.

- With increasing η, less generous policies – including the current one – move up in the ranking.

- This happens because a more elastic labour demand moderates the increase in equilibrium wages, therefore implying higher equilibrium marginal tax rates.

- When η approaches ∞, the alternative between conditional or non-conditional transfers seems to become less important than the generosity of the transfer.

References

Aaberge, R. (2007) Gini’s Nuclear Family, *Journal of Economic Inequality*, 5 (3), 305-322.

Aaberge, R. and U. Colombino (2013) Designing Optimal Taxes with a Microeconometric Model of Household Labour Supply, *Scandinavian Journal of Economics*, 115(2), 449–475.

Aaberge, R., U. Colombino and S. Strøm (1999) Labor Supply in Italy: An Empirical Analysis of Joint Household Decisions, with Taxes and Quantity Constraints, *Journal of Applied Econometrics*, 14 (4), 403-422.

Aaberge, R., J.K. Dagsvik and S. Strøm (1995) Labour Supply Responses and Welfare Effects of Tax Reforms, *Scandinavian Journal of Economics,* 97 (4), 635-659.

Colombino, U. (2011) Designing a universal income support mechanism in Italy. An exploratory tour, Department of Economics and Statistics Cognetti De Martiis, Working Paper No. 12/2011.

Colombino, U. (2013) A new equilibrium simulation procedure with discrete choice models, *International Journal of Microsimulation*, 6(3), 25-49.

Dagsvik, J. (2000) Aggregation in Matching Markets, *International Economic Review*, 41(1), 27-57.

Dagsvik, J. (1994) Discrete and Continuous Choice, Max-Stable Processes, and Independence from Irrelevant Attributes, *Econometrica*, 62(5), 1179-1205.

Kalb, G. (2000) Accounting for Involuntary Unemployment and the Cost of Part-Time Work, Working Paper No. BP-35, Centre of Policy Studies, Monash University, Australia.

King, M. (1983) Welfare Analysis of Tax Reforms Using Household Data, *Journal of Public Economics*, 21, 183-214.

Kornstad, T. and T. Thoresen (2007) A Discrete Choice Model for Labor Supply and Childcare, *Journal of Population Economics*, 20(4), 781-803.

Van Soest, A. (1995) Structural models of family labour supply, *Journal of Human Resources*, 30 (1), 63-88.